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Older or Wealthier?
The Impact of Age Adjustments on the Wealth Inequality Ranking of Countries

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Abstract

Differences in individual wealth holdings are widely viewed as a driving force of economic inequality. However, as this finding relies on cross-section data, we may confuse older with wealthier. We propose a new method to adjust for age effects in cross-sections, which eliminates transitory wealth inequality due to age, yet preserves inequality arising from other factors. This new method is superior to existing methods, like the much used Paglin-Gini, which is shown to have several problems. A new cross-country comparable database reveals that the choice of method is empirically important: Existing methods yield erroneous wealth inequality rankings of countries.

(*JEL*: D31, D63, D91, E21. *Keywords*: Wealth inequality, Life cycle, Age adjustments, Gini coefficient.)

1 Introduction

New sources of cross-country comparable micro data suggest that individual wealth holdings vary substantially within and across countries.¹ In most coun-

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¹See Davies and Shorrocks (2000), Davies et al. (2006), Sierminska et al. (2006), and Wolff (1996) for recent evidence on wealth inequality within and across countries.

tries, the Gini coefficient for wealth is reported to be twice that of income. Moreover, the world distribution of wealth is found to be much more concentrated than
the world distribution of income (Bourguignon and Morrison, 2002; Milanovic,
2002, 2005; Sala-i-Martin, 2006; Davies et al., 2006). Due to data availability,
however, research on wealth inequality across countries are based on cross-section
data. This is argued to be problematic as both theoretical models and empirical
results suggest a strong age—wealth relationship (see e.g. Davies and Shorrocks,
2000). The age—wealth profile is firmly established as increasing during the working lifespan and usually declining somewhat after retirement. Hence, a snapshot
of wealth inequality within a country runs the risk of providing a misleading picture of the differences in lifetime wealth of its citizens. As age—wealth profiles
differ across countries, the wealth inequality ranking of countries may also be affected by transitory wealth differences attributable to life cycle factors. For these
reasons, it has long been argued that age adjustments of inequality measures
based on cross-section data are necessary (see e.g. Atkinson, 1971).

We propose a new method to adjust for age effects, which unlike existing methods, captures that individuals differ both with respect to age and with respect to other wealth generating factors. For example, an individual's education is not only an important determinant of his wealth, but is also strongly correlated with his age. Existing methods assume that the unconditional distribution of mean wealth by age represents the age effects and will, therefore, not only eliminate wealth inequality attributable to age but also differences owing to factors correlated with age, such as education. By contrast, the method proposed in this paper eliminates inequality due to age, yet preserves inequality arising from other factors. To this end, a multivariate regression model is employed, allowing us to isolate the net age effects while holding other determinants of wealth constant. Next, we derive a new, age-adjusted Gini coefficient, where perfect equality requires that each individual receives a share of total wealth equal to the proportion of wealth he would hold if all wealth generating factors except age were the same for everyone in the population.

Our new method is applied to cross-section data from Canada, Germany, Italy, Sweden, the United Kingdom, and the United States, collected from the new, cross-country comparable Luxembourg Wealth Study (LWS) database. We find that existing methods which attempt to adjust for age, yield erroneous wealth inequality ranking of countries. In particular, the country ranking revealed by

the much used Paglin–Gini is shown to be seriously misleading.² A battery of robustness checks support our results.

This is the first study to examine the impact of age adjustments on wealth inequality ranking of countries. However, several studies have investigated the effect of adjusting for age effects on wealth and income inequality. Paglin (1975) studied the effect of age adjustment on the distribution of income and wealth in the United States; his adjustment had dramatic consequences for income and wealth inequality estimates and their time trend. By contrast, Pudney (1993) suggests that only a small part of observed income and wealth inequality in China can be explained by age effects. Other studies that have attempted to adjust for age effects on income inequality estimates include Mookherjee and Shorrocks (1982) for the United Kingdom as well as Danziger et al. (1977), Minarik (1977), Nelson (1977), Friesen and Miller (1983), Formby et al. (1989), and Bishop et al. (1997) for the United States. All the above studies use methods which fail to adjust properly for age effects.

Section 2 sets out the proposed method to identify and adjust for age effects. Section 3 describes data and clarifies definitional issues. Section 4 reports estimation results and age-adjusted wealth inequality measures. Section 5 demonstrates the failure of existing methods in properly adjusting for age effects, and Section 6 concludes.

