The Evolution of Global Absolute Intergenerational Mobility

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Abstract

Chetty et al. (2014*a*) proposed a new measure of absolute intergenerational income mobility: the fraction of children with greater real-terms income than their parents. Chetty et al. (2017) studied this empirically using United States income data. They found that this measure of absolute mobility has decreased over the last four decades. Here we establish a methodology for providing reliable estimates of absolute intergenerational mobility and use it to present new estimates of the evolution of absolute intergenerational mobility in different countries and globally. In particular, we demonstrate that detailed panel data, which are scarce and partially unreliable for many countries, are unnecessary for producing reliable estimates of absolute mobility. We find that globally, children born in the 1980s have a 65%–70% chance of having higher incomes than their parents'. We also find that in several developed countries absolute intergenerational mobility decreased during the second half of the 20th century due to increasing income inequality and decreasing growth rates. However, the sources of this decrease differ. We develop a reduced-form statistical model which quantitatively captures the relationship between absolute mobility, income growth, income inequality and relative intergenerational mobility. In particular, we show that absolute and relative mobility are inversely related.

Keywords: Mobility, inequality, copula modeling
JEL Codes: C23, D31, E24, H0, J62

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1 Introduction

The question whether next generations will be better off than previous ones has been dominating much of the public economic and political debate in the past few decades across the globe. The "growing public perception that intergenerational income mobility [. . .] is declining in the United States" (Chetty et al., 2014*b*, p. 141) and the argument that "people's frustrations [. . . are] rooted in the fear that their kids won't be better off than they were" (Obama, 2013) led scholars to quantify absolute intergenerational mobility – "the likelihood a child will be financially better off than their parent at around the same age." (Halikias and Reeves, 2016)

Intergenerational mobility is typically divided into two classes: relative and absolute. Relative measures gauge children's propensity to occupy a different position in the income distribution than their parents. Absolute measures gauge their propensity to have higher incomes than their parents in real terms. These two classes of mobility also "capture different normative concepts" (Chetty et al., 2014*a*, p. 1560) and "attaching a precise normative significance to 'income mobility' is difficult because of the multidimensionality of this concept" (Fields and Ok, 1999, p. 588). This may create different and possibly contradictory pictures of ostensibly the same phenomenon.

While relative intergenerational income mobility has been studied for decades (Becker and Tomes, 1979; Borjas, 1992; Piketty, 2000; Mazumder, 2005; Aaronson and Mazumder, 2008; Lee and Solon, 2009; Hauser, 2010; Corak, 2013; Chetty et al., 2014*b*; Berman, 2017), investigations of absolute intergenerational income mobility remain "scarce, mainly because of the lack of large, high-quality panel data sets linking children to their parents" (Chetty et al., 2017, p. 398). Chetty et al. (2014*a*) introduced a new measure of absolute mobility, which we denote by A: the fraction of children with higher inflation-adjusted incomes than their parents at the same age, capturing the chances of children to have a higher standard of living than their parents. Chetty et al. (2017) studied the historical evolution of absolute mobility in the United States, finding that it has fallen from around 90% for children born in 1940 to 50% for children born in the 1980s (see Fig. 1). These findings are consistent with the study by Isaacs (2007) based on the Panel Study of Income Dynamics (2017).

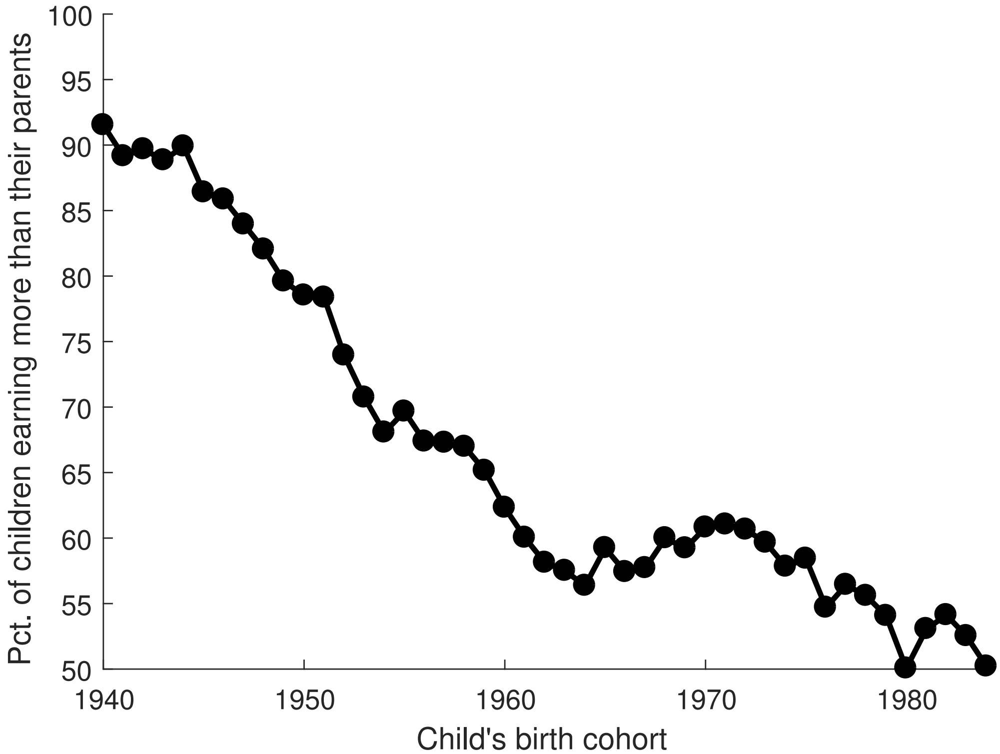

Figure 1: Trends in absolute mobility in the United States. Source: Chetty et al. (2017).

Here, we expand the estimation of absolute mobility trends to several developed and developing countries and to the global level. Following Chetty et al. (2017), our approach combines the marginal income distributions for parents and children and their copula, *i.e.* the joint distribution of parent and child income ranks. We first show that the estimates of absolute mobility depend mainly on the marginal income distributions, while the copula plays only a minor role in determining absolute mobility, within plausible limits. Notably, we find that the long run evolution of absolute mobility cannot be explained by plausible changes in the copula, but only in the marginal distributions.

These observations make the estimation of absolute mobility possible even for countries in which panel data are very scarce. We then combine the available data on intergenerational copulas and historical income distributions from The World Wealth and Income Database (2017) and provide absolute mobility estimates for several developed and developing countries.

We find a decrease in the probability of children to earn more than their parents in France and in the United States for post-war birth cohorts. Unlike the findings on the United States (Chetty et al., 2017), we find that the slow economic growth of the past several decades is the key factor for the decrease in absolute mobility in France, rather than increasing income inequality. We also find that despite higher growth rates in recent decades and regardless of "the American Dream" ethos, the absolute intergenerational mobility in the United States for late 1970s and early 1980s birth cohorts is among the lowest within the group of countries we consider. In China, India and Poland we find very high absolute mobility rates, which reflect the high growth rates characterizing these economies in the recent decades. Despite the high income inequality in China and India (Piketty, Yang and Zucman, 2017; Chancel and Piketty, 2017), the levels of absolute mobility remain very high.

We also provide a statistical model, based on the approximation of the intergenerational joint income distribution as bivariate log-normal, which enables the derivation of closed-form expressions for absolute mobility as a function of income growth, income inequality and relative intergenerational mobility. We demonstrate that this simplified model describes well the long run evolution of absolute mobility, unlocking powerful theoretical and empirical techniques. Notably, we find that the co-movement of absolute and relative mobility should not, in general, be expected and that these two types of mobility are inversely related. We also use the model to provide estimates of absolute mobility in additional countries, for which less data on marginal distributions are available.

Using the simplified model we describe an interplay between income growth and income inequality in their effect on absolute mobility. For example, we can consider two economies

– one grows slowly, at 1.5% per year, and the other fast, at 4.5% per year. In the first, the top 10% income share increases moderately from 39% to 45% over a period of 30 years and in the second it increases sharply from 39% to 66%. In both economies, however, children would have a 60% chance to earn more than their parents (assuming a rank correlation of 0.4).

The same methodology can be used to estimate not only absolute intergenerational mobility, but also intragenerational mobility, without needing detailed panel data. For example, developed countries have gone through a recovery from a major crisis during the past decade, which has benefited most of the population but not the entire population. In the United States, although national income per adult increased in 13% between 2009 and 2016, using data from The World Wealth and Income Database (2017); Panel Study of Income Dynamics (2017) we estimate that only 55% of the adults enjoyed a higher standard of living in 2016 than in 2009. A thorough analysis of intragenerational mobility, however, is left for future work.

Our main contribution is demonstrating that data on marginal income distributions, which are widely available, can be used to estimate absolute intergenerational mobility without the need for high-quality panel data sets, which remain unavailable for most countries and for most birth cohorts. We also offer a theoretical study of the relationship between the canonical measures of intergenerational income mobility and show empirically that a simple model of a bivariate log-normal income distribution can describe adequately the long run dynamics of absolute mobility.

