Heterogeneous-Agent Model of Household Mortgages in Luxembourg: Responses to the Covid-19 Shock

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Main problem

Covid-19 lockdowns

↓

Asymmetric income shocks

↙ ↘

less "social" professions **lighter**	more "social" professions **harder**

↘ ↙

Heterogeneity matters!

Utilizing cross-sectional information

Task

↓

Can we spot households that are "weak links"?

↙ ↘

less "social" professions	more "social" professions
lighter	**harder**

↘ ↙

Will house trades lead to a drop in house prices?

Will low house prices lead to bank balance-sheet weaknesses?

Main goal

- Create a **model that raises red flags** about **mortgage markets before data and standard regressions do**.

- **Complementary diagnostic tool, not substitute**.

Why?

Diagnosing risk early

$\downarrow$

Prevention policies

Why a model?

Laboratory for: ↗ **stress tests**
↘ **policy evaluation**

What we do

- **Dig into how house prices are formed: House prices $\longrightarrow$ affect collateral values in mortgage markets.**

model ambitions: $\nearrow$ **package and endogenize cross-sectional info & trades**

$\searrow$ **address expectations on cross-sectional dynamics**

Main model features

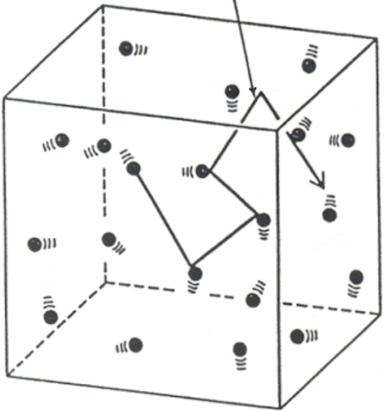
income process/distribution
interest rate
covid-19 income shock (+ shock expectations)

} → **exogenous**

mortgage distribution
homeownership/home-size distribution
net-worth distribution
house price and mortgage collateral
liquid-wealth distribution
consumption choices
impulse-response dynamics of all the above

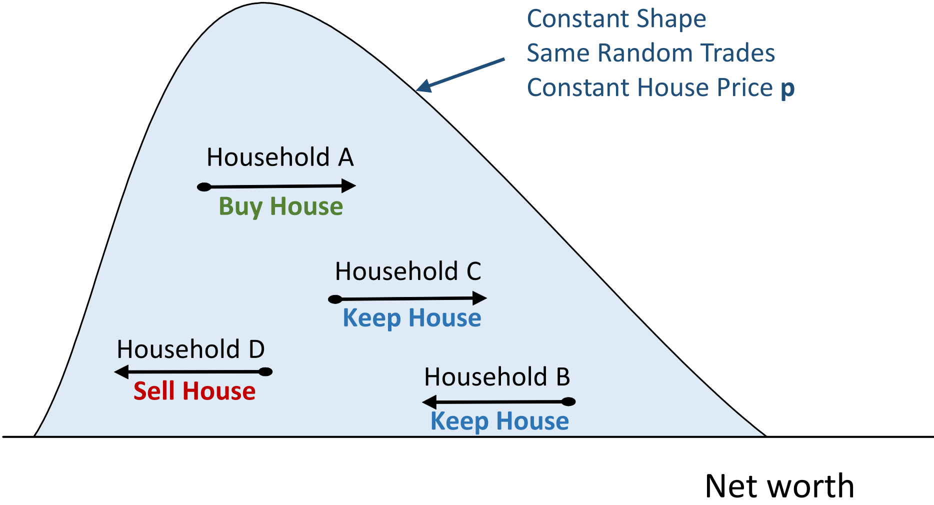
} → **endogenous**

Model overview: stationary equilibrium

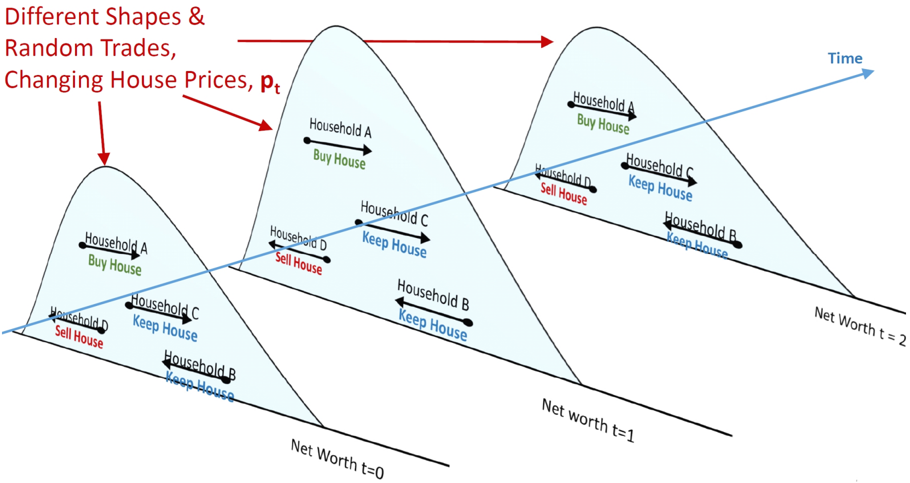


A gas may be pictured as a collection of widely spaced molecules in continuous, chaotic motion.

