When do the parameters of aversion to income inequality and aversion to rank inequality provide the same social welfare assessments?

****

The theory and empirical evidence from the Luxembourg Income Study database.

Stanislaw Maciej Kot & Piotr Paradowski

Paper prepared for the International Conference

"*Income and Wealth Inequality: Drivers and Consequences*"

Gdansk, September 27-29, 2023

THE AIM OF THE PAPER

- Developing the theoretical background of consistent assessments of social welfare when social welfare functions (SWFs) are implied either by the Atkinson index, $\mathbf{A_\varepsilon}$, or by generalised Gini indexes **Gv.**
- The normative parameter ***ε*** reflects *an aversion to income inequality*
- $\mathbf{A_\varepsilon}$ implies
    - $\mathbf{SWF_\varepsilon = \mu(1-A_\varepsilon)}$ **(1)**
- The normative parameter *v* reflects an *aversion to rank inequality*
- $\mathbf{G_v}$ implies
    - $\mathbf{SWF_v=\mu(1-G_v)}$ (2)
- $\mathbf{SWF_\varepsilon}$ and $\mathbf{SWF_v}$ are cardinalisations of welfare functions

MOTIVATION

There is inconsistency in assessing social welfare

Two sources of inconsistency :

1. Due to different methodologies, i.e. based on SWF_{ε} or SWF_{v}
2. Due to an unknown range of values of ε and v within each of the methodologies.

Some preliminaries…

- Welfare economics seeks an answer to whether a given policy provides a higher (lower) economic welfare of society than an alternative policy (Kakwani and Son, 2022, p.95).
- A social welfare function describes how individuals' economic welfare is aggregated into the economic welfare of society. SWF specifies normative judgments by assigning weights to individuals (ibidem, p. 95).
- *SWF* has a cardinal representation in the form of the *equally distributed equivalent income* (EDEI) (Kolm, 1969; Atkinson, 1970; Sen, 1973).
- EDEI is "…the level of income which, if distributed equally to all individuals, would generate the same welfare (average utility) as the existing distribution." (Lambert, 2001,p.95).

Who assesses social welfare embodied in a given income distribution?

- Economic theory delegates the assessment of social welfare to an abstractive *social decision-maker* (SDM).
- Every SDM has an individual Social Evaluation Function, also called the Social Welfare Function (SWF).
- Every member of a society may play the role of SDM.
- Thus, there can be as many distinct SWFs as society members (Champernowne & Cowell, 1998, p.88).

Social decision maker averse to income inequality

- Let the positive valued random variable X with the distribution function F(x) and a finite mean μ describe a society's income distribution.
- Let $\mathbf{\{SDM_{\varepsilon}\}_{\varepsilon\epsilon(0,\infty)}}$ be the family of **SDMs** who are averse to income inequality.
- Assume the individual welfare (utility of income) function of the form
  - $$\boldsymbol{u(x) = \begin{cases} \frac{x^{1-\varepsilon}}{1-\varepsilon}, for\ \varepsilon \neq 1 \\ lnx, for\ \varepsilon = 1 \end{cases}}, x>0$$
- The EDEI, ξ_{ε}**,** is a solution to the equation u(ξ_{ε})=E[u(X)]
- The Atkinson (1970) inequality index
  - $$\boldsymbol{A_{\varepsilon} = \frac{\mu-\xi_{\varepsilon}}{\mu}} \qquad \textbf{(3)}$$
- EDEI, ξ_{ε} is a cardinal measure of SWF_{ε}
  - $$\xi_{\varepsilon} = \mu(1 - A_{\varepsilon}), \quad \varepsilon>0 \qquad (1')$$

Social decision maker averse to rank inequality

- $G_{v,}$, the generalised Gini index is defined as:
  - $$\boldsymbol{G_v = 1 - v(v-1)\int_0^1 (1-p)^{v-2} L(p)\,dp}$$, **v>1, p ϵ [0,1]** **(4)**
- where *L(p)* is the Lorenz curve and the normative parameter *v* reflects *aversion to rank inequality.* For *v*=2, G_2 is the ordinary Gini index.
- Let $\{\mathbf{SDM_v}\}_{v\epsilon(1,\infty)}$ be the family of **SDMs** averse to rank inequality.
- $\mathbf{SDM_v}$ evaluates income distribution with SWF_v of the form
  - ξ_v=μ(1-G_v), v>1 (2')
- where ξ_v is EDEI, i.e. a cardinal measure of SWF_v.