1.1 Outline of the paper

The paper is structured as follows. In Section 2 we discuss the analysis methodology and address the necessity of panel data for producing reliable estimates of absolute mobility.

In Section 3, we follow the methodology described in Section 2 and present the evolution of absolute mobility in different countries and globally. Section 4 presents a reduced-form model for the relationship between absolute mobility, income growth, income inequality and relative intergenerational mobility. Finally, the results and their implications are discussed and concluded in Section 5.

2 Panel data and absolute intergenerational mobility

In an ideal setting, measuring the rate of absolute mobility – the fraction of children with greater real-terms income than their parents – is trivial. For every birth cohort of children we could trace back their parents and compare their incomes at a certain age. However, such data are usually available for small samples and do not cover the whole income distribution or available for a very limited range of birth cohorts. Notably, in many developing countries, such data are rare. As a result, estimating typical measures of relative mobility for different countries over a long period of time would have been impossible or very unreliable. Such measures mainly depend on the copula of the joint income distribution of parents and children.

However, in the case of absolute intergenerational mobility, one is able to provide reliable estimates with narrow confidence intervals, even in the absence of historical detailed panel data. The reason is double. First, as we demonstrate below, the structure of realistic copulas is roughly similar. When two realistic copulas differ in some relative mobility measure, they are very likely to differ proportionally in other relative mobility measures, which are theoretically independent from one another. The practical implication of this observation is that collapsing the copula into a single representative measure of relative mobility is empirically justified.

Second, as we shall also demonstrate below, the sensitivity of the absolute mobility estimates to plausible changes in the copula is low. In particular, plausible changes in relative mobility measures cannot explain long term changes in absolute mobility. Therefore, assuming a fixed copula in time will provide meaningful and reliable estimates of absolute mobility.

2.1 Empirical copulas and measures of relative mobility

We first use empirical copulas measured for different birth cohorts, different countries and for both pre-tax and post-tax incomes and compare them in terms of different measures of relative mobility. Our aim is to demonstrate that although relative mobility is measured by theoretically distinct measures, in practice, differences in one measure translate into proportional changes in other measures.

We consider copulas as transition (doubly stochastic) matrices $P \in \mathscr{P}(N)$, where p_{ij} represents the probability of transferring to quantile j (child) for those starting in quantile i (parent) and N is the number of quantiles. We consider four standard measures of relative mobility:

- Spearman's rank correlation (Spearman, 1904) (or rank-rank slope, RRS), defined as

$$\rho_S(P) = \frac{12\sum_{i=1}^{N}\sum_{j=1}^{N} ijp_{ij} - 3N(N+1)^2}{N(N^2-1)} \tag{2.1}$$

- Bartholomew's index (Bartholomew, 1967) (average absolute jump), defined as

$$B(P) = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} |i-j|\, p_{ij} \tag{2.2}$$

- Average absolute non-zero jump, defined as the average absolute jump normalized while excluding the trace of P, or

$$NZ(P) = \frac{N \cdot B(P)}{N - tr(P)} \tag{2.3}$$

- Shorrocks' trace index (Shorrocks, 1978), defined as

$$S(P) = \frac{N - tr(P)}{N - 1} \tag{2.4}$$

The different measures are mathematically related, however they are not linearly dependent. Specifically, it is possible to construct matrices which have the same trace index, but very different rank correlation, average absolute jump measure or Bartholomew's index and vice versa. Bartholomew (1967); Shorrocks (1978); Atkinson and Bourguignon (1982); Atkinson (1983) provide several constructive examples demonstrating the differences between such measures, describing mathematical constructions of copulas such that one measure is preserved while others may change.

For our analysis of relative intergenerational mobility measures we use 10 empirical transition matrices estimated using survey and tax data covering pre-tax incomes in Denmark, Finland, Norway, Sweden, UK and USA (Jäntti et al., 2006) for different birth cohorts in each; post-tax incomes in Germany, UK and USA (Eberharter, 2014); de-identified federal income tax returns for pre-tax incomes in USA (Chetty et al., 2014*a*).

Figure 2 depicts the relationship between the relative mobility measures calculated for the various empirical transition matrices. It demonstrates that despite the a priori independence of the relative mobility measures, they are, in fact, almost linearly related across time and countries, both for pre- and post-tax incomes. We conclude, therefore, that the shape of the copulas is similar and they can be practically summarized by a single parameter. We use the rank correlation since in many countries the estimated relative mobility is simply reported using the rank correlation or the intergenerational earnings elasticity, rather than providing the entire copula. In addition, the rank correlation proves to be empirically robust compared to other measures (Chetty et al., 2014*a*).

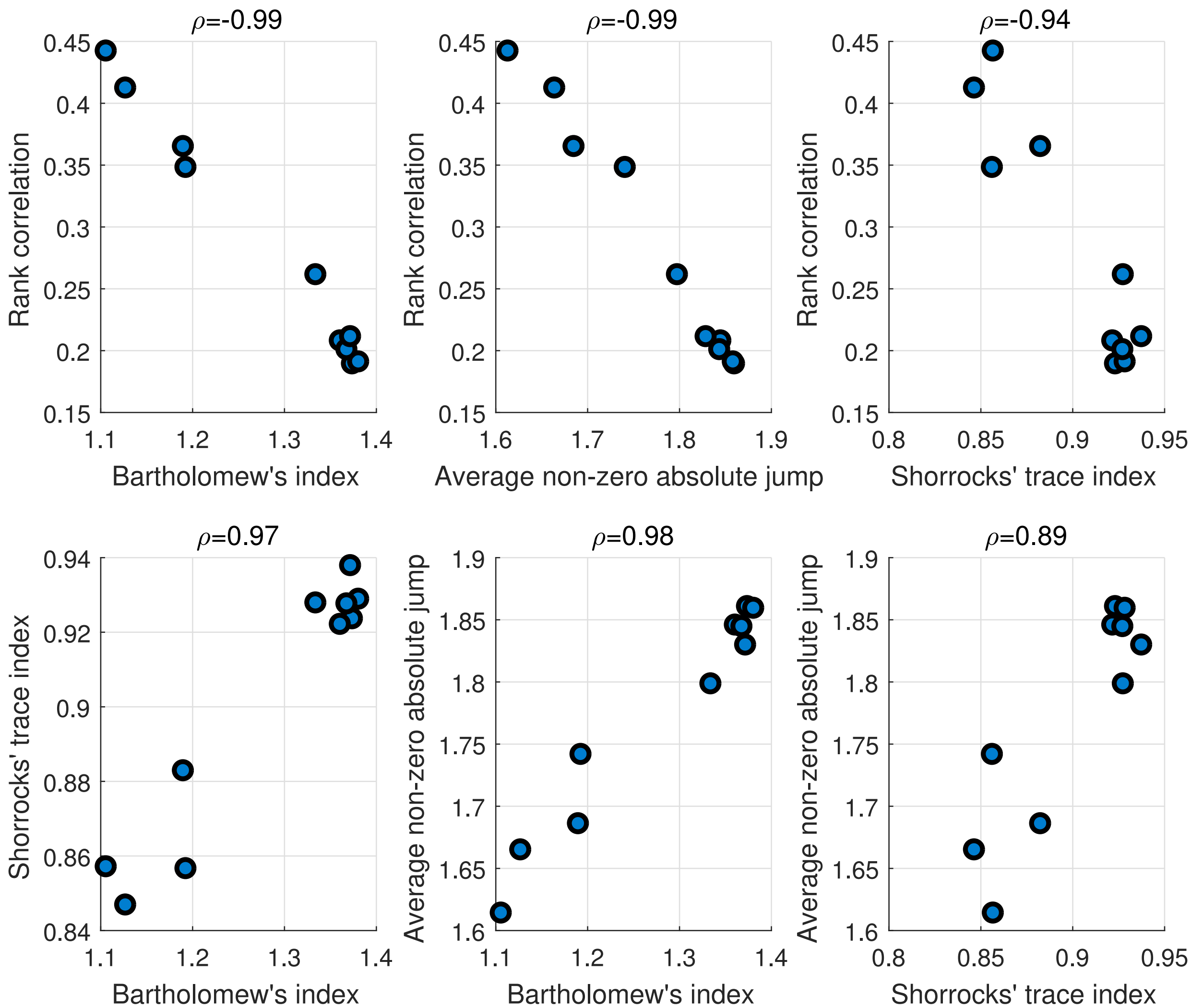

Figure 2: The relationship between relative mobility indices in empirical copulas.

2.2 Empirical copulas and absolute mobility

Figure 2 demonstrates that a single parameter characterizing relative intergenerational mobility can practically describe empirical intergenerational copulas. We will now demonstrate, in addition, that different empirical copulas lead to very similar estimates of absolute mobility. This observation allows us to argue that the marginal income distributions and a single observation of a relative mobility measure, such as the rank correlation, can provide robust estimates of absolute mobility for various countries.