Model overview: stationary equilibrium



Model overview: distribution dynamics



Model

- Momentary utility: $U(c_t, h_t)$

$$\begin{aligned} c_t &\rightarrow \text{consumption} \\ h_t &\rightarrow \text{housing stock (e.g. square meters)} \end{aligned}$$

- Assets:

Two asset types:

↗ **housing stock, if homeowner;** $h_t > 0$

↘ **mortgage,** $b_t < 0$ **if** $h_t > 0$, **or liquid wealth;** $b_t \geq 0$ **if** $h_t = 0$

- For simplicity, no modeling of rental market and the interest rate, r, is the same for $b_t < 0$ and $b_t > 0$ and constant.

Model

- The **value of a house** is $p \cdot h$.

- A mortgage needs a **down payment**, calculated as a **constant proportion**, $\theta \in (0, 1)$, of the value of the house. Therefore, the **mortgage limit** is,

$$\boxed{-b_t \leq \theta p h_t} \quad (1)$$

- **Mortgages are collateralized and the house price matters!**

Model

- **We assume a local law of large numbers, that there are potentially infinite individuals located at any particular point.**

lumpy housing choice $\rightarrow h_t \in \{0\} \cup [h_{\min}, \infty)$

- **Despite that individuals may make lumpy, discrete adjustments in their housing units (e.g. moving from a larger house to a smaller house), we will focus on the adjustments of this locally aggregative housing market.**

Model

- Household problem (the only one in partial equilibrium):

$$\mathcal{P}_H \begin{cases} \max\limits_{(c_t,b_t,h_t)_{t\geq 0}} E_0\left[\int_0^\infty e^{-\rho t} U\left(c_t, h_t\right) dt\right] \\ \text{subject to:} \\ \dot{b}_t + p\dot{h}_t = rb_t + w_t - c_t \\ w_t \in \{w_1, .., w_J\} \text{ with } \Pr\{w_{t+dt} = w_k \mid w_t = w_j\} = \lambda_{j,k} dt \\ -b_t \leq \theta p h_t \\ h_t \in \{0\} \cup [h_{\min}, \infty) \\ \text{given } h_0 \in \{0\} \cup [h_{\min}, \infty) \quad , \quad b_0 \in \mathbb{R} \\ \lim\limits_{t\to\infty} E_t\left(\lambda_t b_t\right) = 0 \end{cases}$$

Concern: too many dimensions

Model

- Reducing both the state-space dimensions and the choice-space dimensions.

Net worth a_t

$$\boxed{a_t \equiv b_t + ph_t} \tag{2}$$

$f(h_t)$ pecuniary value of owned housing services in terms of consumable good

$$\boxed{U(c_t, h_t) = u(q_t), \text{ with } q_t \equiv c_t + f(h_t)} \tag{3}$$

Model

- We obtain the branch function,

$$h(a) = \begin{cases} 0 & , \quad a < \frac{p}{\phi} h_{\min} \\ \frac{\phi}{p} a & , \quad a \in \left[\frac{p}{\phi} h_{\min} \, , \, \frac{p}{\phi} h_{\max} \right) \\ h_{\max} & , \quad a \geq \frac{p}{\phi} h_{\max} \end{cases}$$

- Let,

$$F(a) \equiv f(h(a)) - rph(a)$$

and the budget constraint becomes:

$$\boxed{\dot{a}_t = ra_t + w_t + F(a_t) - q_t} \tag{4}$$

Hamilton-Jacobi-Bellman (HJB) equation

- Let the value function that corresponds to the j-th gridpoint of w be denoted by $V_j(a)$. The HJB equation is given by,

$$\rho V_j(a) = \max_q \left\{ u(q) + V_j'(a)\left[ra + w_j + F(a) - q\right] + \sum_{k=1}^{J} \lambda_{j,k}\left[V_k(a) - V_j(a)\right] \right\}$$

- Solution:

$$s_j(a) \equiv ra + w_j + F(a) - \underbrace{q_j(a)}_{\text{optimal choice}}$$

Hamilton-Jacobi-Bellman (HJB) equation

- Matrix representation of HJB (finite-differences solution technique)

$$\frac{V_{i,j}^{n+1} - V_{i,j}^{n}}{\Delta} =$$

$$= -\rho V_{i,j}^{n+1} + U_{i,j}^{n} + \frac{V_{i+1,j}^{n+1} - V_{i,j}^{n+1}}{\Delta a} S_{F,i,j}^{+} + \frac{V_{i,j}^{n+1} - V_{i-1,j}^{n+1}}{\Delta a} S_{F,i,j}^{-}$$
$$+ \sum_{k=1}^{J} \lambda_{j,k} \left(V_{i,k}^{n+1} - V_{i,j}^{n+1} \right) \quad , \quad j = 1, \ldots J. \tag{5}$$