A CONSISTENT WELFARE ASSESSMENT

- All SDM_{ε}s and SDM_{v}s may offer any assessments of social welfare.
- The question is: „Among all social decision-makers, are there the pairs

(SDM_{ε}, SDM_{v}) providing a consistent assessment of social welfare?”

- By consistent assessment of welfare we mean that:
  - $SWF_{\varepsilon} = SWF_{v}$ (5)
- Using (1) and (2), Eq. (4) becomes
  - $\mu(1 - A_{\varepsilon}) = \mu(1 - G_{v})$, $\varepsilon > 0$, $v > 1$ (6)
- or, equivalently
  - $A_{\varepsilon} = G_{v}$ (7)
- Notice that searching for a pair (SDM_{ε}, SDM_{v}) is equivalent to searching for the pair (ε, v).
- We will see that the problem in question resembles the problem of entanglement of particles in Quantum Physics.

WELFARE ENTANGLEMENT

When a person is randomly selected as **SDM**, we don't know if the person is $\mathbf{SDM_{\varepsilon}}$ or $\mathbf{SDM_{v}}$.

- If she were $\mathbf{SDM_{\varepsilon}}$, her assessment would be $\mathbf{SWF_{\varepsilon}} = \boldsymbol{\mu}(\mathbf{1} - \boldsymbol{A_{\varepsilon}})$
- If she were $\mathbf{SDM_{v}}$, her assessment would be $\mathbf{SWF_{v}} = \boldsymbol{\mu}(\mathbf{1} - \boldsymbol{G_{v}})$

Welfare entanglement. For a consistent welfare assessment, we should impose the condition:

$$\mathbf{SWF_{\varepsilon}} = \mathbf{SWF_{\mathit{v}}}$$

or equivalently

$$\boldsymbol{A_{\varepsilon}} = \boldsymbol{G_{v}}$$

This condition generates the pairs $(\mathbf{SDM_{\varepsilon}}, \mathbf{SDM_{v}})$ such that if we know ε, we **automatically** know *v*

Thus, a solution to this nonlinear equation creates *the **welfare entanglement*** of $\mathbf{SDM_{\varepsilon}}$ and $\mathbf{SMD_{v}}$

Non-entangled **SDMs** may offer any assessments of social welfare.

QUANTUM ENTANGLEMENT

Quantum superposition is the idea that particles exist in multiple states at once.

When a measurement is performed, it is as if the particle selects one of the states in the superposition.

Quantum entanglement is the phenomenon that occurs when a pair of generated particles, or a group of particles, interact in a way such that if one quantum state of a particle is known, the quantum state of any entangled particles is known **automatically**.

The non-entangled particles may have any quantum states

EXAMPLES OF WELFARE ENTANGLEMENT

Fig.1. The welfare entanglement of SDM_{ε} & SDM_{v} in Brazil 2016: *ε* against *v*