Chetty et al. (2017) used linear programming for constructing plausible copulas, used for checking the robustness of their estimates. We use, instead, the same 10 empirical copulas described above, which cover various countries and birth cohorts, as well as pre- and post-tax incomes. Using the same marginal distributions used for the United States absolute mobility estimates as in Chetty et al. (2017) (based on the United States census and Current Population Surveys (CPS) data), we estimated the United States absolute mobility, each time using a different empirical copula.

The results are presented in Fig. 3. They demonstrate that estimating the absolute mobility in the United States with different copulas, which may be very different from the one characterizing the United States, results in very similar evolution in time. The estimates obtained using the various empirical copula differ from the benchmark estimates of Chetty et al. (2017) by 3.5% on average. Therefore, we conclude that long run changes which are larger than several percent, cannot be explained by plausible changes in the copula, but only by changes in the marginal distributions.

3 Estimating absolute mobility globally

Following the discussion on the sensitivity of absolute mobility to changes in the copula and assuming that empirical copulas can be represented using a single parameter such as the rank correlation, we present estimates of trends in absolute mobility for various countries – the United States, France, China, India, Russia and Poland. We assume that the rank correlation in each country is fixed in time and in order to take into account plausible changes

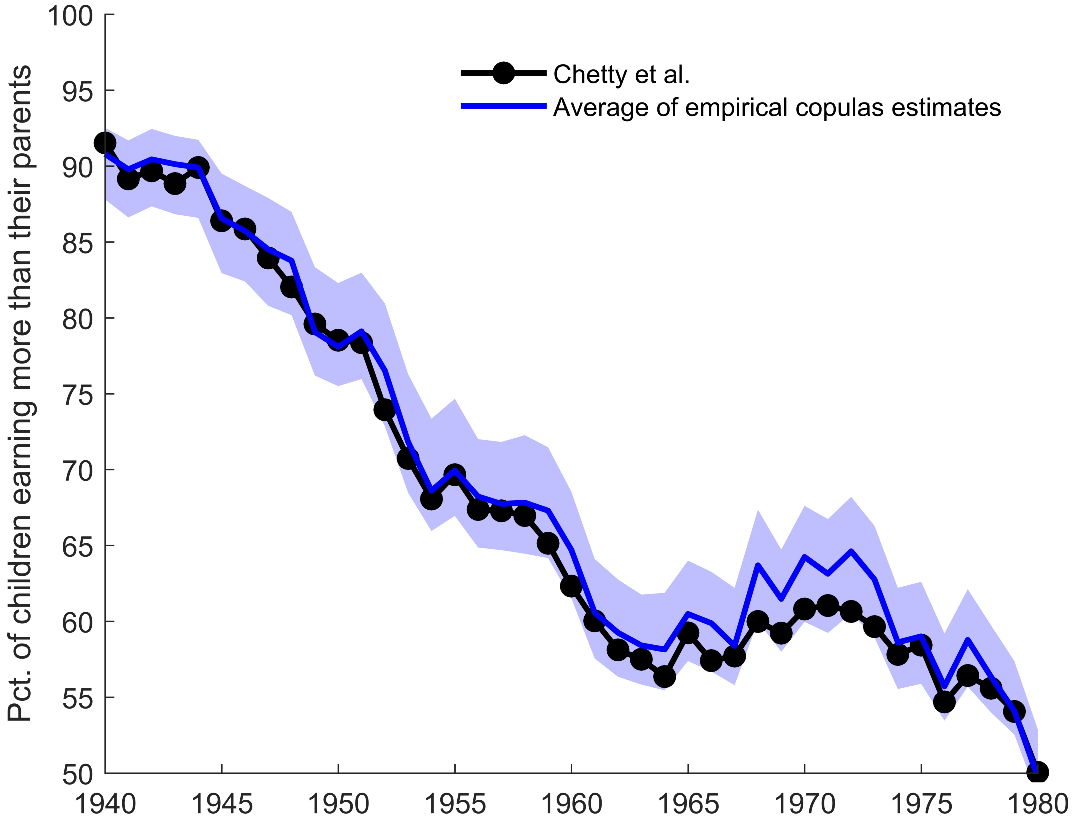

Figure 3: Trends in absolute mobility in the United States, estimated using 10 empirical copulas. The shaded blue area is the area covered by the various absolute mobility estimates. The blue curve is the arithmetic mean of all 10 estimates in each year. The black circles are the baseline estimates reported in Chetty et al. (2017).

in time or the lack of accurate data, we consider a range of values. The correlation values used are presented in Tab. 1.[1] We also estimate the absolute intergenerational mobility globally, based on the data of Bourguignon and Morrisson (2002) used for estimating the relative mobility of the world population.[2]

In order to get the marginal income distributions we use data from The World Wealth and Income Database (2017) and the generalized Pareto curve interpolation method (Blanchet,

[1]For the countries in which the interegeneratoinal elasticity was reported, rather than the rank correlation, we use the relationship $\rho = \frac{\sigma_p}{\sigma_c}\beta$, where σ_p and σ_c are the standard deviations of the parents and children marginal income distributions and β is the estimated interegeneratoinal income elasticity (see (Trivedi and Zimmer, 2007) and Section 4.1)

[2]In Bourguignon and Morrisson (2002) it is assumed that there is no mobility within countries, *i.e.* that the composition of deciles in each country remains the same, and the relative mobility is due to the different growth rates of deciles in various countries. Therefore such estimation of mobility is very conservative and the real rank correlation is bounded from above by the estimated rank correlation of 0.8.

Table 1: Rank correlation values used in the absolute mobility analysis

Country	Nominal value	Lower limit	Upper limit	Source
China	0.31	0.27	0.35	Fan, Yi and Zhang (2015)
France	0.42	0.4	0.5	Lefranc and Trannoy (2005)
India	0.65	0.4	0.9	Hnatkovska, Lahiri and Paul (2013)
Poland	0.4	0.2	0.6	Brzeziński, Jancewicz and Letki (2013)
Russia	0.35	0.27	0.53	Denisova and Kartseva (2017)
USA	0.3	0.29	0.31	Chetty et al. (2014*a*)
World	–	–	0.8	Based on Bourguignon and Morrisson (2002)

Fournier and Piketty, 2017) to obtain simulated samples (the sample size was $N = 5 \cdot 10^5$, large enough to reduce the statistical uncertainty of the estimates to practically zero) which capture the entire pre-tax income distribution.

Since the available historical data include all adult population and it is not possible to restrict the data to 30-year-olds only, these results differ from those obtained above (Fig. 3) conceptually. However, as long as the income growth and inequality among 30-year-olds is similar to that of the entire adult population, than absolute mobility estimates will be very close even if the entire adult population is considered. In most countries, this would have a small effect. For example, in the United States, the difference between the absolute mobility estimates using the census-based marginal distributions used by Chetty et al. (2017) and using The World Wealth and Income Database (2017) marginal distributions is lower than 2%, excluding several birth cohorts in the mid 1940s, in which the difference is 6%–8%. The small difference is driven by two effects – income growth is slightly lower among 30-year-olds than among the entire adult population, while inequality among 30-year-olds is also lower. The effects of these on absolute mobility almost cancel out.

This difference, however, may be large in some countries and requires micro-data sets which include age. Such data sources are usually based on surveys and are therefore limited in their ability to capture the entire income distribution adequately, particularly the top income group. Therefore, no single data source would be perfect for the current task. On the other hand, in order to obtain a good understanding of the long run evolution of absolute mobility, it is possible using the historical data of the entire adult population, which we follow in our analysis. We also note that this data limitation does not invalidate the methodology. Overcoming it only requires acquiring the marginal distributions for 30-year-olds only, while also capturing well the top of the distribution.

The results are presented in Fig. 4. They indicate that the documented decrease in absolute mobility rates found in the United States also occurred in France. Using the detailed historical data on the income distribution in France, it is also possible to show that the absolute mobility in France increased rapidly for the children born in the 1920s–1940s, the direct beneficiaries of the *Trente Glorieuses*. The decrease of absolute mobility from the 1940s birth cohorts onward is mainly due to decreasing income growth rates rather than rising inequality (see Section 3.1).

The cases of China and India are unique in comparison to the other countries analyzed. The estimated high absolute mobility reflects the high growth rates in both. The absolute mobility levels remain high despite increasing income inequality (Piketty, Yang and Zucman, 2017; Chancel and Piketty, 2017) and are also reinforced by the low relative intergenerational mobility (Hnatkovska, Lahiri and Paul, 2013; Corak, 2013; Fan, Yi and Zhang, 2015) (see Section 4).

Absolute mobility levels in Poland are also found to be relatively high, especially among developed countries. This echoes the findings on the successful transition of Poland into a market economy (Piatkowski, 2018). In Russia, however, this transition was accompanied by rapid increase of income inequality (Novokmet, Piketty and Zucman, 2017) and negative or slow growth during a large part of the 1990s and early 2000s. The levels of absolute mobility for children born in the late 1960s and the 1970s in Russia is therefore very low.

Globally, absolute mobility is between 65% to 70% for the early 1980s birth cohorts.