- Written in a compact way as:

$$\boxed{\frac{v^{n+1} - v^{n}}{\Delta} = -\rho v^{n+1} + u^{n} + \underbrace{\left(A_1 + \hat{A}\right)}_{\mathbf{A}} v^{n+1}} \tag{6}$$

Hamilton-Jacobi-Bellman (HJB) equation

- Matrix A_1 in (6) is an $(I \cdot J) \times (I \cdot J)$ matrix with array,

$$A_1 = \begin{bmatrix} x_1 & y_1 & 0 & \cdots & 0 \\ \psi_2 & x_2 & y_2 & \cdots & 0 \\ 0 & \psi_3 & x_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & x_N \end{bmatrix} \tag{7}$$

- Matrix $\hat{A}$ in (6) is an $(I \cdot J) \times (I \cdot J)$ matrix with array (here, $J = 2$),

$$\hat{A} = \begin{bmatrix} -\lambda_{1,2} & 0 & \underbrace{\cdots}_{I-2} & 0 & \color{red}{\lambda_{1,2}} & 0 & \cdots & 0 \\ 0 & -\lambda_{1,2} & \underbrace{\cdots}_{I-2} & 0 & 0 & \color{red}{\lambda_{1,2}} & \cdots & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \vdots & -\lambda_{1,2} & \vdots & \vdots & \vdots & \color{red}{\lambda_{1,2}} \\ \color{blue}{\lambda_{2,1}} & 0 & \underbrace{\cdots}_{I-2} & 0 & -\lambda_{2,1} & 0 & \cdots & 0 \\ 0 & \color{blue}{\lambda_{2,1}} & \underbrace{\cdots}_{I-2} & 0 & 0 & -\lambda_{2,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \color{blue}{\lambda_{2,1}} & 0 & \cdots & \cdots & -\lambda_{2,1} \end{bmatrix} \tag{8}$$

Asymptotic distribution of net worth

- Use Forward Kolmogorov equations (example $J = 2$, $g_j(a)$ is the conditional density if $w_t = w_j$):

$$0 = -\frac{d\left[s_1(a)\,g_1(a)\right]}{da} - \lambda_{1,2}g_1(a) + \lambda_{2,1}g_2(a)$$

and

$$0 = -\frac{d\left[s_2(a)\,g_2(a)\right]}{da} - \lambda_{2,1}g_2(a) + \lambda_{1,2}g_1(a)$$

Asymptotic distribution of net worth

- the asymptotic distribution $g(a) = (g_1(a), g_2(a))$ can be obtained through solving,

$$\boxed{0 = \mathbf{A}^T g} \tag{9}$$

with g being given by,

$$g = \begin{bmatrix} g_{1,1} \\ \vdots \\ g_{I,1} \\ - - - \\ \vdots \\ - - - \\ g_{1,J} \\ \vdots \\ g_{I,J} \end{bmatrix} \tag{10}$$

Market clearing: house prices

- A house price, p, clears the market if,

$$\sum_{k=1}^{J} \int_{h_{\min}}^{\infty} h \left[\int_{-\boldsymbol{\theta} \mathbf{ph}}^{\infty} g_k\left(h, b\right) db \right] dh = \underbrace{H^S}_{\text{aggregate housing supply}}$$

with $g_k\left(h, b\right)$ denoting the conditional joint distribution of housing and mortgage/liquid assets $\left(h, b\right)$, conditional on having income w_k.

Model

- Income process in discrete time (Guvenen et al., 2020):

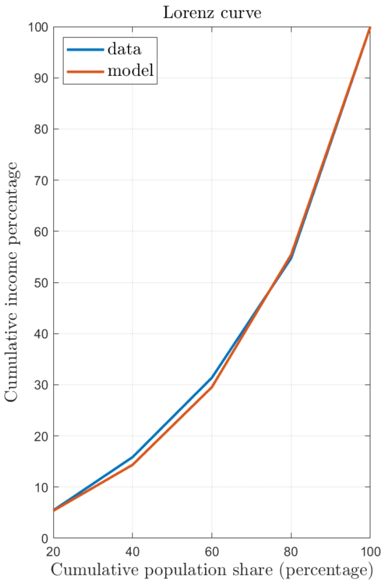
$$w_t = \rho_w w_{t-1} + \varepsilon_t$$

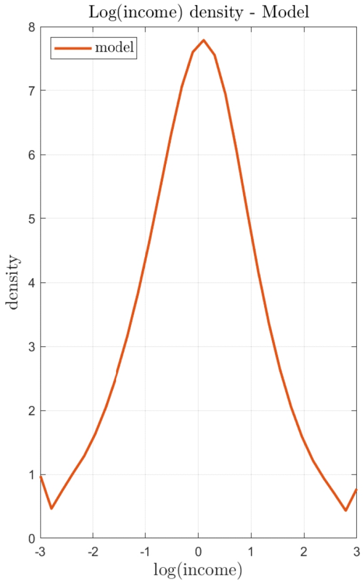
where:

$$\varepsilon_t \sim \begin{cases} N\left(\mu_1, \sigma_1^2\right) & , \quad \text{with Pr } p_1 \\ N\left(\mu_2, \sigma_2^2\right) & , \quad \text{with Pr } 1 - p_1 \end{cases}$$

values we used: $\rho_w = 84.85\%$, $\mu_1 = 3.36\%$, $\sigma_1 = 5\%$, $\mu_2 = -7.84\%$, $\sigma_2 = 130\%$, $p_1 = 70\%$