2 A new method for age adjustment

The proposed method for age adjustment of inequality may be described as a three-step procedure. First, a new age adjusted Gini coefficient (AG) is derived. Second, a multivariate regression model is employed, allowing us to isolate the net age effects while holding other determinants of wealth constant. Third, the wealth distribution that characterizes perfect equality in age adjusted wealth is determined. Below, we describe the three steps, before showing that AG can be viewed as a generalization of the classical Gini coefficient (G).

 $^{^2}$ The Paglin–Gini (Paglin, 1975, 1977, 1979, 1989) was subject to three rounds of comments and replies in the *American Economic Review*, has numerous citations, and continuous to be subject to controversy.

2.1 A new age adjusted Gini coefficient

Consider a society consisting of n individuals where every individual i is characterized by the pair $(w_i, \widetilde{w_i})$, where w_i denotes the actual wealth level and $\widetilde{w_i}$ is the equalizing wealth level. If actual and equalizing wealth is the same for all individuals and all individuals live equally long, there is perfect equality of lifetime wealth in this society. As will be clear when we define the equalizing wealth level formally in Section 2.3, the equalizing wealth is the same for all individuals belonging to the same age group in this society; it is a function of individual i's age, but not of any other individual characteristics. If none of the wealth generating factors (but age) are correlated with age, the equalizing wealth is simply the mean wealth of each age group. Further, if there are no age effects on wealth, the equalizing wealth will be equal to the mean wealth for all individuals in the society.

The joint cross-sectional distribution Y of actual and equalizing wealth is given by:

$$Y = [(w_1, \widetilde{w_1}), (w_2, \widetilde{w_2}), ..., (w_n, \widetilde{w_n})],$$

Let Ξ denote the set of all possible joint distributions of actual and equalizing wealth, such that the sum of actual wealth equals the sum of equalizing wealth. Suppose that the social planner imposes the following modified versions of the standard conditions on an inequality partial ordering defined on the alternatives in Ξ , where $A \leq B$ represents that there is at least as much age-adjusted inequality in B as in A. Let μ denote the mean wealth of the population as a whole. Let the distributions of differences, Δ_i 's, between actual wealth w_i and equalizing wealth \widetilde{w}_i for the two distributions ($\Delta_i(A) = w_i(A) - \widetilde{w}_i(A)$) and $\Delta_i(B) = w_i(B) - \widetilde{w}_i(B)$) be sorted in an ascending order.

Condition 1. Scale Invariance: For any a > 0 and $A, B \in \Xi$, if A = aB, then $A \sim B$.

Condition 2. Anonymity: For any permutation function $\rho: n \to n$ and for $A, B \in \Xi$, if $(w_i(A), \widetilde{w_i}(A)) = (w_{\rho(i)}(B), \widetilde{w_{\rho(i)}}(B))$ for all $i \in n$ then $A \sim B$.

Condition 3. Unequalism: For any $A, B \in \Xi$ such that $\mu(A) = \mu(B)$, if $\Delta_i(A) = \Delta_i(B)$ for every $i \in n$, then $A \sim B$.

³See Almås et al. (2007) for analogous conditions imposed to study equality of opportunity.

Condition 4. Generalized Pigou–Dalton: For any $A, B \in \Xi$, if there exist two individuals s and k such that $\Delta_s(A) < \Delta_s(B) \le \Delta_k(B) < \Delta_k(A)$, $\Delta_i(A) = \Delta_i(B)$ for all $i \ne s, k$, and $\Delta_s(B) - \Delta_s(A) = \Delta_k(A) - \Delta_k(B)$, then $A \succ B$.

Scale invariance states that if all actual and equalizing wealth levels are rescaled by the same factor, then the level of age-adjusted inequality remains the same. Anonymity implies that the ranking of alternatives should be unaffected by a permutation of the identity of individuals. Unequalism entails that the social planner is only concerned with how unequally each individual is treated, defined as the difference between his actual and equalizing wealth. Finally, the generalized version of the Pigou–Dalton criterion states that any fixed transfer of wealth from an individual i to an individual j, where $\Delta_i > \Delta_j$, reduces age adjusted inequality.