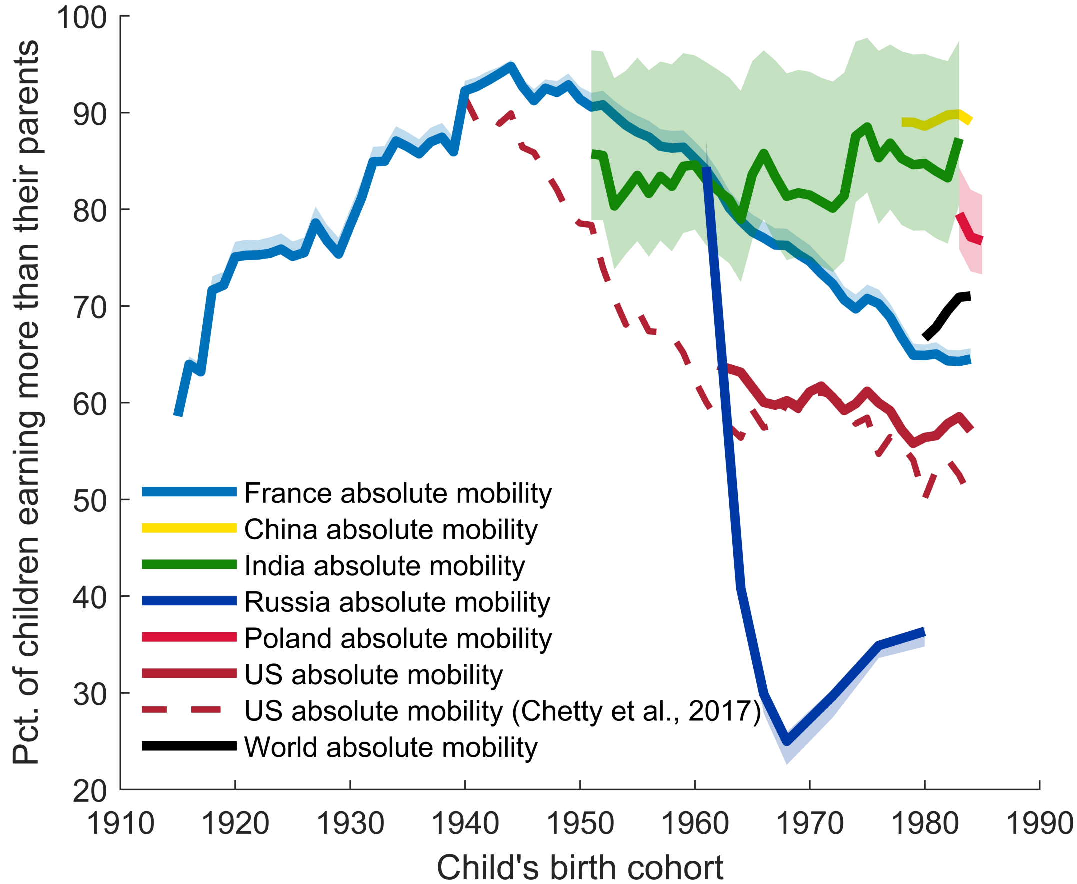

Figure 4: Evolution of absolute mobility in France, China, India, Russia, Poland, United States and globally. The shaded areas take into account the lower and upper limits of the rank correlation, which in some countries has a negligible effect.

However, this is a lower bound, since the rank correlation used for estimation (0.8) is an upper bound of the actual rank correlation.

3.1 Absolute mobility decomposition

Figure 4 shows absolute intergenerational mobility decreasing in several countries and stable in others. It is possible to decompose the dynamics of the absolute mobility estimates in order to understand the sources for its long run trend.

As noted, the rank correlation, hence relative mobility, is assumed to be fixed in time and it was demonstrated that plausible changes in relative mobility cannot be the source of long run changes in absolute mobility. Therefore, the observed trend can be attributed to two factors – the generally decreasing income growth rates and the generally increasing income inequality. For each country analyzed we wish to understand what is the contribution of those factors to the absolute intergenerational mobility trend. For that purpose, we produce, in addition to the baseline estimate of each country, two counterfactual calculations: one in which the shape of the income distribution is kept constant in time and similar to the earliest distribution in the data, but with the average income changing according to its real historical values; and another, in which the distribution shape changed according to historical data, but the income growth rate was fixed in time and equal to the average income growth rate over the whole period analyzed.

Such calculations are presented in Fig. 5 for France and India. They demonstrate fundamentally different dynamics. In India, absolute mobility is largely high and stable. However, if income inequality would have stayed at 1950s levels, absolute mobility would have increased to above 95% for children born in the 1980s. Alternatively, if annual growth was fixed at 2.5%, much lower than the growth rates of the recent two decades, but inequality would have indeed increased, absolute mobility would have increased to less than 75% for children born in the 1980s. The stable absolute mobility is the result of two processes – increasing income growth rates and increasing income inequality.

France, however, demonstrates different dynamics. The absolute mobility evolution is predominantly governed by the decreasing growth rates. The increasing income inequality in France during the past several decades cannot explain but only a small portion of the decrease in absolute mobility.

We formalize the different contributions by calculating the fraction of the overall change in absolute mobility produced by each counterfactual. Those contributions are presented in Tab. 2.

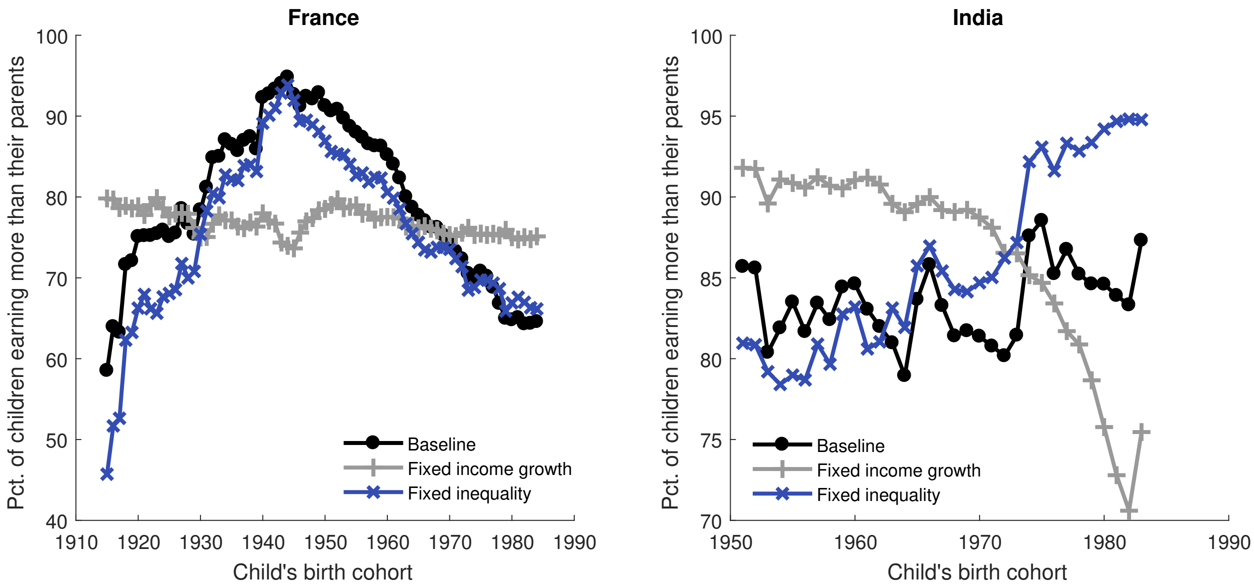

Figure 5: Counterfactual calculations of absolute mobility in France and India.

Table 2: Growth and inequality changes contribution to evolution of absolute mobility

Country	Change in absolute mobility (*pp*)	Growth contribution (*pp*)	Inequality contribution (*pp*)
China	0.2	0	0.2
France	6	7.8	−1.8
India	1.6	10.5	−8.9
Poland	−2.9	−0.5	−2.4
Russia	−48	−39	−9
USA	−6.8	−2.3	−4.5
World	4.5	5.5	−1

In France and in Russia the change in absolute mobility is mainly due to the decreasing growth rates, as also graphically demonstrated in Fig. 5 for France. In addition, within the global population there is a small increase in absolute mobility which is mainly due to growth and only little affected by the mild change in income inequality.

As explained, inequality plays a much larger role in the absolute mobility evolution in the United States and India. In China and in Poland, in which the absolute mobility evolution is described over less than 10 birth cohorts, the overall change is very small, so that the contribution of each factor to it may be misleading, and left in Tab. 2 for consistency only.

4 Growth, inequality, relative mobility and absolute mobility

The global estimates of absolute mobility presented in Fig. 4 raise the need for understanding the trends detected in different countries. Since the absolute mobility depends on marginal distributions and their copula, it is possible to reframe this dependence using growth, quantifying the change in the average income between the marginal distributions, inequality, describing the intergenerational change in the shape of the marginal distributions and relative mobility, embedded in the copula. This is specifically important for understanding what are the drivers of the observed trends in the different countries, as captured in Tab. 2.