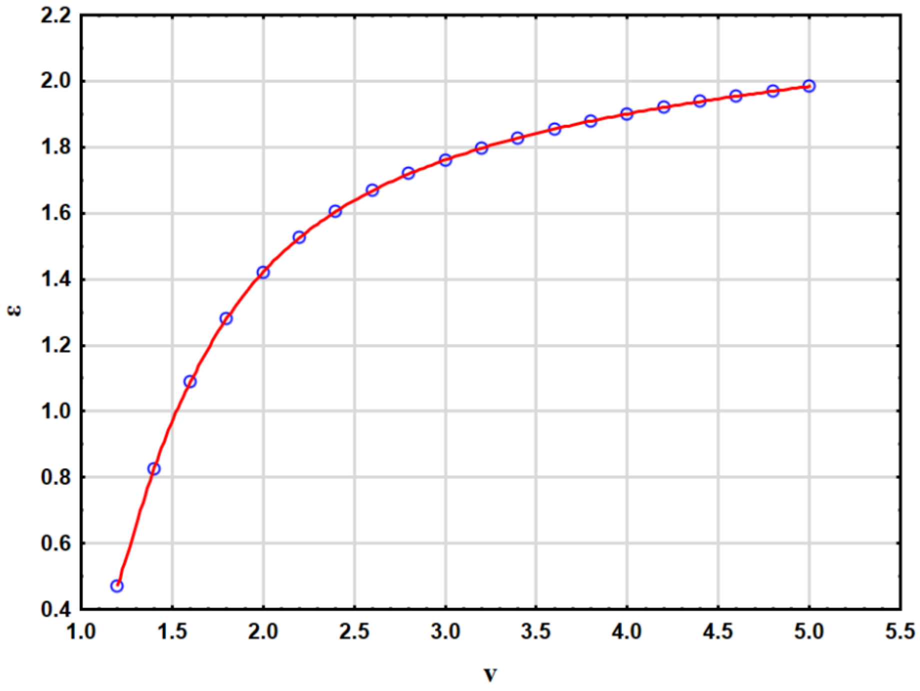
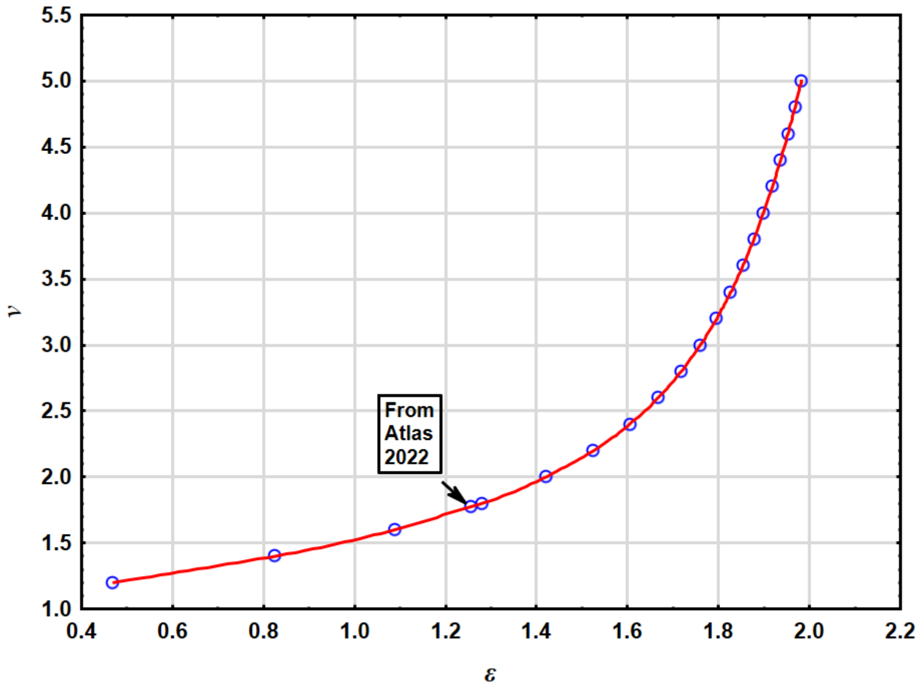


Fig.2. The welfare entanglement of SDM_{ε} & SDM_{v} in Brazil 2016: *v* against *ε*



In Fig. 1, the measured values of *v* reveal entangled *ε.* For instance, for *v*=2, *ε*= 1.42161.
In Fig.2, the measured values of *ε* reveal entangled *v.* For instance, for *ε*=1.2558 (from Kot & Paradowski, 2022) the entangled *v*=1.7745.

BENCHMARK ENTANGLEMENT

- Quantum systems can become entangled through various types of interactions.
- We now demonstrate an entanglement of social decision-makers based on *benchmark incomes* originated by Hoffman (2001) and independently put forward by Lambert and Lanza (2006) and Corvalan (2015).
- Imagine an unequal two-person society, with incomes $x_1 < x_2$. Rising x_1 by a small amount, which does not change the rank, results in falling inequality. On the other hand, if x_2 rises, inequality will also rise. Thus, for an *n*-member society, a specific income level, say x^*, dividing these effects, must exist.
- Hoffman (2001) called x^* as „a relative poverty line”.
- Lambert and Lanza (2006) prove the existence of x^* -called by the authors *the benchmark income*- for a general class of inequality measures.

BENCHMARK ENTANGLEMENT (cont.)

- For the Atkinson index (3), the benchmark income, x_ε^*, has the form:
  - $x_\varepsilon^* = \begin{cases} \mu(1 - A_\varepsilon)^{(\varepsilon-1)/\varepsilon}, \; for \; \varepsilon \neq 1 \\ \mu, for \;\; \varepsilon = 1 \end{cases}$ (8)
- For the generalised Gini index G_v (4), the benchmark income x_v^* is
  - $x_v^* = F^{-1}\left(1 - [(1 - G_v)/v]^{1/(v-1)}\right)$ (9)
- Foster and Székely (2000, 2008) proposed the Atkinson index A_ε growth elasticity for computing pro-poorness.
- Lambert and Lanza (2006) demonstrated that "… all growth taking place entirely below x_ε^* counts as pro-poor, whilst growth taking place entirely above x_ε^* may or may not do so, depending on its effect on μ."
- This property holds for any assumed poverty line.

