Heterogeneous-Agent Model of Household Mortgages in Luxembourg: Responses to the Covid-19 Shock

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November 2023

(Supported by FNR, project MORTGAGE COVID19-14741841CK)
Main problem

Covid-19 lockdowns

Asymmetric income shocks

less “social” professions lighter

more “social” professions harder

Heterogeneity matters!
Utilizing cross-sectional information

Task

Can we spot households that are “weak links”?

less “social”
professions
lighter

more “social”
professions
harder

Will house trades lead to a drop in house prices?
Will low house prices lead to bank balance-sheet weaknesses?
Main goal

- Create a **model** that **raises red flags** about **mortgage markets** before data and standard regressions do.

- Complementary diagnostic tool, not substitute.
Why?

Diagnosing risk early

↓

Prevention policies
Why a model?

Laboratory for:

- stress tests
- policy evaluation
What we do

- Dig into how house prices are formed: House prices \(\rightarrow\) affect collateral values in mortgage markets.

Model ambitions:
- Package and endogenize cross-sectional info & trades
- Address expectations on cross-sectional dynamics
Main model features

- income process/distribution
- interest rate
- covid-19 income shock (+ shock expectations)

\[
\begin{align*}
\text{income process/distribution} & \quad \text{interest rate} \\
\text{covid-19 income shock (+ shock expectations)} & \quad \text{exogenous}
\end{align*}
\]

- mortgage distribution
- homeownership/home-size distribution
- net-worth distribution
- house price and mortgage collateral
- liquid-wealth distribution
- consumption choices
- impulse-response dynamics of all the above

\[
\begin{align*}
\text{mortgage distribution} & \quad \text{homeownerships/home-size distribution} \\
\text{net-worth distribution} & \quad \text{house price and mortgage collateral} \\
\text{liquid-wealth distribution} & \quad \text{consumption choices} \\
\text{impulse-response dynamics of all the above} & \quad \text{endogenous}
\end{align*}
\]
Model overview: stationary equilibrium

A gas may be pictured as a collection of widely spaced molecules in continuous, chaotic motion.
Model overview: stationary equilibrium

Constant Shape
Same Random Trades
Constant House Price $p$

Net worth

Household A
Buy House

Household C
Keep House

Household D
Sell House

Household B
Keep House
Model

- **Momentary utility:** \( U(c_t, h_t) \)
  
  \[ c_t \rightarrow \text{consumption} \]
  
  \[ h_t \rightarrow \text{housing stock (e.g. square meters)} \]

- **Assets:**

  \( h_t \) if homeowner; \( h_t > 0 \)

  Two asset types:

  - **Mortgage,** \( b_t < 0 \) if \( h_t > 0 \), or
  - **liquid wealth,** \( b_t \geq 0 \) if \( h_t = 0 \)

- For simplicity, no modeling of rental market and the interest rate, \( r \), is the same for \( b_t < 0 \) and \( b_t > 0 \) and constant.
The **value of a house** is \( p \cdot h \).

A mortgage needs a **down payment**, calculated as a **constant proportion**, \( \theta \in (0, 1) \), of the value of the house. Therefore, the **mortgage limit** is,

\[
-b_t \leq \theta ph_t
\]

(1)

**Mortgages are collateralized and the house price matters!**
We assume a local law of large numbers, that there are potentially infinite individuals located at any particular point.

\[ \text{lumpy housing choice} \rightarrow h_t \in \{0\} \cup [h_{\text{min}}, \infty) \]

Despite that individuals may make lumpy, discrete adjustments in their housing units (e.g. moving from a larger house to a smaller house), we will focus on the adjustments of this locally aggregative housing market.
Model

- Household problem (the only one in partial equilibrium):

\[
\max_{(c_t, b_t, h_t)_{t \geq 0}} E_0 \left[ \int_0^\infty e^{-\rho t} U(c_t, h_t) \, dt \right] \\
\text{subject to:} \\
\dot{b}_t + \phi \dot{h}_t = rb_t + w_t - c_t \\
w_t \in \{w_1, ..., w_J\} \text{ with } \Pr \{ w_{t+dt} = w_k \mid w_t = w_j \} = \lambda_{j,k} \, dt \\
-b_t \leq \theta \phi h_t \\
h_t \in \{0\} \cup [h_{\text{min}}, \infty) \\
given \ h_0 \in \{0\} \cup [h_{\text{min}}, \infty) , \ b_0 \in \mathbb{R} \\
\lim_{t \to \infty} E_t (\lambda_t b_t) = 0
\]

Concern: too many dimensions
Model

- Reducing both the state-space dimensions and the choice-space dimensions.