Asymptotic distribution of income process





Model Calibration

Preference parameters

$\rho = 6\%$ (contains crude demographic birth-death process)

$$u(q) = \frac{q^{1-\gamma}}{1-\gamma}, \quad \gamma = 2,\ f(h) = \eta h^{\alpha}, \quad \eta = 0.2,\ \alpha = 85\%$$

Policy-related parameters

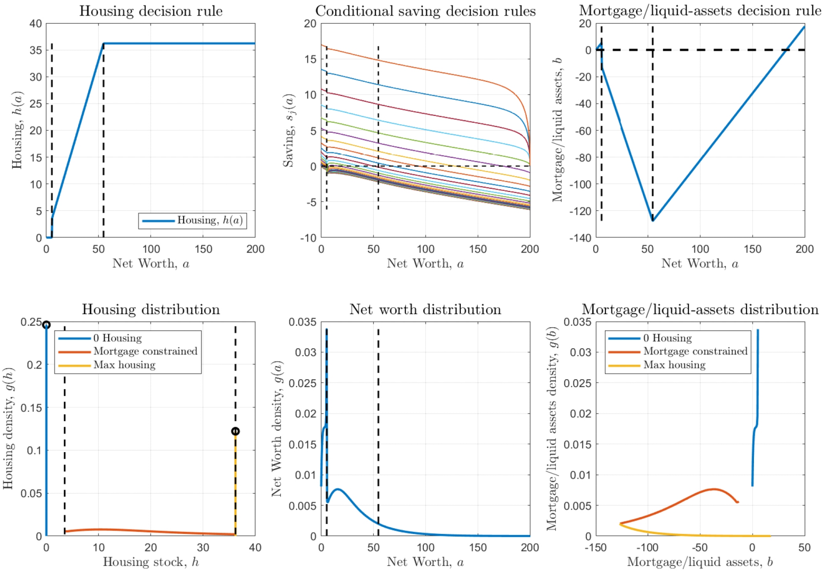
$\theta = 70\%$ (down payment 10% + 20% birth-death process)

$$r = 2\%$$

$$h_{\min} = 3.5$$

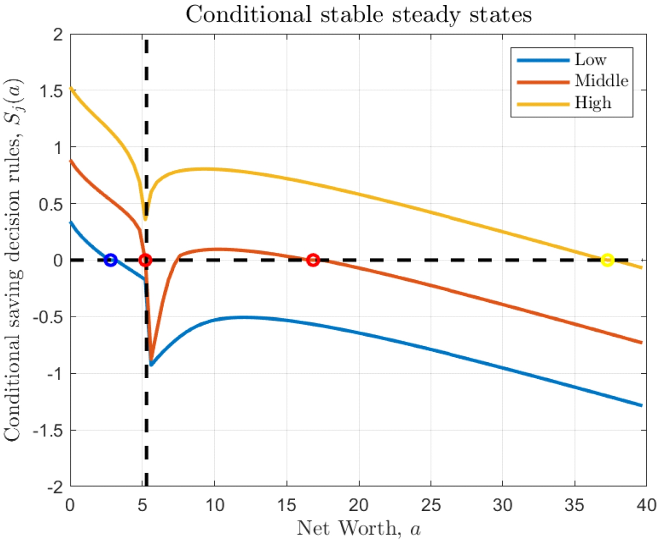
$H^s = 15$ (housing supply – important for normalizing the model)

Results

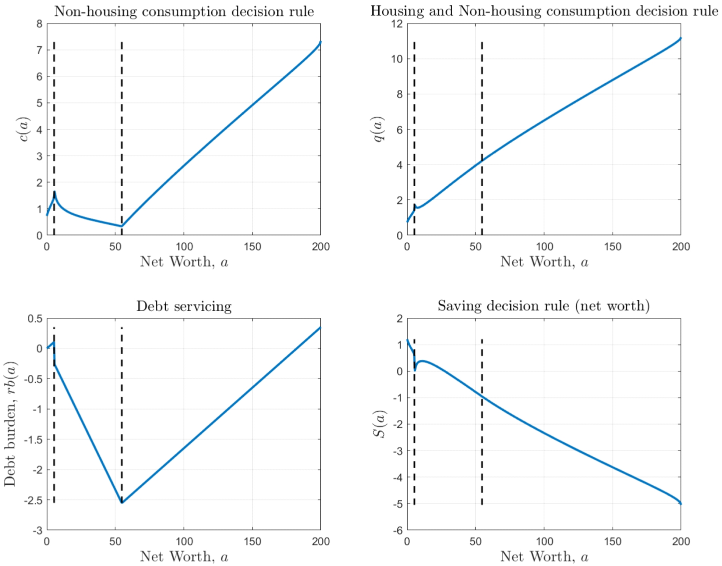


Housing decision rule
Housing, $h(a)$
Net Worth, a
Conditional saving decision rules
Saving, $s_j(a)$
Net Worth, a
Mortgage/liquid-assets decision rule
Mortgage/liquid assets, b
Net Worth, a
Housing distribution
0 Housing
Mortgage constrained
Max housing
Housing density, $g(h)$
Housing stock, h
Net worth distribution
Net Worth density, $g(a)$
Net Worth, a
Mortgage/liquid-assets distribution
0 Housing
Mortgage constrained
Max housing
Mortgage/liquid assets density, $g(b)$
Mortgage/liquid assets, b