In order to characterize the dependence of absolute mobility on growth, inequality and relative mobility we present a reduced-form statistical model for intergenerational mobility. For this purpose we also introduce the intergenerational earnings elasticity (IGE), a canonical measure of relative intergenerational mobility. It is defined as the elasticity of the logarithm of child income with respect to the logarithm of parent income and we denote it by β (Mulligan, 1997; Lee and Solon, 2009; Chetty et al., 2014*a*). IGE, like the rank correlation, is a measure of immobility rather than of mobility: the larger it is, the stronger the relationship between parent and child incomes. Therefore, $R_1 \equiv 1 - \beta$ is used as a measure of relative mobility. Similarly, we denote $R_2 \equiv 1 - \rho_S$ as the measure of relative mobility corresponding to the rank correlation ρ_S.

We present a theoretical study of the absolute mobility measure A, with particular focus on the relationship between A and the two measures of relative mobility, R_1 and R_2. We find that under a general model of the joint log-income distribution, absolute mobility is inversely related to both relative mobility measures. In addition, this model allows us to directly incorporate growth and inequality and describe their relationship with absolute mobility.

Although Chetty et al. (2014*a*, p. 1574) argued that "the income distribution is not well approximated by a bivariate log-normal distribution", we find that the decline in absolute mobility in the United States is. While such a simplified model may not explain adequately some aspects of mobility and inequality, for the purpose of estimating long run absolute mobility trends, this model is satisfactory.

Using the bivariate normal model it is possible to obtain closed-form expressions for the dependencies of A on R_1 and on R_2, as well as on the marginal income distributions means and standard deviations, hence on growth and inequality. This helps elucidate possible mechanisms by which changes in the marginal distributions of parent and child incomes influence absolute mobility, whose discussion has been hitherto limited.

4.1 Model

Our starting point is a population of N parent-child pairs. We denote by Y_p^i and Y_c^i the respective inflation-adjusted incomes of the parent and the child (at the same age) in family $i = 1 \dots N$. We assume the incomes are all positive and define the log-incomes $X_p^i = \log Y_p^i$ and $X_c^i = \log Y_c^i$.

The intergenerational earnings elasticity is defined as the slope (β) of the linear regression

$$X_c = \alpha + \beta X_p + \epsilon\,, \tag{4.1}$$

where α is the regression intercept and ϵ is the error term.

The rate of absolute mobility, A, introduced in (Chetty et al., 2014*a*, p. 1563) is the fraction of children earning more than their parents in real terms, equal to the probability $P\left(X_c - X_p > 0\right)$.

One hypothetical sample of the joint parents and children log-income distribution is presented in Fig. 6. Fig. 6 also depicts how A and β are defined – the blue line is $y = x$, hence the rate of absolute mobility is defined as the fraction of parent-child pairs which are above it. The red line is the linear regression $y = \alpha + \beta x$, for which β is the IGE.

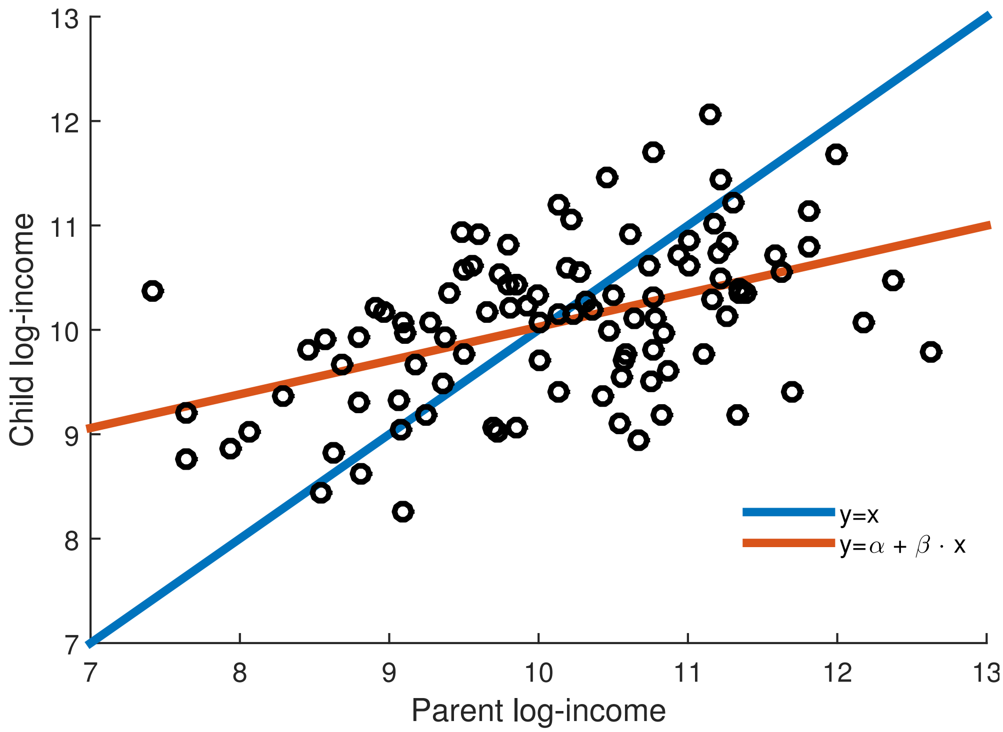

Figure 6: An illustration of the absolute and relative mobility measures. The black circles are a randomly chosen sample of 100 parent-child log-income pairs, assuming a bivariate normal distribution. The parameters used were $\mu_p = 10.1$, $\sigma_p = 0.78$ (for the parents marginal distribution) and $\mu_c = 10.25$, $\sigma_c = 1.15$ (for the children marginal distribution) with correlation of $\rho = 0.6$. The resulting α and β were 1.8 and 0.84, respectively.

The log-normal distribution is a widely used approximation of empirical income distributions (Pinkovskiy and Sala-i-Martin, 2009) and has a mechanistic basis as the long run attractor distribution for quantities undergoing random multiplicative growth Aitchison and Brown (1957); Adamou and Peters (2016). Thus, a simple and plausible model for the joint distribution of parent and child log-incomes is the bivariate normal distribution. In this model, the marginal income distributions of both parents and children are log-normal and the correlation between their log-incomes is defined by a single parameter ρ. The marginal log-income distributions of the parents and the children follow $\mathcal{N}\left(\mu_p, \sigma_p^2\right)$ and $\mathcal{N}\left(\mu_c, \sigma_c^2\right)$, respectively. Hence the joint distribution is fully characterized by 5 parameters: μ_p, σ_p, μ_c, σ_c and ρ.

The choice of model may substantially affect the analysis of the absolute and relative mobility measures. Other possible models for the joint income distribution of parents and children can include marginal distributions which are not log-normal, such as beta and gamma distributions, as well as other types of copula. In the bivariate log-normal model the copula is Gaussian, while other types found useful for various empirical applications (Trivedi and Zimmer, 2007), such as the Clayton, the Gumbel and the Plackett copula families (Bonhomme and Robin, 2009), may prove to be a better description of the relationship between the marginal income distributions. In their study of intergenerational mobility in France, Bonhomme and Robin (2009) argue that the Gaussian copula "tends to underestimate the dependence in the middle of the distribution, that is, the probabilities of remaining in the second, third, and fourth quintiles" and show that the empirical copula is best estimated by the Plackett copula. In Appendix B we show that the choice of copula model, assuming a similar rank correlation, has practically no effect on absolute mobility. This observation is consistent with our observations on empirical copulas (see Section 2), showing that absolute mobility estimates are insensitive to plausible changes in the copula.

4.2 Theoretical results

We first address the properties of the bivariate log-normal approximation. We derive closed-form expressions for the measures of mobility – A, R_1, and R_2 – in terms of the model distribution parameters.

Proposition 1 *For a bivariate normal distribution with parameters μ_p, σ_p (for the parents marginal distribution) and μ_c, σ_c (for the children marginal distribution) assuming correla-*

tion ρ, the relative mobility R_1 is

$$R_1 = 1 - \frac{\sigma_c}{\sigma_p}\rho\,. \tag{4.2}$$

Following Prop. 1 it is also possible to derive the rate of absolute mobility as a function of the distribution parameters and the IGE:

Proposition 2 *For a bivariate normal distribution with parameters μ_p, σ_p (for the parents marginal distribution), μ_c, σ_c (for the children marginal distribution) and correlation ρ, the rate of absolute mobility is*

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(2R_1 - 1\right) + \sigma_c^2}}\right)\,, \tag{4.3}$$

where Φ is the cumulative distribution function of the standard normal distribution.

Equation (4.3) enables the calculation of A assuming the knowledge of the marginal income distributions and R_1 (or β). It is therefore also possible to calculate A in terms of R_2, by replacing R_1 and R_2 according to the known relationship between them in a bivariate log-normal model (Trivedi and Zimmer, 2007).