BENCHMARK ENTANGLEMENT (cont.)

- The following condition will guarantee a consistent evaluation of benchmark incomes by SDM_ε and SDM_v:
  - $x_\varepsilon^* = x_v^*$ (10)
- Notice that Eq. (10) imposes the *benchmark entanglement* between **SDM_ε** and **SDM_v**. Fig. 3 illustrates such an entanglement for Brazil in 2016.

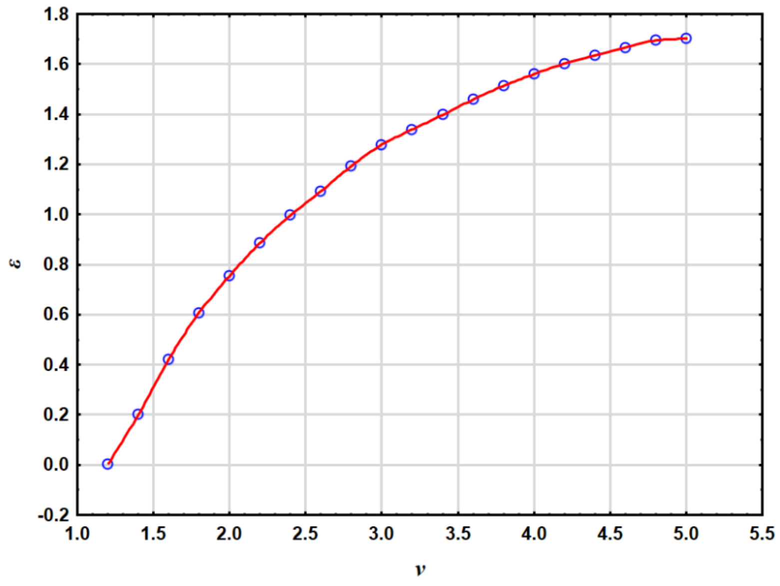


Fig. 3. The benchmark entanglement of SDM_ε and SDM_v in Brazil 2016.

A double-entangled social decision-makers

- Now, we ask whether a randomly selected social decision-maker can be both *welfare-entangled* and *benchmark-entangled*. If so, the following system of nonlinear equations should have a unique solution with respect to ε and v, namely:
  - $$\begin{cases} A_\varepsilon = G_v \\ x_\varepsilon^* = x_v^* \end{cases}$$, for $\varepsilon \geq 0$ and $v > 1$ (11)
- The system (11) can be solved numerically.
- Within an economic framework, the unique solution (ε^*, v^*) to the system (11) identifies a *single pair* of social decision-makers who consistently assess social welfare and benchmark incomes.
- The pair (ε^*, v^*) may serve as a *standard of value judgements*. Analysts should use normative parameters ε^* and v^* when assessing social welfare, inequality and other distributional issues.

EMPIRICAL ILLUSTRATION OF THE DOUBLE ENTANGLEMENT

Country	Year	v*	ε*	$GA_{v\varepsilon}$	EDEI	x*	z*
Brazil	2016	2.09173	1.47251	0.48859	5243	8267	0.80640
Chile	2017	3.23201	2.16581	0.59387	5856	8877	0.61568
Colombia	2016	2.45709	1.73716	0.54177	3930	6159	0.71809
Dominican Rep.	2007	2.56597	1.75826	0.59868	2850	4789	0.67454
Guatemala	2014	1.80579	1.68175	0.37928	3787	5028	0.82422
Mexico	2016	2.92947	1.95123	0.57503	3987	6182	0.65890
Panama	2016	1.86041	1.39508	0.42907	9684	14472	0.85323
Paraguay	2016	1.52746	1.01984	0.35954	7603	11768	0.99137
Peru	2016	1.50416	0.91867	0.30827	5935	8864	1.03317
Uruguay	2016	1.76528	1.75393	0.31875	9969	12408	0.84790

Source: Luxembourg Income Study (LIS) Database.

FINAL REMARKS

- The pair *(ε^*,v^*)* may serve as a *standard of value judgements*. Analysts should use normative parameters ε^* and v^* when assessing social welfare, inequality and other distributional issues.

- Following this ethical recommendation guarantees a consistent assessment of social welfare within the two distinct methodologies, a consistent assessment of pro-poor growth and avoiding paradoxical consequences of transfers.

Thank you