The new age-adjusted Gini coefficient is based on a comparison of the absolute values of the differences in actual and equalizing wealth between all pairs of individuals, and is defined as

$$AG = \frac{\sum_{j} \sum_{i} |(w_i - \widetilde{w}_i) - (w_j - \widetilde{w}_j)|}{2\mu n^2}.$$
 (1)

It is straightforward to see that AG satisfies Conditions 1–4. Note that these conditions are similar to those underlying the classical Gini coefficient in all respects but one: The equalizing wealth is not given by the mean wealth in the society as a whole, but depends on the age of the individuals.

It is straightforward to construct age-adjusted Lorenz curves based on the distribution of differences $(w_i - \widetilde{w}_i)$. Hence, it is by no means necessary to focus on the Gini coefficient – other inequality indices that are based on the Lorenz curve, such as the Bonferroni index, can also form the basis for age adjustments.

2.2 Identifying the net age effects

Suppose that the wealth level of individual i at a given point in time, depends on the age group a that he belongs to as well as his lifetime resources given as a function h of a vector X of individual characteristics

$$w_i = f(a_i)h(X_i). (2)$$

For simplicity, f is specified as a function of age alone but could in general also reflect other aspects affecting individuals' life cycle behavior, such as their time preferences, interest rates, and the rules governing retirement.

The functional form of f depends on the underlying model of wealth accumulation. In the simplest life cycle model, there is no uncertainty, individuals earn a constant income until retirement age, and the interest rate as well as the rate of time preference is zero. In this model, the wealth of an individual increases up to retirement and declines afterwards. If the earnings profile is upward sloping, the model predicts borrowing in the early part of the life cycle. The fact that this is not always observed could be explained by credit market imperfections. Introducing lifetime uncertainty and noninsurable health hazard induces the elderly to hold assets for precautionary purposes, which reduces the rate at which wealth declines during retirement. If the sole purpose of saving is to leave a bequest to one's children, individuals behave as if their horizons were infinite and wealth does not decline with age.

Empirically, we can specify a flexible functional form of f, yielding the wealth generating function

$$\ln w_i = \ln f(a_i) + \ln h(X_i) = \delta_i + X_i' B, \tag{3}$$

where δ_i gives the percentage wealth difference of being in the age group of individual i relative to some reference age group, holding all other variables constant. Due to the right skewness combined with the sparse tail of the wealth distribution, our log-linear specification is preferable to a linear specification. As net wealth may be negative, we therefore add to each wealth observation a constant equal to the absolute value of the minimum wealth observation when estimating the log-linear specification. This is simply a matter of adjusting the location of the distribution. Equation (3) is estimated by OLS separately for each country. The key assumption underlying this estimation is that there are no omitted factors correlated with age that determine individual wealth holding. In that case, we obtain consistent estimates of the net age effects on wealth.

It is important to emphasize that the objective of the estimation of equation (3) is not to explain as much variation as possible in wealth holdings, but simply to get an empirically sound estimate of the effects of age on wealth. Drawing on the findings of Jappelli (1999) and Hendricks (2007) of variables correlated with wealth, X includes education and sex in our baseline specification. When

performing the robustness analysis, we extend the set of controls to include number of children, industry and occupation of household head, region of residence, marital status, immigration status, and spouse's characteristics. The reason for not including these variables in the baseline specification is twofold. First, we do not have data on all the variables for every country in the study. In addition, some of the variables are potentially endogenous to individuals' wealth holding. In any case, our results are robust to the inclusion of the additional controls.

Existing age-adjusted inequality measures, discussed in detail in Section 5, implicitly assumes a stationary economy, implying no cohort or time-specific effects. Consequently, they risk confounding age effects with cohort and time-specific effects, as these factors are perfectly collinear in a cross-section. A novelty of this paper is that we make an effort to separate age effects from cohort/timespecific effects. Jappelli (1999) and Kapteyn et al. (2005) explore reasons why different cohorts accumulate different amounts of wealth. They find that productivity growth is the primary determinant of differences in wealth across cohorts; productivity growth generates differences in permanent incomes across cohorts, which feeds into the wealth accumulation of individuals belonging to different generations. Following Masson (1986), we assume that the age cross-sections and the cohort profiles of wealth (in constant prices) coincide except for a constant state of real growth. If wealth grows at the rate g, then the typical profile for any given cohort is (1+g) times larger than that for the one-year-older cohort. Thus, we inflate each individual's wealth value in the cross-section by the factor $(1+g)^{age}$. Mirer (1979) shows that under commonly accepted assumptions in the life cycle theory, the growth rate of wealth is equal to the growth rate of income between successive cohorts. To adjust the observed wealth levels for economic growth across cohorts, we use an annual growth rate of 2.5 percent. Our results are robust to other choices of growth rates.