Corollary 1 *Under Prop. 2 notations, the rate of absolute mobility A is*

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - \frac{4\sigma_c}{\sigma_p}\sin\left(\frac{\pi(1-R_2)}{6}\right)\right) + \sigma_c^2}}\right)\,. \tag{4.4}$$

where Φ is the cumulative distribution function of the standard normal distribution.

Our next step is to test whether the bivariate log-normal model for the joint income distribution is empirically sound. For that purpose we compare the model prediction for the historical rate of absolute mobility in the United States and France with the historical rate reported by Chetty et al. (2017) and the results presented in Section 3.

We use pre-tax national income per adult data and the top 10% income share data to obtain μ_p, σ_p, μ_c and σ_c every year. Such data are available for long periods of time and for various countries in The World Wealth and Income Database (2017). Since in the bivariate log-normal model, the marginal log-income distributions are normal, these parameters can be obtained directly and no fit is required. The Lorenz curve of the log-normal distribution $\log \mathcal{N}\left(\mu, \sigma^2\right)$ is $\Phi\left(\Phi^{-1}(z) - \sigma\right)$ (Cowell, 2011). Therefore, the top 10% income share s corresponds to

$$\sigma = \Phi^{-1}(0.9) - \Phi^{-1}(1 - s) \ . \tag{4.5}$$

If we denote the per-adult pre-tax income as m, it follows that the parameter μ is

$$\mu = \log(m) - \frac{\sigma^2}{2} \, . \tag{4.6}$$

First, we demonstrate the similarity between the absolute mobility estimates presented in Section 3 to those resulting in the log-normal approximation. These are presented for France and the United States in Fig. 7. Assuming similar rank correlations to those used in Section 3, we find that the difference between the estimated absolute mobility values is 2.2 percentage points on average in France and 3.9 percentage points on average in the United States. The log-normal approximation overstates the effect of income inequality, which produces a downward bias in the mobility estimates. However, this bias affects very little the long run evolution of the absolute mobility.

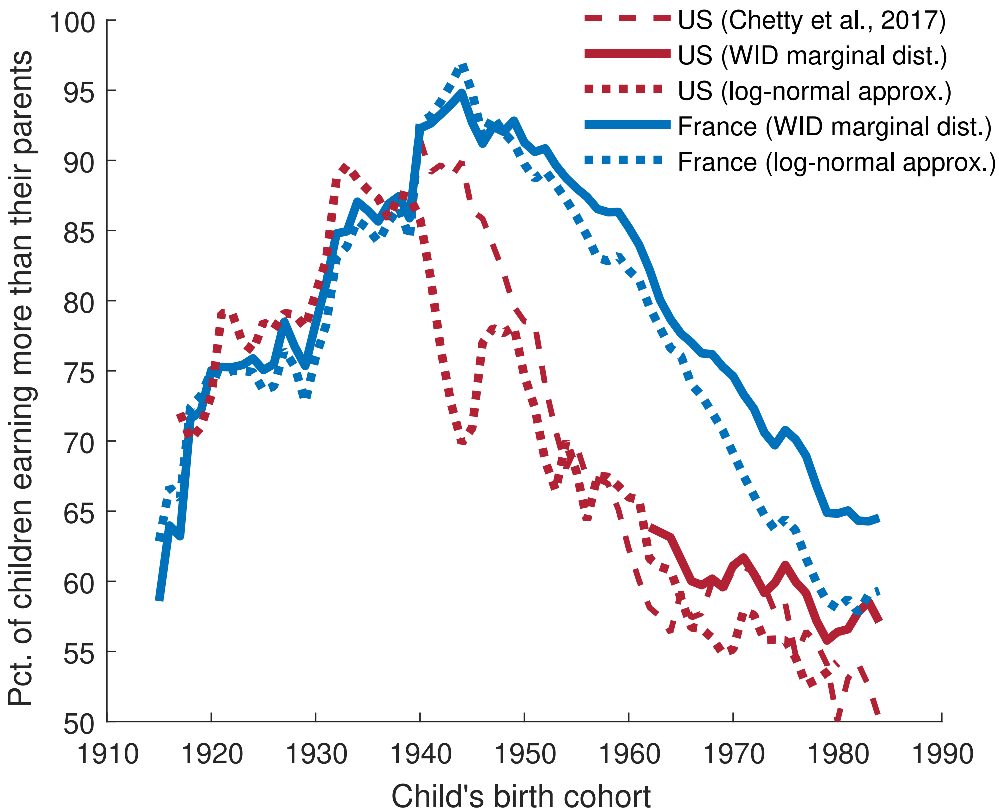

Figure 7: Estimated absolute mobility in France and the United States, based on the generalized Pareto curve interpolation method and on The World Wealth and Income Database (2017) data (solid lines) and assuming the log-normal approximation (dotted lines). We assumed a rank correlation of 0.42 in France and 0.3 in the United States as done in Section 3. The dashed red line is the estimated absolute mobility in the United States estimated by Chetty et al. (2017).

Figure 7 demonstrates that despite its comparative methodological naïvety, the bivariate log-normal model can be used to describe the long run evolution of absolute mobility. Following this observation we produce absolute mobility estimates for Sweden, Denmark and the United Kingdom, as presented in Fig. 8, using data for the pre-tax national income per adult and the top 10% income share data from The World Wealth and Income Database (2017), as done in Fig. 7.

Similarly to France and the United States, we find that other developed countries experienced a decrease in absolute mobility for post-war birth cohorts. The evolution of absolute mobility in Sweden is very similar to the observed evolution in France (compare with Fig. 7)

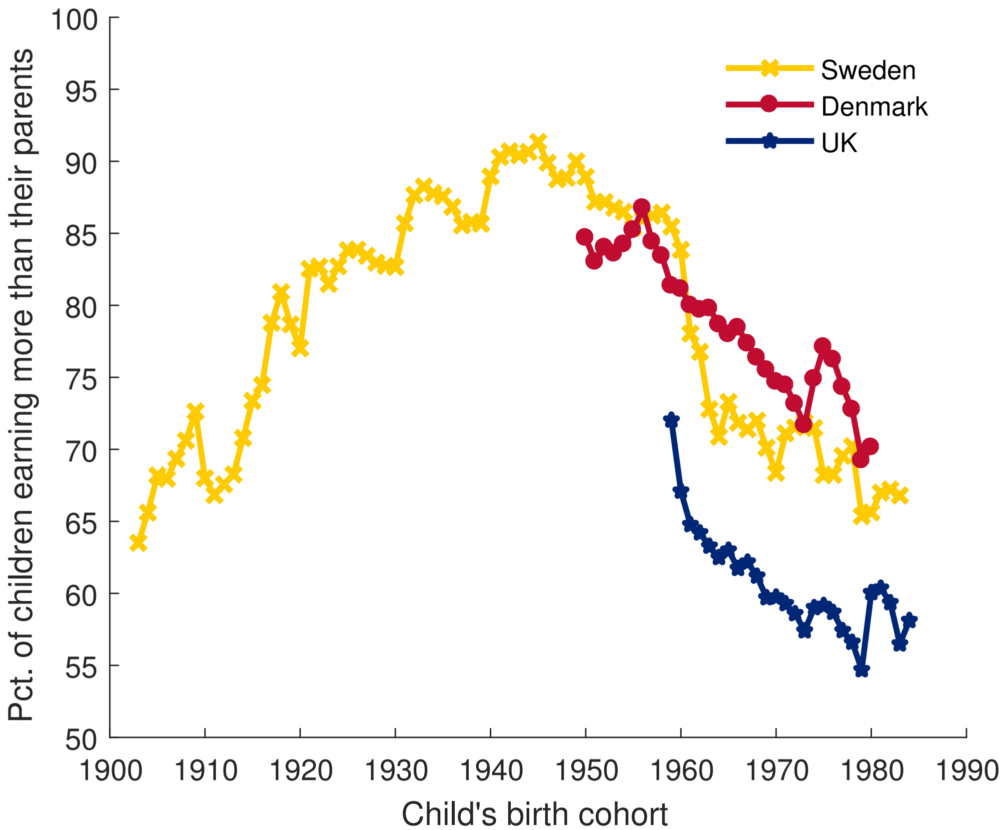

Figure 8: Estimated absolute mobility in the Sweden, Denmark and the United Kingdom assuming the bivariate log-normal model and assuming fixed rank correlation of 0.2, 0.19 and 0.21 (Jäntti et al., 2006), respectively.

and likewise, it is largely driven by the lower growth rates of the past several decades. The case of the UK is similar to that of the United States, in which the combination of increasing income inequality and decreasing income growth led to a fast decrease in absolute mobility.

4.3 The relationship between absolute and relative mobility

We can also use the properties derived to further study the measures of mobility – A, R_1 and R_2. Eqs. (4.3–4.4) demonstrate that the rate of absolute mobility can be explicitly described as a function of the relative mobility measures R_1 and R_2. Fig. 9 shows A as a function of R_1 and R_2 for different birth cohorts in the United States. It shows that the bivariate normal model – with positive income growth and inequality changes consistent with data, but absent other effects – predicts an inverse relationship between absolute and relative mobility.

