Demographic Change and Intergenerational Wealth Transmission

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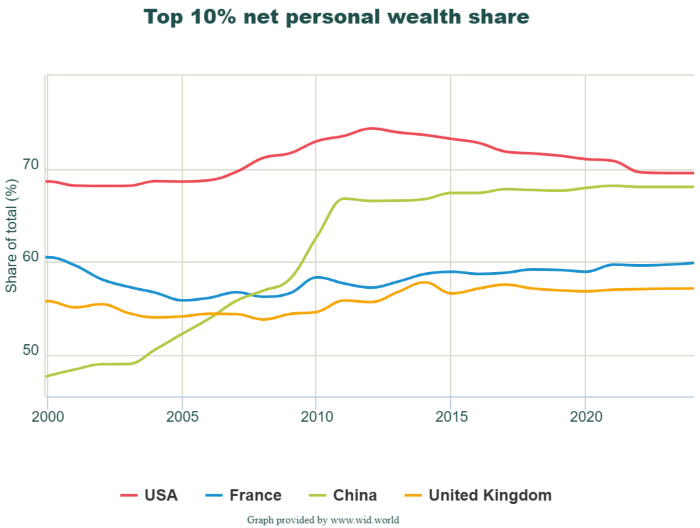
Outline

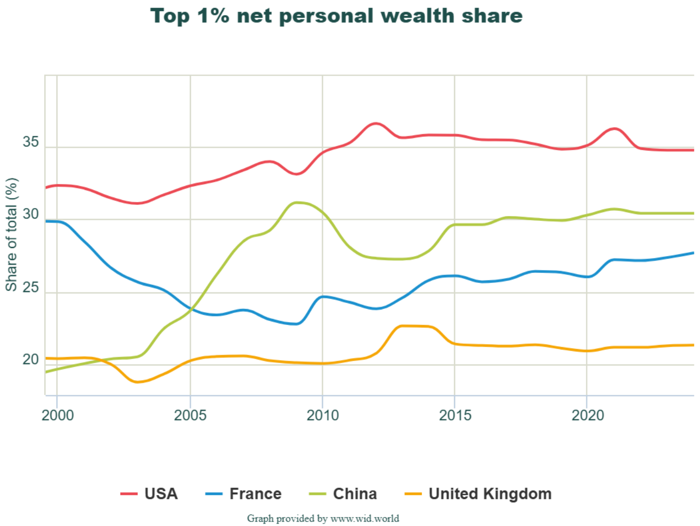
Why a concern with wealth?

- Important component of individual wellbeing
  - housing ownership
  - security in old age
- Core of political economy questions
  - wealth and power
  - the focus on the top 1% (Alvaredo et al. 2013 , Mankiw 2013)
- Key to long-run inequality
  - asset ownership at heart of models
- Changing inequality patterns over recent years

Wealth share of the rich



Wealth share of the very rich



Source: World Inequality Report 2018, 4.2.1, http://wir2018.wid.world

Equilibrium distribution in practice?

- Long-term evidence suggests periods of equilibrium
  - With abrupt changes from world events (Piketty and Zucman 2015)
- Effect of shocks?
  - across the board: recessions, booms
  - distributional: income, wealth inequality
- Shocks from policy?
  - equilibrium may still be relevant
  - give picture of the long run

Main theme

- Much of the literature focuses on the effect of market forces:
  - upper tail – role of financial asset prices
  - other key assets such as houses
  - lower tail – extent to which poor are credit constrained.
- Focus on non-market forces underlying distribution of wealth
  - forces dividing wealth: gifts, bequests from parents to children
  - forces uniting wealth: marriage
- Also consider the effect of outside intervention

Literature: approaches to family factors

- Literature: assumptions about family composition?
  - Atkinson (1980),Blinder (1976): all families have two children
  - Stiglitz (2015): asexual; same number of children for all
- Literature: equilibrium analysis?
  - assume equilibrium distribution (Banerjee and Newman 1991,Galor and Zeira 1993,Laitner 1979)
  - a characteristic of the equilibrium distribution like its variance (Atkinson 1980)
  - simulate over limited number of generations (Blinder 1976)
- The approach here:
  - families are heterogeneous in size
  - people differ in length of life
  - discuss the equilibrium wealth distribution

Thinking and models

- Why models?
  - help to develop arguments
  - even if not written down
  - writing it down can help clarify
- Models and economics
  - many economic models unnecessarily complicated
  - issue today is **necessarily** complicated
- Modelling requirements
  - clarity of purpose
  - clarity about components

Modelling components

- **Time**
  - discrete (counting days, years, generations)
  - continuous (any length)
  - both (?)
- **People**
  - who are taking decisions?
  - what are their motivation?
- **Mechanisms**
  - what implements the actions?
  - what is the driving force?