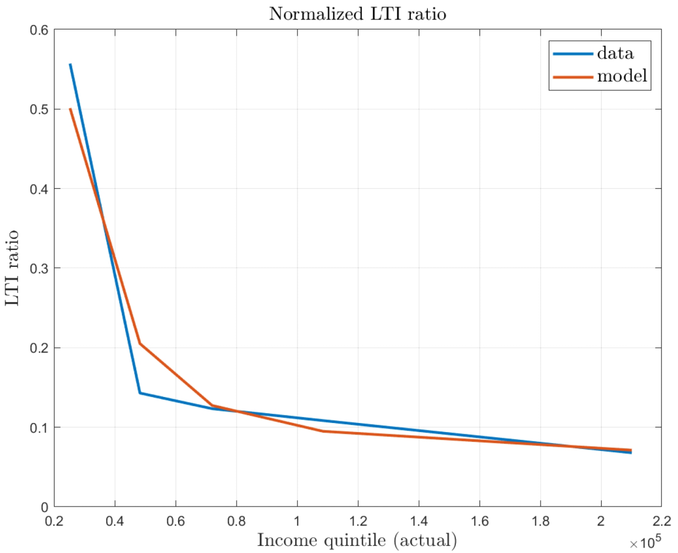
Results


Conditional stable steady states
Conditional saving decision rules, $S_j(a)$
Net Worth, a
Low
Middle
High
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20
25
30
35
40

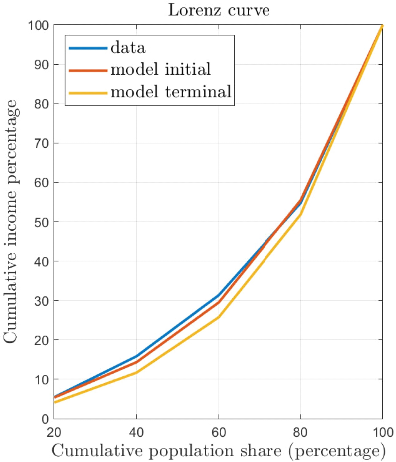
Results

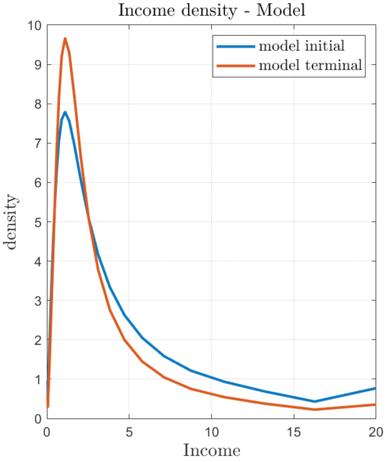

Non-housing consumption decision rule
$c(a)$
Net Worth, a
Housing and Non-housing consumption decision rule
$q(a)$
Net Worth, a
Debt servicing
Debt burden, $rb(a)$
Net Worth, a
Saving decision rule (net worth)
$S(a)$
Net Worth, a

Results on the main target: vulnerability

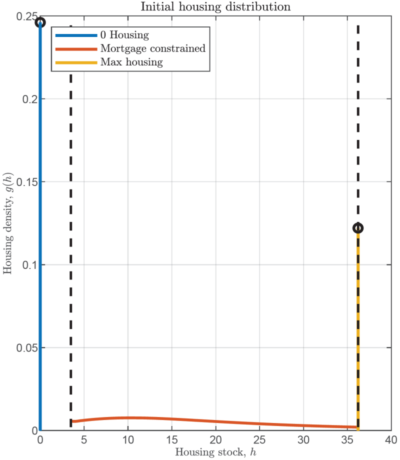


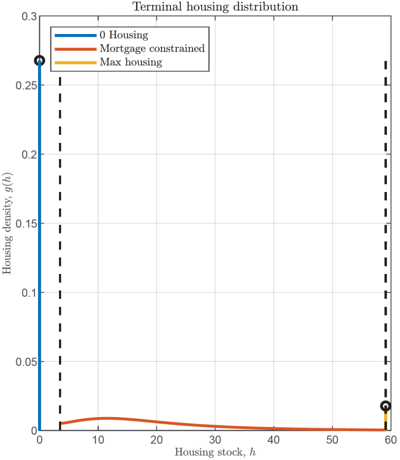
Shocks as stress tests: permanent departure of companies from Luxembourg