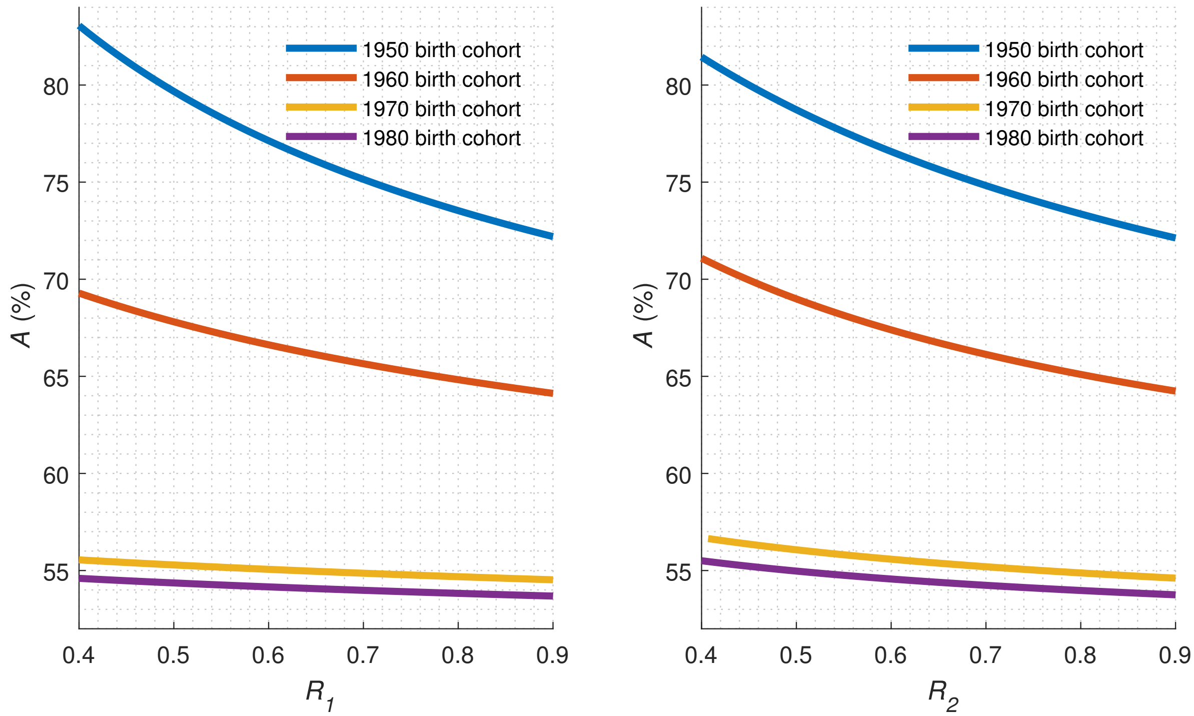

Figure 9: The theoretical relationship between the rate of absolute mobility, R_1 (left) and R_2 (right), assuming the bivariate normal log-incomes model for different birth cohorts in the United States. This demonstrates the inverse relationship between absolute and relative mobility measures.

Proposition 2 illustrates that the rate of absolute mobility increases with increasing income growth and decreases with increasing income inequality, as described by Chetty et al. (2017). However, it also demonstrates that an additional mechanism can be at play, since the rate of absolute mobility decreases with increasing relative mobility.

The direct implication is that if the relative mobility in the United States had been decreasing during the past few decades, as some argue (Aaronson and Mazumder, 2008; Putnam, Frederick and Snellman, 2012), the decrease in absolute mobility is less significant than reported by Chetty et al. (2017).

Figure 9 seemingly stands in contrast to the findings about absolute intergenerational mobility in the United States, France, Sweden and Denmark (see Fig. 4), since the absolute and relative mobility in those countries were found to be ordered similarly. However, in practice, had the relative mobility in the United States been higher, the difference in absolute mobility rates with respect to the other countries would have become even larger.

The low relative intergenerational mobility in China contributes to its high levels of absolute intergenerational mobility. Had it been perfectly mobile in relative terms (*i.e.* rank correlation of 0), the estimated average absolute intergenerational mobility would have decreased by 4 percentage points only. This demonstrates that despite the inverse relationship described, in practice, as also observed in Section 2, the effect of changes in relative mobility on absolute mobility, is usually minor.

4.4 Absolute mobility and median incomes

Chetty et al. (2017) also find that the share of children earning more than the median parent declined from 92% in the 1940 birth cohort to 45% in the 1984 cohort (Katz and Krueger, 2017). This alternative measure of absolute mobility moves almost identically to A across cohorts in the United States (Katz and Krueger, 2017). Defining the share of children earning more than the median parent as $\tilde{A}$, it simply follows in the bivariate log-normal model that $\tilde{A}$ is defined as

$$\tilde{A} \equiv \Phi\left(\frac{\mu_c - \mu_p}{\sigma_c}\right) , \tag{4.7}$$

where Φ is the cumulative distribution function of the standard normal distribution.

Using $\tilde{A}$ has obvious advantages over A. In particular, they can be "directly computed from standard public-use cross-sectional household survey data and do not require data that longitudinally link children to parents" (Katz and Krueger, 2017, p. 382). However, $\tilde{A}$ would be close to A only if the IGE is close to 1/2:

Proposition 3 *For a bivariate normal distribution with parameters μ_p, σ_p (for the parents marginal distribution), μ_c, σ_c (for the children marginal distribution) and assuming IGE of β, then*

$$A = \tilde{A} \iff \beta = \frac{1}{2}\,. \tag{4.8}$$

It is therefore no surprise that for the United States A and $\tilde{A}$ are relatively similar – Aaronson and Mazumder (2008) estimate the IGE for the 1950–1970 birth cohorts at 0.46–0.58. In countries such as Denmark or Finland, in which the IGE is substantially lower than 0.5 (Corak, 2013), using $\tilde{A}$ neglects the part high relative mobility plays in determining A. This will lead to significant overestimation of absolute intergenerational mobility. For example, in Denmark, the average estimated A for 1950–1980 birth cohorts is 78.7% (see Fig. 8). If $\tilde{A}$ is considered, the average value is 87%. For the United States this difference would be considerably smaller: 70.7% and 72.8%, respectively. Setting aside the normative question of which measure of absolute intergenerational mobility is of most interest, we emphasize that $\tilde{A}$ cannot be used as a proxy for A, unless the IGE is close to 0.5.

5 Discussion

Our findings highlight a decreasing absolute mobility trend in all developed countries. The sources of this trend, however, differ from country to country. Specifically, in the United States, the rising income inequality is the main contributor for decreasing absolute mobility. In other countries, such as France or Sweden, a similar evolution of absolute mobility is almost fully explained by the decrease of income growth rates. We also found that in China and India, despite high income inequality, the rapid economic growth leads to very high absolute intergenerational mobility.

From an empirical perspective, our findings imply that using widely available data on marginal income distributions and limited data on relative intergenerational mobility, it is possible to produce estimates of absolute intergenerational mobility without the need for high-quality panel data sets, which remain unavailable in most countries. The structure of realistic copulas is found to be roughly similar and the practical implication of this observation is that collapsing the copula into a single representative measure of relative mobility is empirically justified for the purpose of estimating absolute mobility rates. In addition, the sensitivity of the absolute mobility estimates to the value of relative mobility measures is low and plausible changes in relative mobility measures cannot explain the long run evolution of absolute mobility. Therefore, assuming a fixed copula in time will provide meaningful and reliable estimates of absolute mobility. We also find that a model as simple as a bivariate log-normal distribution is satisfactory for describing the dynamics of absolute mobility with high confidence.

Our findings join recent work on global inequality and the effects of globalization (Bourguignon, 2015; Milanovic, 2016; Rodrik, 2017; Alvaredo et al., 2017) to describe another angle of the effect of global trends. Since absolute mobility decreases with both increasing inequality and decreasing growth, but increases when relative mobility is low, it captures an important normative concept of the economic prospects of young adults. The recent global "populism wave", mainly in developed countries, is claimed to be linked to globalization and inequality and their relationship (Milanovic, 2016; Rodrik, 2017). We argue that this relationship is partially captured by absolute intergenerational mobility. This can explain, for example, similarities in the recent political trends in France and the United States. Both countries exhibited the so-called "populism wave" during the presidential elections of 2016 and 2017, respectively. While inequality in the United States has been sharply increasing during the past few decades to levels last observed during the 1920s, inequality in France remained largely stable and at much lower levels. Gross domestic product growth, however, was almost consistently higher in the United States than in France during the same period.

The measure of absolute intergenerational mobility captures an aspect of the chances of young adults to achieve a higher standard of living than their parents. Since this happens both when growth is low and when income inequality is very high, the absolute mobility trend demonstrates a possible partial explanation for the common observed political phenomena of the recent years – as we find that in both countries, absolute mobility has been decreasing during the past few decades at a roughly similar rate.