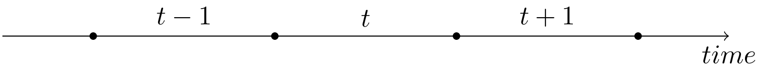
Time and families

- Time:
  - People can live for 3 ages (1 young, 2 middle age, 3 old age)
  - Children in period $t-1$ become adults in period t
  - Adults live for 1 or 2 ages; survive to old age with probability π_2
- **Adults**
  - Each family has two adults: take decisions jointly, pool wealth
  - Have at least one child, but no more than K
- **Children**
  - Proportion of families with k children: $p_k < 1$, $\sum_{k=1}^{K} p_k = 1$
  - Population stationarity, $\sum_{k=1}^{K} kp_k = 2$

Maximisation problem: 1

- People choose:
  - B_t : bequests
  - C_t : consumption in period 2
  - C'_t : consumption in period 3
  - E_t : earnings ($\frac{\bar{E}-E_t}{\bar{E}}$ is leisure)
- Assume annuity A_t purchased in period 2
  - perfect insurance against uncertain length of life
  - means survival into period 3
  - fair annuity: $A_t = \pi_2 C'_t$
- Wealth acquired per adult given by $W_t = E_t + I_t$
  - $I_t \geq 0$: Inheritance received
- Budget constraint:
  - $C_t + \frac{A_t+B_t}{1+r} \leq W_t$
  - $C_t + \frac{\pi_2 C'_t}{1+r} + \frac{B_t}{1+r} \leq W_t$
  - r: interest rate

Timeline



Age	0	Young	30	Middle age	60	Old	90
Period of life		1		2		3	
Surv prob	1		π_1		π_2		0
Inheritance			I_t				
Decisons				C_t, E_t, B_t, A_t			

Maximisation problem: 2

- Utility function: $$\gamma \ln\left(B_t+\bar{B}\right) + [1-\gamma]\left[\ln\left(C_t-\bar{C}\right)+\delta\pi_2\ln\left(C_t'-\bar{C}\right)\right] + \nu\ln\left(\frac{\bar{E}-E_t}{\bar{E}}\right)$$
- Parameters
    - γ: relative weight put on bequests rather than own consumption
    - δ: relative weight put on future consumption relative to present
    - ν: weight put on leisure.
    - $\bar{B}\geq 0$ captures the potential base aversion to altruism
    - $\bar{C}\geq 0$: precommitted consumption in each period
    - $\bar{E}>0$: maximum possible earnings during middle age
- Problem is maximise utility subject to...
    - budget constraint: $C_t+\frac{\pi_2 C_t'}{1+r}+\frac{B_t}{1+r}\leq W_t$
    - constraints on variables: $B_t, C_t, C_t'\geq 0$; $0\leq E_t\leq\bar{E}$

Solution

- The solution has two cases, determined by the size of inheritance
  - critical value of inheritance, $\hat{I}$.

Case 1: $I_t \geq \hat{I}$. For high inheritance $E_t = 0$

Case 2: $I_t < \hat{I}$. For low inheritance $E_t > 0$

- Critical value of inheritance, $\hat{I}$ depends on
  - preference parameters $\overline{B}, \overline{C}, \nu, \gamma$
  - survival probability π_2
  - external factors $\overline{E}$, interest rate r
- $\hat{I} := \frac{\xi}{\nu}\bar{E} - \frac{\overline{B}}{1+r} + \left[1 + \frac{\pi_2}{1+r}\right]\overline{C},$
  - $\xi := 1 + [1-\gamma]\,\delta\pi_2$
- Examine detailed solution in the two cases...