The assumption of a stationary economy also implies no intracohort mobility in individual wealth holdings, which has been criticized by e.g. Johnson (1977) and Friesen and Miller (1983). By conditioning on individual characteristics, the assumption of parallel age—wealth profiles may be more reasonable for AG than for existing age-adjusted inequality measures. However, just as any other study measuring inequality using cross-section data, this paper admittedly comes short of fully accounting for the effects of intracohort mobility. Yet, it is reassuring that several studies suggest that accounting for mobility has little impact on country rankings by income inequality (see e.g. Burkhauser and Poupore, 1997; Aaberge

2.3 Defining equalizing wealth

Identifying the net age effect is only part of the job; we also need to find a consistent way of adjusting for age effects when there are other wealth generating factors. There is a considerable literature concerning the problem of how to adjust for some, but not all, income generating factors when the income function is not additively separable (see e.g. Bossert and Fleurbaey (1996) and Kolm (1996)). The problem of adjusting for age effects on wealth is analogous. To eliminate wealth differences attributable to age but preserve inequality arising from all other factors, we employ the so-called general proportionality principle proposed by Bossert (1995) and Konow (1996), and further studied in Cappelen and Tungodden (2007). Then, the absence of age-adjusted inequality requires that any two individuals belonging to a given age group have the same wealth level. Moreover, in any situation where everyone has the same wealth generating factors except age, there should be no lifetime wealth inequality.⁴

More formally, the equalizing wealth level of individual i depends on his age as well as every other wealth generating factor of all individuals in the society, and is formally defined as:

$$\widetilde{w}_i = \frac{\mu n \sum_j f(a_i) h(X_j)}{\sum_k \sum_j f(a_k) h(X_j)} = \frac{\mu n e^{\delta_i}}{\sum_k e^{\delta_k}},\tag{4}$$

where e^{δ_k} gives the net age effect of belonging to the age group of individual k after integrating out the effects of other wealth generating factors correlated with age. No age-adjusted inequality corresponds to every individual i receiving \widetilde{w}_i , which is the share of total wealth equal to the proportion of wealth an individual from his age group would hold if all wealth generating factors except age were the same for everyone in the population. If there is no age effect on wealth, the equalizing wealth level is equal to the mean wealth level in the society.

⁴In a study of income inequality in the United States, Bishop et al. (1997) use a method to make age adjustments which disregard that the underlying income function is not additively separable. First, they estimate a multiplicative separable income function, which can be expressed as $lnY = \alpha_0 + \beta Age + Z'\gamma + \epsilon$, where α_0 is a constant, Age is the age and Z is a set of controls. Next, they use the prediction $\ln Y^* = \ln Y - \beta Age$ as their age-adjusted income measure. However, the net age effect is given by $\frac{dY}{dAge}$ which is generally different from $\beta = \frac{d \ln Y}{dAge} = \frac{dY}{Y} \frac{1}{dAge}$, as Y is a function of Z. If Z is correlated with Age then Bishop et al.'s approach will fail to capture the net age effects.

2.4 Relationship to the classical Gini-coefficient

From equation (1), it is straightforward to see that that AG is a generalization of the classical Gini coefficient (G). Both measures are based on a comparison of the absolute values of the differences in the actual and equalizing wealth levels between all pairs of individuals. The distinguishing feature is how equalizing wealth is defined. For G, the equalizing wealth level is assumed to be μ , implying not only equal lifetime wealth but also a flat age-wealth profile. However, a flat age—wealth profile runs counter to both consumption needs over the life cycle as well as productivity variation depending on human capital investment and experience. Indeed, the relationship between wealth and age can produce wealth inequality at a given point in time even if everyone is completely equal in all respects but age. As transitory wealth differences even out over time, a snapshot of inequality produced by G runs the risk of producing a misleading picture of actual variation in lifetime wealth. In comparison, AG abandons the assumption of a flat age-wealth profile and allows equalizing wealth to depend on the age of the individuals. By doing this, AG purges the cross-section measure of inequality of its intra-age or life-cycle component. If there are no age effects on wealth, the age-wealth profile is flat and AG coincides with G as $\mu = \widetilde{w}_i$ for all individuals in every age group.⁵