The seemingly counter-intuitive inverse theoretical relationship found between absolute and relative mobility stems from a fundamental conceptual difference between the two categories of mobility. It exposes the problems that can arise if both are treated as measuring similarly the same phenomenon. In particular, absolute mobility is very sensitive to across-the-board economic growth. For example, during the Middle Ages – when relative mobility rates were low because social class and profession were predominantly inherited (Goldthorpe, 1987; Clark, 2014) – even the slightest positive or negative income growth would result in very high or very low absolute mobility. A misleading picture of intergenerational mobility may arise if the basic properties of these measures are overlooked. Therefore, empirically addressing this measure of intergenerational mobility requires careful delineation of the phenomena of interest and the manner in which quoted measures reflect them.

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A Proofs

A.1 Proof of proposition 1

First, by definition, the correlation ρ, between X_p and X_c equals to their covariance, divided by $\sigma_p \sigma_c$

$$\rho = \frac{\mathrm{Cov}\left[X_p, X_c\right]}{\sigma_p \sigma_c} . \tag{A.1}$$

β can be directly calculated as follows, by the linear regression slope definition:

$$\beta = \frac{\sum_{i=1}^{N} \left(X_p^i - \bar{X}_p\right) \left(X_c^i - \bar{X}_c\right)}{\sum_{i=1}^{N} \left(X_p^i - \bar{X}_p\right)} , \tag{A.2}$$

where $\bar{X}_p$ and $\bar{X}_c$ are the average parents and children log-incomes, respectively.

It follows that

$$\beta = \frac{\mathrm{Cov}\left[X_p, X_c\right]}{\sigma_p^2} . \tag{A.3}$$

We immediately obtain

$$\beta = \frac{\sigma_c}{\sigma_p} \rho \tag{A.4}$$

and therefore

$$1 - \beta = R_1 = 1 - \frac{\sigma_c}{\sigma_p} \rho \tag{A.5}$$

$\blacksquare$

A.2 Proof of proposition 2

We start by defining a new random variable $Z = X_c - X_p$. It follows that calculating A is equivalent to calculating the probability $P(Z > 0)$.

Subtracting two dependent normal distributions yields that

$$Z \sim \mathcal{N}\left(\mu_c - \mu_p, \sigma_p^2 + \sigma_c^2 - 2\mathrm{Cov}\left[X_p, X_c\right]\right) , \tag{A.6}$$

so according to Prop. 1

$$Z \sim \mathcal{N}\left(\mu_c - \mu_p, \sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2\right) . \tag{A.7}$$

If follows that

$$\frac{Z - \left(\mu_c - \mu_p\right)}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}} \sim \mathcal{N}\left(0, 1\right) , \tag{A.8}$$

so we can now write

$$
\begin{aligned}
&P\left(Z>0\right)= \\
&P\left(\frac{Z-(\mu_c-\mu_p)}{\sqrt{\sigma_p^2\left(1-2\beta\right)+\sigma_c^2}}>-\frac{\mu_c-\mu_p}{\sqrt{\sigma_p^2\left(1-2\beta\right)+\sigma_c^2}}\right)= \\
&\Phi\left(\frac{\mu_c-\mu_p}{\sqrt{\sigma_p^2\left(1-2\beta\right)+\sigma_c^2}}\right)=\Phi\left(\frac{\mu_c-\mu_p}{\sqrt{\sigma_p^2\left(2R_1-1\right)+\sigma_c^2}}\right),
\end{aligned} \tag{A.9}
$$

where Φ is the cumulative distribution function of the standard normal distribution.

■

A.3 Proof of corollary 1

For the bivariate log-normal model it is known that (Trivedi and Zimmer, 2007)

$$
1-R_2=\frac{6\arcsin\frac{\rho}{2}}{\pi}. \tag{A.10}
$$

hence, using Eq. (4.2)

$$
R_2=1-\frac{6\arcsin\frac{\sigma_p\beta}{2\sigma_c}}{\pi}. \tag{A.11}
$$

Substituting into Eq. (4.3) we obtain

$$
A=\Phi\left(\frac{\mu_c-\mu_p}{\sqrt{\sigma_p^2\left(1-\frac{4\sigma_c}{\sigma_p}\sin\left(\frac{\pi(1-R_2)}{6}\right)\right)+\sigma_c^2}}\right). \tag{A.12}
$$

■

A.4 Proof of proposition 3

Following Eq. (4.3)

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\,(1-2\beta)+\sigma_c^2}}\right). \tag{A.13}$$

Following Eq. (4.7)

$$\tilde{A} = \Phi\left(\frac{\mu_c - \mu_p}{\sigma_c}\right) \tag{A.14}$$

Therefore

$$\tilde{A} = A \iff \frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\,(1-2\beta)+\sigma_c^2}} = \pm\frac{\mu_c - \mu_p}{\sigma_c}. \tag{A.15}$$

We then obtain

$$\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\,(1-2\beta)+\sigma_c^2}} = \pm\frac{\mu_c - \mu_p}{\sigma_c} \iff \sigma_c = \pm\sqrt{\sigma_p^2\,(1-2\beta)+\sigma_c^2} \iff \beta = \frac{1}{2}. \tag{A.16}$$

■

B Log-normal model sensitivity to the copula model

In general, within the methodology presented, the copula model choice may affect the estimated absolute mobility. In order to demonstrate that as long as the rank correlation is the same the copula model effect on estimated absolute mobility is, in practice, insignificant, we compare four copula models – Gaussian, which is the copula in the bivariate log-normal model, as well as the Clayton, the Gumbel and the Plackett copula families (Trivedi and Zimmer, 2007; Bonhomme and Robin, 2009). In their study of relative mobility in France, Bonhomme and Robin (2009) argue that the Gaussian copula "tends to underestimate the dependence in the middle of the distribution, that is, the probabilities of remaining in the second, third, and fourth quintiles" and show that the empirical copula is best estimated by the Plackett copula.

Figure 10 demonstrates that the differences between the absolute mobility estimates when using different copula models, while assuming the same rank correlations, are negligible. The average difference between each of the time series was less than 1 percentage point, *i.e.* an effect of less than 2%.

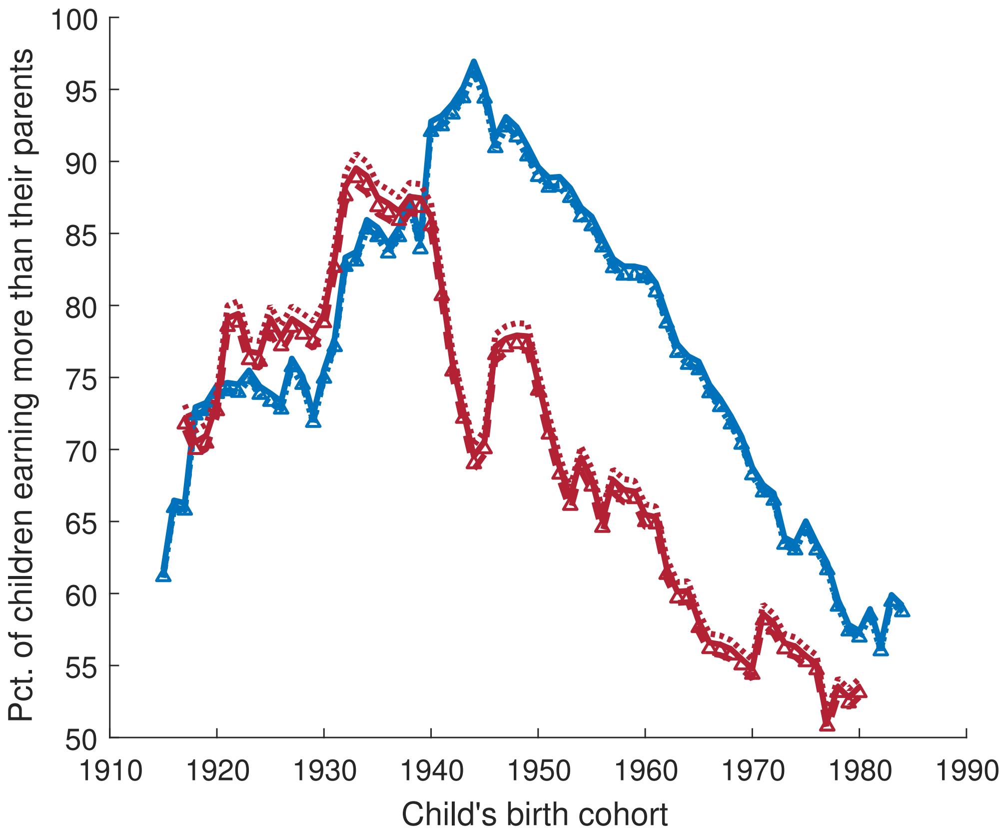

Figure 10: The copula model effect on the absolute mobility in France (blue) and the United States (red). The copula models used were Gaussian (solid lines), Clayton (dashed), Gumbel (dotted) and Plackett (triangles).