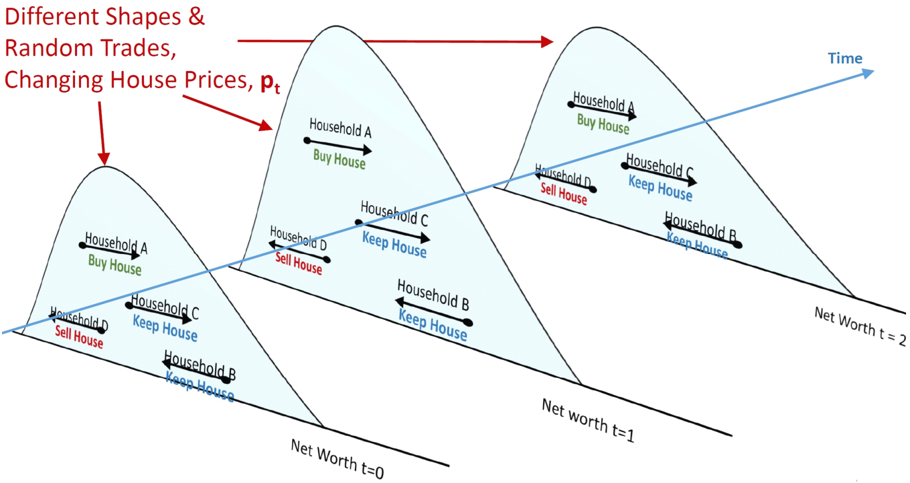


Shocks as stress tests: permanent departure of companies from Luxembourg

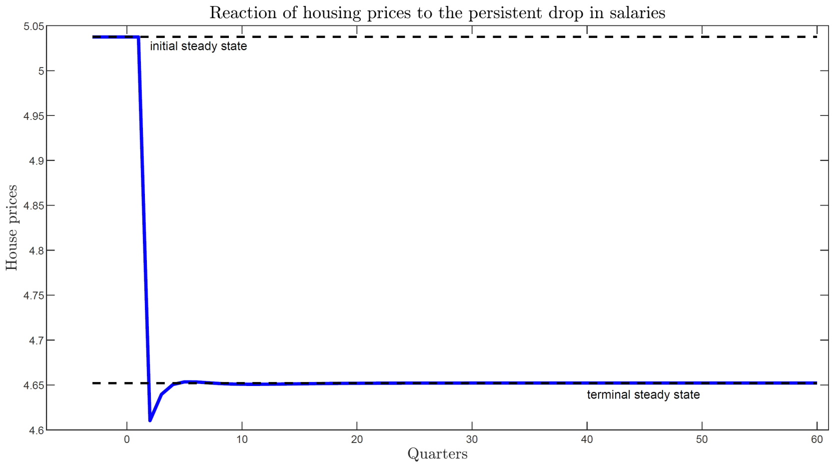




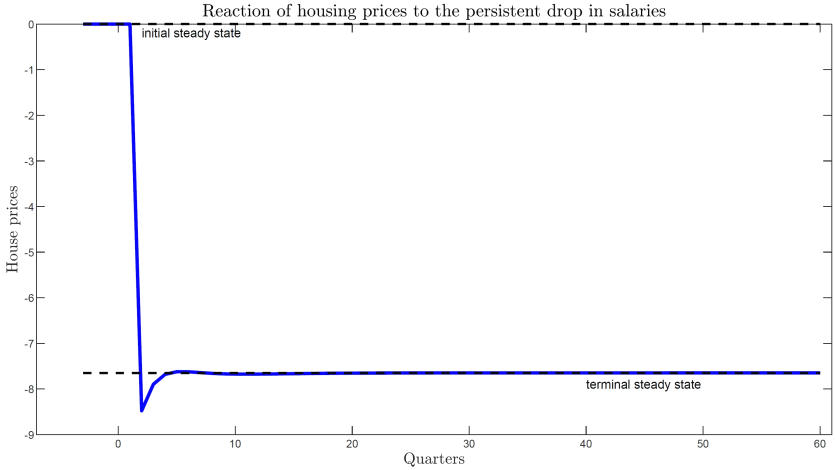
Transition idynamics



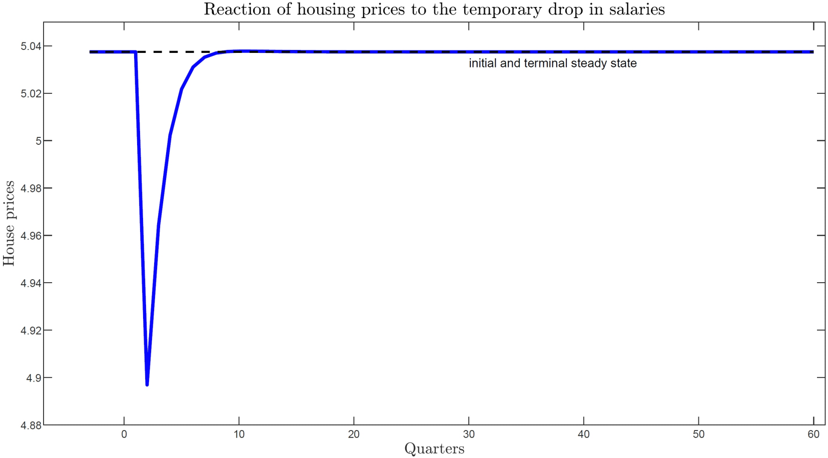
Permanent departure of companies from Luxembourg: price transition