Case 1 (high-inheritance) solution

- $E_t = 0$
- If no-one survives to the third age

$$\begin{aligned} C_t &= [1-\gamma]\left[I_t + \frac{\bar{B}}{1+r}\right] + \gamma \bar{C} \\ B_t &= \max\left\{[1+r]\gamma I_t - [1-\gamma]\bar{B} - \gamma[1+r]\bar{C}, 0\right\}. \end{aligned}$$

- In general

$$\begin{aligned} C_t &= \frac{1-\gamma}{\xi}\left[I_t + \frac{\bar{B}}{1+r}\right] + \left[1 - \frac{1-\gamma}{\xi}\left[1 + \frac{\pi_2}{1+r}\right]\right]\bar{C} \\ B_t &= \max\left\{\frac{[1+r]\gamma}{\xi} I_t - \frac{[\xi-\gamma]}{\xi}\bar{B} - \frac{\gamma}{\xi}[1+r+\pi_2]\bar{C}, 0\right\} \end{aligned}$$

- If the parameters $\bar{B}$ and $\bar{C}$ were zero:
  - consumption, bequests proportional to inheritance

Case 2 (low-inheritance) solution

- In general

$$
\begin{aligned}
E_t &= \frac{\xi \bar{E} - \nu \left[I_t + \frac{\bar{B}}{1+r} - \left[1 + \frac{\pi_2}{1+r}\right] \bar{C} \right]}{\xi + \nu}, \\
C_t &= \frac{1-\gamma}{\xi+\nu} \left[I_t + \bar{E} + \frac{\bar{B}}{1+r} \right] + \left[1 - \frac{1-\gamma}{\xi+\nu} \left[1 + \frac{\pi_2}{1+r} \right] \right] \bar{C}, \\
B_t &= \max \left\{ \frac{[1+r]\gamma}{\xi+\nu} [I_t + \bar{E}] - \frac{[1-\gamma][1+\delta\pi_2] + \nu}{\xi+\nu} \bar{B} \right. \\
&\quad \left. - \frac{\gamma}{\xi+\nu} [1 + r + \pi_2] \bar{C}, 0 \right\}
\end{aligned}
$$

Comparative statics of individual

- Demographic changes affect decisions in two ways.
    - children from larger families get a lower inheritance
    - longevity is associated with an increased π_2
- Inheritance effect
    - $\frac{\partial C_t}{\partial I_t} > 0, \frac{\partial B_t}{\partial I_t} > 0$
    - (in case 2) $\frac{\partial E_t}{\partial I_t} < 0$
- Longevity effect
    - $\frac{\partial C_t}{\partial \pi_2} < 0, \frac{\partial B_t}{\partial \pi_2} < 0, \frac{\partial \hat{I}}{\partial \pi_2} > 0$
    - (in case 2) $\frac{\partial E_t}{\partial \pi_2} > 0$

** How the model works, individual level

- Common-sense responses to two big changes
- Effect of greater inheritance
  - increases consumption
  - increases later bequest
  - reduces earnings, if the person is working
- Effect of greater longevity
  - reduces consumption
  - reduces later bequest
  - increases critical inheritance level
  - increases earnings, if the person is working

Simple inheritance mechanics

- Child will be a worker if $I_{t+1} < \hat{I}$
- Get a corresponding value of wealth $\hat{W}$:
  - condition for a low inheritance is $W_t < \frac{k}{2\beta}\hat{W}$
  - where $\beta := \gamma[1+r]$
  - "bequest factor" = bequest-preference-weight $\times$ growth factor
- Child's wealth in the two cases is:

  high inheritance $W_{t+1} = I_{t+1} = \frac{2}{k}B_t$

  low inheritance $W_{t+1} = \frac{2}{k}B_t + E_t$
- Use this with the equation for I_{t+1} to get a fundamental relation

** Simple inheritance mechanics

- Child will be a worker if inheritance is too low (below $\hat{I}$)
  - corresponding critical value of wealth $\hat{W}$
  - compute this from $\hat{I}$
- You get low inheritance if...
  - ...$\hat{W}$ is high (because $\hat{I}$ is high)
  - ...you're from a big family (large k)
  - ...there's a low bequest factor (small β)
- In the two inheritance cases we have:

  high inheritance: child's wealth is proportional to bequests
  low inheritance: child's wealth is linear in bequests
- So wealth of gen $t+1$ is linked to wealth of gen t