Net worth $a_t$

$$a_t \equiv b_t + ph_t$$  \hspace{1cm} (2)

$f(h_t)$ pecuniary value of owned housing services in terms of consumable good

$$U(c_t, h_t) = u(q_t), \text{ with } q_t \equiv c_t + f(h_t)$$  \hspace{1cm} (3)
Model

- We obtain the branch function,

\[ h(a) = \begin{cases} 
0 & , \quad a < \frac{p}{\phi} h_{\text{min}} \\
\frac{\phi}{p} a & , \quad a \in \left[ \frac{p}{\phi} h_{\text{min}}, \frac{p}{\phi} h_{\text{max}} \right] \\
h_{\text{max}} & , \quad a \geq \frac{p}{\phi} h_{\text{max}}
\end{cases} \]

- Let,

\[ F(a) \equiv f(h(a)) - rph(a) \]

and the budget constraint becomes:

\[ \dot{a}_t = ra_t + w_t + F(a_t) - q_t \]  \hspace{1cm} (4)
Let the value function that corresponds to the $j$-th gridpoint of $w$ be denoted by $V_j(a)$. The HJB equation is given by,

$$\rho V_j(a) = \max_q \left\{ u(q) + V'_j(a) \left[ ra + w_j + F(a) - q \right] \right\} + \sum_{k=1}^{J} \lambda_{j,k} \left[ V_k(a) - V_j(a) \right]$$

Solution:

$$s_j(a) \equiv ra + w_j + F(a) - \underbrace{q_j(a)}_{\text{optimal choice}}$$
Hamilton-Jacobi-Bellman (HJB) equation

- Matrix representation of HJB (finite-differences solution technique)

\[
\frac{V_{i,j}^{n+1} - V_{i,j}^n}{\Delta} = -\rho V_{i,j}^{n+1} + U_{i,j}^n + \frac{V_{i+1,j}^{n+1} - V_{i,j}^{n+1}}{\Delta a} S_{F,i,j}^+ + \frac{V_{i,j}^{n+1} - V_{i-1,j}^{n+1}}{\Delta a} S_{F,i,j}^- \\
+ \sum_{k=1}^{J} \lambda_{j,k} \left( V_{i,k}^{n+1} - V_{i,j}^{n+1} \right), \quad j = 1, \ldots J.
\]

(5)

- Written in a compact way as:

\[
\frac{v^{n+1} - v^n}{\Delta} = -\rho v^{n+1} + u^n + \left( A_1 + \hat{A} \right) v^{n+1}
\]

(6)
Matrix $A_1$ in (6) is an $(I \cdot J) \times (I \cdot J)$ matrix with array,

$$
A_1 = \begin{bmatrix}
    x_1 & y_1 & 0 & \cdots & 0 \\
    \psi_2 & x_2 & y_2 & \cdots & 0 \\
    0 & \psi_3 & x_3 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & \cdots & x_N
\end{bmatrix}
$$

(7)
Matrix $\hat{A}$ in (6) is an $(I \cdot J) \times (I \cdot J)$ matrix with array (here, $J = 2$),

$$
\hat{A} = 
\begin{bmatrix}
-\lambda_{1,2} & 0 & \cdots & 0 & \lambda_{1,2} & 0 & \cdots & 0 \\
0 & -\lambda_{1,2} & \cdots & 0 & 0 & \lambda_{1,2} & \cdots & 0 \\
0 & 0 & \ddots & 0 & 0 & 0 & \ddots & 0 \\
\vdots & \vdots & \ddots & -\lambda_{1,2} & \vdots & \vdots & \ddots & \lambda_{1,2} \\
\lambda_{2,1} & 0 & \cdots & 0 & -\lambda_{2,1} & 0 & \cdots & 0 \\
0 & \lambda_{2,1} & \cdots & 0 & 0 & -\lambda_{2,1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \lambda_{2,1} & 0 & \cdots & \cdots & -\lambda_{2,1} \\
\end{bmatrix}
$$

(8)
Use Forward Kolmogorov equations (example $J = 2$, $g_j(a)$ is the conditional density if $w_t = w_j$):

\[ 0 = -\frac{d [s_1(a) g_1(a)]}{da} - \lambda_{1,2} g_1(a) + \lambda_{2,1} g_2(a) \]

and

\[ 0 = -\frac{d [s_2(a) g_2(a)]}{da} - \lambda_{2,1} g_2(a) + \lambda_{1,2} g_1(a) \]
Asymptotic distribution of net worth