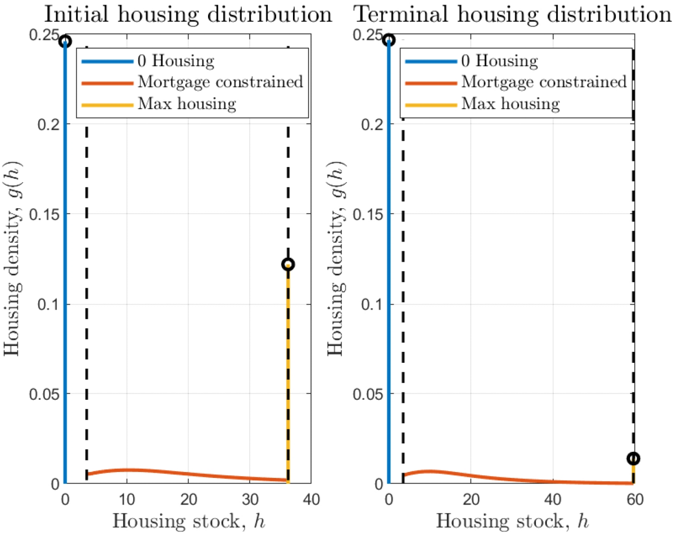
Permanent departure of companies from Luxembourg: price transition (%)



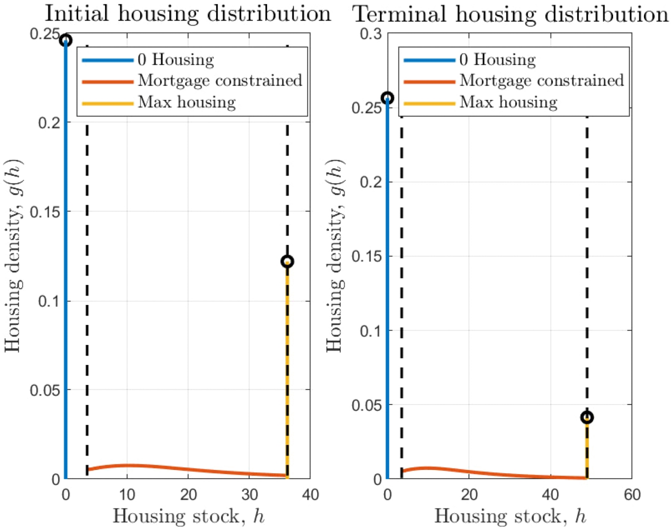
Temporary drop in incomes (of mostly lower-middle class households – no more than 2% drop)



Prudential policy: increasing the down payment from 10% to 20%, implies decrease in house prices by 7.8%



Monetary policy: decreasing the mortgage rate from 2% to 1.75%, implies increase in house prices by 8.8%



Important: detrended model's covid-time predictions

House prices grow too fast in Luxembourg

↓

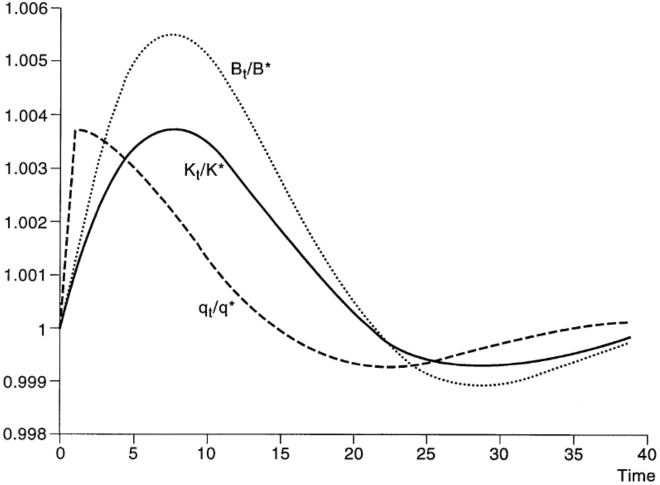
We are not sure why (immigration), say the trend is 10%

↙ ↘

if prudential policy
down payment ↑ to 20%
house-price slowdown:
10% - 7.8% = 2.2%