Wealth dynamics and inheritance

- There's a parent-to-child wealth relation $W_{t+1} = g_k(W_t)$

  high inheritance : $g_k(W_t) = \frac{2\beta}{k}\left[\frac{W_t - \hat{W}_0}{\xi}\right]$

  low inheritance : $g_k(W_t) = \frac{\nu\hat{I}}{\xi+\nu} + \frac{2\beta}{k}\max\left\{\frac{W_t - \hat{W}_0}{\xi+\nu}, 0\right\}$
- Here $\hat{W}_0$ is valuation of the pre-commitment items $\bar{B}, \bar{C}$
- Each g_k is a line in (W_t, W_{t+1})-space
- The line has two kinks, at lower and upper values of W_t:

lower $W_t = \hat{W}_0$: divides zero- and positive-inheritance zones

upper $W_t = \hat{W}_k$: divides low- and high-inheritance zones

Cut-down version with $\pi_2 = \bar{B} = \bar{C} = 0$

- A restricted model: only two periods (Cowell and Van de gaer 2025)
- Let p_k be prop of families with k children
  - family structure $\mathbf{p} = \{p_1, ..., p_K\}$ defines a Markov process
  - will there be an equilibrium of the process?
  - if so, what will it look like?

Theorem: for all $\mathbf{p}$: (1) a globally stable equilibrium exists if and only if $0 \leq \beta \leq 1$. (2) in equilibrium, there is a non-zero lower bound on wealth

- Two scenarios in equilibrium depend on the bequest factor β:
  1. if $0 \leq \beta \leq 1/2$: a finite upper bound to wealth; everybody works
  2. if $1/2 < \beta \leq 1$: no finite upper bound: some rentiers are present

The process g_k and equilibrium (cut-down version)

- Focus on scenario 2, where there are people who do not work
    - simplifies the analysis
    - gives us a strikingly clear result
- What happens to top end of the wealth distribution?
    - the rentier (idle rich) part
    - mechanics are given by $W_{t+1} = g_k(W_t) = \frac{2\beta}{k} W_t$
- Equilibrium requires $F_*(W) = \sum_{k=1}^{K} \frac{kp_k}{2} F_*\left(\frac{k}{2\beta} W\right)$
    - focus on the interval $\mathbb{W}_1 := \left[\frac{K\hat{W}}{2\beta}, \infty\right)$

Equilibrium distribution (cut-down version)

- Focusing on $\mathbb{W}_1$ gives clear result on shape of the distribution

Theorem: the equilibrium distribution must satisfy $F_*(W) = 1 - AW^{-\alpha}$ where A is a constant and α is a root of the equation $\beta^{-\alpha} = \sum_{k=1}^{K} p_k \left[\frac{k}{2}\right]^{1-\alpha}$

- Interpretation
  - in equilibrium we have a Pareto distribution!
  - the higher is α, the lower is inequality
- What drives inequality?
  - the family structure $\mathbf{p} = \{p_1, ..., p_K\}$
  - in particular p_1, the proportion of "little emperors"

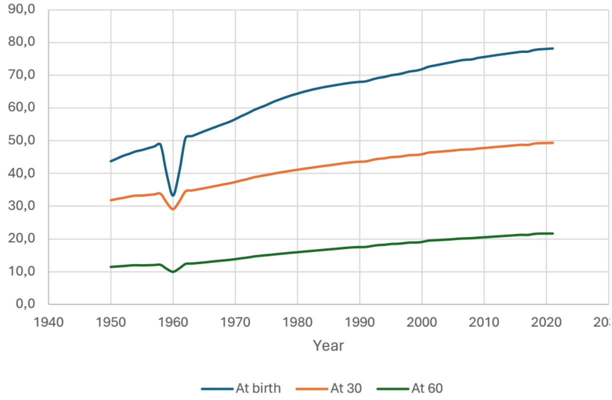
The g_k relation (general)

- There's a kinked line $W_{t+1} = g_k(W_t)$ for every $k = 1, \ldots, K$
- Each of these lines has two kinks:
  lower ($W_t = \hat{W}_0$): between 0-inheritance and positive-inheritance workers
  upper ($W_t = \hat{W}_k$): between workers (low-inheritance) and rentiers (high-inheritance) zones
- If slope of the $g_1(W_t)$-line is above 1, no upper bound on wealth
  - all W can be reached through a succession of one-child families
- Lower-bound wealth $\underline{W}$ is where $W_t = g_K(W_t)$
- If the probability π_2 increases
  1. kink point of every $g_k(\cdot)$-function moves to the northeast
  2. slope of the rentier branch of the $g_k(\cdot)$-function decreases
  3. intercept of the worker branch of the $g_k(\cdot)$-function increases.