- the asymptotic distribution \( g(a) = (g_1(a), g_2(a)) \) can be obtained through solving,

\[
0 = A^T g
\]  \hspace{1cm} (9)

with \( g \) being given by,

\[
g = \begin{bmatrix}
g_{1,1} \\
\vdots \\
g_{l,1} \\
\_ \_ \_ \\
\_ \_ \_ \\
\_ \_ \_ \\
g_{1,l} \\
\vdots \\
g_{l,l}
\end{bmatrix}
\]  \hspace{1cm} (10)
A house price, \( p \), clears the market if,

\[
\sum_{k=1}^{J} \int_{h_{\text{min}}}^{\infty} h \left[ \int_{-\theta ph}^{\infty} g_k (h, b) \, db \right] \, dh = H^S
\]

with \( g_k (h, b) \) denoting the conditional joint distribution of housing and mortgage/liquid assets \((h, b)\), conditional on having income \( w_k \).
Income process in discrete time (Guvenen et al., 2020):

\[ w_t = \rho_w w_{t-1} + \varepsilon_t \]

where:

\[ \varepsilon_t \sim \begin{cases} 
N(\mu_1, \sigma_1^2) , & \text{with Pr } p_1 \\
N(\mu_2, \sigma_2^2) , & \text{with Pr } 1 - p_1 
\end{cases} \]

values we used: \( \rho_w = 84.85\%, \mu_1 = 3.36\%, \sigma_1 = 5\%, \mu_2 = -7.84\%, \sigma_2 = 130\%, p_1 = 70\% \)
Asymptotic distribution of income process

Lorenz curve

Log(income) density - Model

Cumulative income percentage

Cumulative population share (percentage)

density

log(income)
Model Calibration

Preference parameters

\[ \rho = 6\% \quad \text{(contains crude demographic birth-death process)} \]

\[ u (q) = \frac{q^{1-\gamma}}{1-\gamma}, \quad \gamma = 2, \quad f (h) = \eta h^\alpha, \quad \eta = 0.2, \quad \alpha = 85\% \]

Policy-related parameters

\[ \theta = 70\% \quad \text{(down payment 10\% + 20\% birth-death process)} \]

\[ r = 2\% \]

\[ h_{\text{min}} = 3.5 \]

\[ H^s = 15 \quad \text{(housing supply – important for normalizing the model)} \]
Conditional stable steady states

- Low
- Middle
- High

Conditional saving decision rules, $S_j(a)$

Net Worth, $a$
Results
Results on the main target: vulnerability

![Normalized LTI ratio graph](image)

- **Normalized LTI ratio**
- **Data**
- **Model**

**Income quintile (actual)**

- **LTI ratio**
- **Income quintile**

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Shocks as stress tests: permanent departure of companies from Luxembourg

Lorenz curve

Cumulative income percentage

Cumulative population share (percentage)

Income density - Model

Density

Income

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Shocks as stress tests: permanent departure of companies from Luxembourg
Different Shapes & Random Trades, Changing House Prices, $p_t$
Permanent departure of companies from Luxembourg: price transition

Reaction of housing prices to the persistent drop in salaries

Initial steady state

Terminal steady state

House prices

Quarters
Permanent departure of companies from Luxembourg: price transition (%)
Temporary drop in incomes (of mostly lower-middle class households – no more than 2% drop)

Reaction of housing prices to the temporary drop in salaries

initial and terminal steady state
Prudential policy: increasing the down payment from 10% to 20%, implies decrease in house prices by 7.8%
Monetary policy: decreasing the mortgage rate from 2% to 1.75%, implies increase in house prices by 8.8%
House prices grow too fast in Luxembourg

We are not sure why (immigration), say the trend is 10%

if prudential policy
down payment \( \uparrow \) to 20%

\[ \text{house-price slowdown: } 10\% - 7.8\% = 2.2\% \]

if monetary policy
mortgage rate \( \downarrow \) to 1.75%

\[ \text{house-price acceleration: } 10\% + 8.8\% = 18.8\% \]
House-price trend: are we at a boom of a credit cycle?

Source: Kiyotaki and Moore (JPE, 1997)
House-price trend: are we at a boom of a credit cycle?
Maybe yes!!!
House-price trend: are we at a boom of a bubble?

House-price trend: are we at a boom of a bubble? Hopefully not, but we must be prudent...
Important: detrended model’s post covid-time (stagflation) predictions

House prices grow too fast in Luxembourg

\[ \downarrow \]

Again (immigration), say the trend is 10%

\[
\begin{align*}
\text{mortg. rate} & \uparrow \text{ from 2\% to 3\%} \\
\text{without indexation} & \\
\text{house-price drop:} & \\
10\% - 26.9\% &= -16.9\%
\end{align*}
\]

\[
\begin{align*}
\text{mortg. rate} & \uparrow \text{ from 2\% to 3\%} \\
\text{with 60\% indexation} & \\
\text{mild house-price drop:} & \\
10\% - 14.8\% &= -4.8\%
\end{align*}
\]
Conclusions

- Parsimonious forward-looking model and portable code

- Useful for the study of inflation (many challenges for adapting to the cross section, needs a task force)

- Things left to be improved about model re-scaling, but successful in pointing at vulnerable households and trades

- The big questions about credit-cycle booms and busts cannot be fully addressed. However, the model sees benign vulnerabilities and provides reasonable policy evaluation.