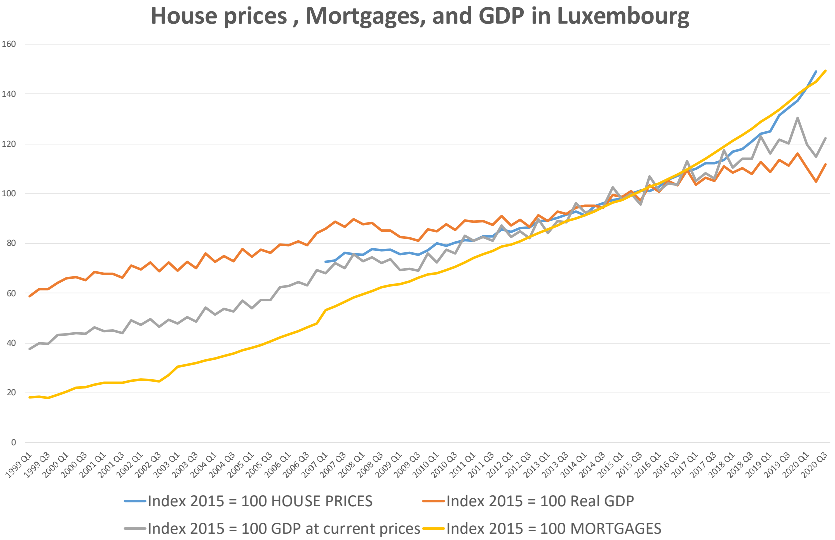
if monetary policy
mortgage rate ↓ to 1.75%
house-price acceleration:
10% + 8.8% = 18.8%

House-price trend: are we at a boom of a credit cycle?

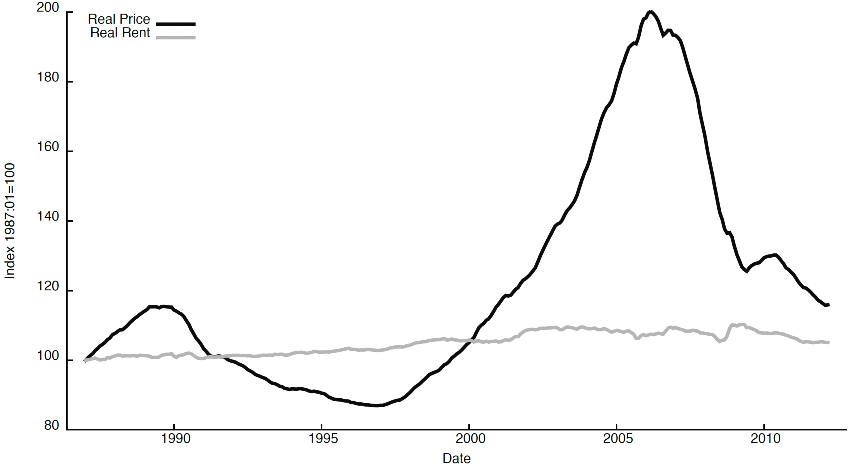


Source: Kiyotaki and Moore (JPE, 1997)

House-price trend: are we at a boom of a credit cycle? Maybe yes!!!

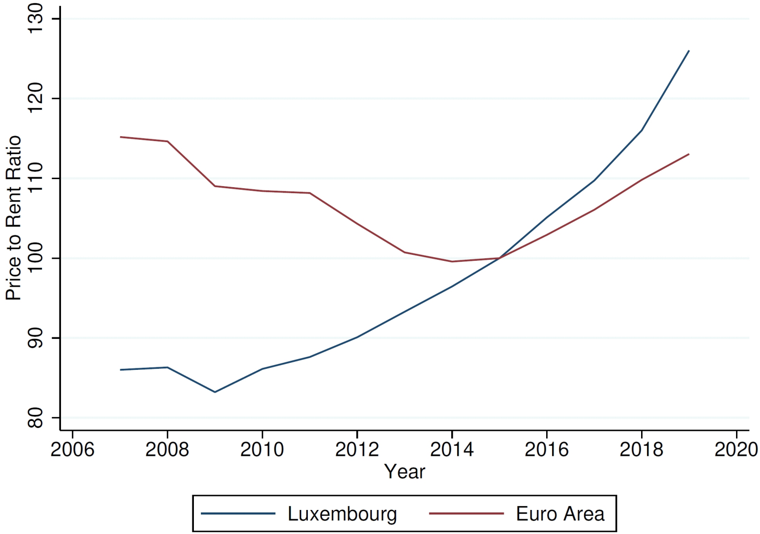


House-price trend: are we at a boom of a bubble?



US Data. Source: Perez and Santos (2018)

House-price trend: are we at a boom of a bubble? Hopefully not, but we must be prudent...



Important: detrended model's post covid-time (stagflation) predictions

House prices grow too fast in Luxembourg

↓

Again (immigration), say the trend is 10%

↙ ↘

mortg. rate ↑ from 2% to 3%
without indexation
house-price drop:
10% - 26.9% = -16.9%

mortg. rate ↑ from 2% to 3%
with 60% indexation
mild house-price drop:
10% - 14.8% = -4.8%

Conclusions

- Parsimonious forward-looking model and portable code

- Useful for the study of inflation (many challenges for adapting to the cross section, needs a task force)

- Things left to be improved about model re-scaling, but successful in pointing at vulnerable households and trades

- The big questions about credit-cycle booms and busts cannot be fully addressed. However, the model sees benign vulnerabilities and provides reasonable policy evaluation.