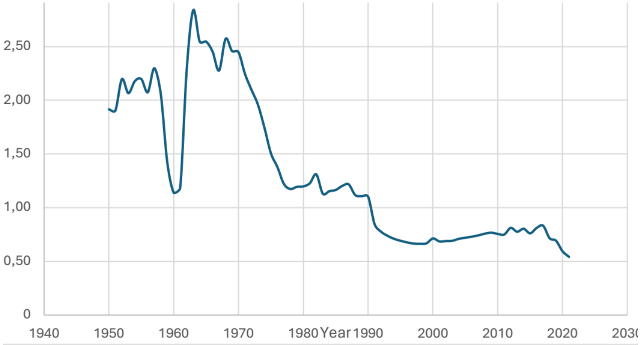
An application

- Important to found the analysis on reality
  - focus on current snapshot distribution isn't appropriate
  - model results are about *equilibrium* distribution
- We don't observe equilibrium
  - can model it from real world data
  - also model out-of equilibrium behaviour
  - use simulation approach
- What basis for a simulation exercise?
  - need big changes in π values
  - need big changes in $\mathbf{p}$ distribution
  - an important country that fits these?
- China

China: life expectancy



China: net reproduction rate



See Wang et al. (2016), Zhang (2017)

Simulation: method and parameters

- Start from an arbitrary W-distribution for 100,000 households
  - simulate the behaviour of the following generations
  -

Basic parameters			
γ	0.4	δ	1
ν	0.3	$\bar{B}$	0.5
$\bar{C}$	0.05	$\bar{E}$	1
r	1.8068		

.

Implied parameters			
$\hat{I}$	4.004	$\hat{W}_1$	2.64
β	0.842	$\hat{W}_2$	4.84
$\hat{W}_0$	0.43	$\hat{W}_3$	7.05

- Take as benchmark Chinese data before and during the One Child Policy

Simulation: China data

(a) Survival probability after period 2

pre-OCP		OCP	
π_{2b}	0.396	π_{2a}	0.631

(b) Distribution of the number of children per woman

pre-OCP			OCP		
# Children	p_{bi}	Cum Freq	# Children	p_{ai}	Cum Freq
0	0	0	0	0	0
1	0.04800	0.0480	1	0.4664	0.4664
2	0.10500	0.1530	2	0.4198	0.8862
3	0.17700	0.3300	3	0.0928	0.9790
4	0.21199	0.5420	4	0.0161	0.9951
5	0.18785	0.7299	5	0.0037	0.9988
6	0.13086	0.8607	6	0.0011	0.9999
7	0.07528	0.9360	7	0.0001	1
8	0.03755	0.9735			
9	0.01648	0.9900			
10	0.00649	0.9965			
11	0.00231	0.9988			
12	0.00079	0.9996			
13	0.00025	0.9999			
14	0.00008	0.9999			
15	0.00003	1			
16	0.00001	1			

Unpacking the simulation: four scenarios

	$\mathbf{p}_a$	$\mathbf{p}_b$
π_{2a}	Scenario a	Scenario d
π_{2b}	Scenario c	Scenario b

a: (OCP): distribution #children $\mathbf{p}_a$, survival prob π_{2a}
b: (pre-OCP): distribution of #children $\mathbf{p}_b$, survival prob π_{2b}
c: counterfactual where only distribution #children changes
d: counterfactual where only survival prob changes

- Journey from *b*efore to *a*fter can be seen as

  either $b \to c$ (OCP effect) then $c \to a$ (ILE effect)

  or $b \to d$ (ILE effect) then $d \to a$ (OCP effect)

- Decompose the historical change from b to a as follows:

  OCP effect: $1/2$ effect $b \to c$ + $1/2$ effect $d \to a$

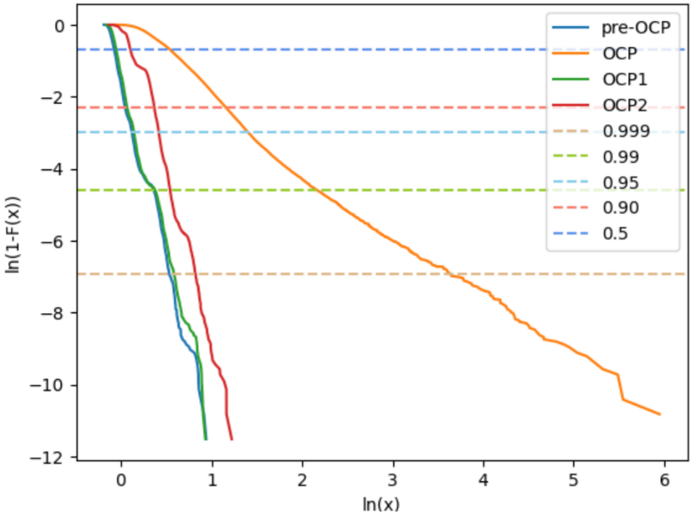
  ILE effect: $1/2$ effect $b \to d$ + $1/2$ effect $c \to a$

Effects of demographic changes $b \to a$

	pre-OCP b	OCP1	OCP2	OCP a
average W	0.954	0.967	1.184	2.201
average E	0.740	0.762	0.717	0.542
average I	0.214	0.214	0.467	1.659
% rentiers	0	0	0	3.9
corr (E, I)	-1	-1	-1	-0.418
share bot 50%	0.469	0.469	0.441	0.316
share top 10%	0.124	0.123	0.133	0.279
share top 1%	0.013	0.017	0.016	0.100
Gini W	0.047	0.047	0.086	0.284

- pre-OCP: long-run equilibrium *b*efore the policy
- OCP1/OCP2: situation after one/two-generation transition
- OCP: long-run equilibrium *a*fter the policy

Equilibrium wealth distributions



Effects of demographic change in China: summary

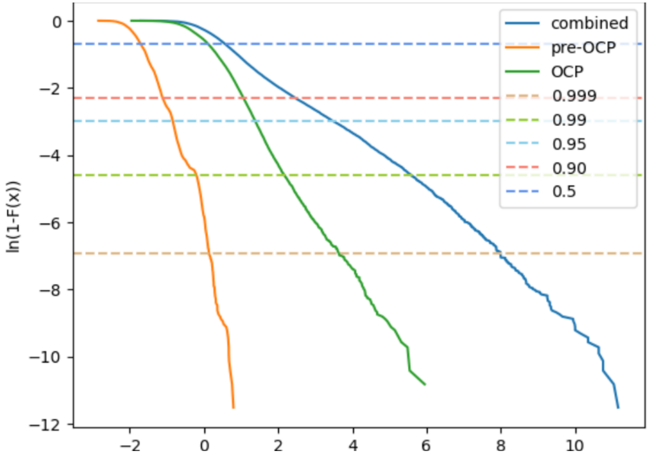
- Important effects after two generations
- Smaller size of families: children receive larger inheritances
- Average inheritances increase from pre-OCP value
  - lowers labour supply
- Gini coefficient almost doubles. Pareto line flattens
- Much reinforced in long-run equilibrium

Decomposition of $b \to a$ changes

	Total	OCP	ILE
average W	1.247	23.813	−22.566
median W	0.795	1.288	−0.493
Gini W	0.246	0.572	−0.326
average E	−0.197	−0.339	0.142
average I	1.444	24.152	−22.708
% rentiers	3.8	18.0	−14.2

- OCP, ILE effects on changes are opposed
- The OCP effect outweighs the ILE effect everywhere

Equilibrium wealth decompositions



OCP effect: shift from "pre-OCP" to "combined" curve

ILE effect: the shift from "combined" to "OCP" curve

Conclusions

- A three-age model gives enough flexibility:
  - to model major life decisions
  - to represent major demographic effects
  - to construct a full family model
- The model leads to an equilibrium distribution
  - depends on k-distribution and bequest factor β
  - takes the Pareto form in the upper tail
  - little emperors increase equilibrium inequality
- The China simulation
  - OCP and ILE have opposing effects
  - OCP is much stronger than ILE:
  - ...inheritances increase dramatically
  - ...earnings decrease somewhat
  - ...lifetime wealth increases
  - ...inequality in lifetime wealth increases substantially

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