3 Data and definitions

The distribution of household wealth within and across countries has recently received much attention. An important reason is the increased availability and quality of data on household wealth. Household surveys of assets and debt have typically suffered from nonsampling errors because of high nonresponse and misreporting rates. On top of this, comparative studies of wealth distributions have been haunted by comparability problems because of methodological and data issues ranging from the basic problem of index numbers to differences in the

⁵Note that the properties of inequality measures based on the Gini coefficient are preserved when applied to distributions with zero and negative values (Amiel et al., 1996). However, if somebody holds negative wealth, it could be the case that G is outside the interval [0,1] and AG is outside the interval [0,2]. The numerical values of these ordinal inequality measures are only of interest as a way of comparing and ordering the distributions by inequality. The fact that they range over different intervals, and that the range may be affected by negative values, is therefore beside the point.

methods and definitions used in the various countries. Today, the data problems are mitigated by oversampling of wealthy people in surveys as well as utilizing supplementary information such as administrative data from tax and estate registers. The LWS — an international project to collect and harmonize existing micro data on household wealth into a coherent database — has reduced the comparability problems. We use the LWS database, and select the following six countries due to data availability: Canada, Germany, Italy, Sweden, the United Kingdom, and the United States.⁶

We follow common practice and focus on the distribution of household net wealth, which refers to material assets that can be sold in the marketplace less any debts, thereby excluding pension rights as well as human capital. The concept of net wealth consists of financial assets and nonfinancial assets net of total debt. Financial assets include deposit accounts, stock, and mutual funds, whilst nonfinancial assets consist of the principal residence and other real estate investments.⁷ Business assets are not included. Total debts refer to all outstanding loans.

This paper uses the household as the economic unit. This is in part because assets are recorded at the household level but also to conform to previous studies of wealth distributions. Households with missing values for wealth, education, age, or sex of household head are dropped. The only exception is Canada, where sex of the household head is never reported. To compare wealth holdings of singles and couples, we assign each married/cohabiting spouse a wealth level equal to his or her net household wealth divided by the square root of two. Robustness analysis demonstrates that our results are unaffected by the choice of equivalence scale.

To define age groups, we follow common practice and rely on information about the age of the household head. To be specific, we define seven age groups: 24 years and younger, 25–34 years, 35–44 years, 45–54 years, 55–64 years, 65–74 years, and 75 years and older.⁸ There are no household heads older than 75 years

⁶See Sierminska et al. (2006) and the LWS homepage http://www.lisproject.org/lws.htm for a detailed description of LWS database.

⁷The self-assessed current value of the principle residence and other real estate investments is reported for all countries except for Sweden, where the tax value is reported inflated by a regional constant. The principle residence represents almost the same share of total assets in Sweden as in it neighbouring country Finland (61 vs. 64 percent).

⁸Formby et al. (1989) and Paglin (1989) discuss the theoretical effects of the choice of the widths of the age groups on age adjustments of inequality. The results of Formby et al. (1989)

in the Swedish data. In all countries, we categorize the education variable into four educational groups. The four groups correspond as close as possible to the following categories: 'High school dropout', 'High school graduate', 'Nonuniversity post-secondary certificate', and 'University degree or certificate'.

In the robustness analysis, we experiment with various specifications of equation (1). First, we include number of children as a control. For all countries but Canada, we have data on marital status as well as on the industry and occupation of household head. For this subset of countries, we run regressions adding these variables to the set of controls. Marital status is divided into four categories: 'single without children', 'single with children', 'couple without children', and 'couple with children'. Industry and occupation are included using the countries' own categories. With the exception of Canada, we also include the characteristics of spouses as a robustness check. For Germany, Italy, Sweden, and the United Kingdom, we have data on region of residence and immigrant status of household head. To examine the sensitivity of our results, we add these variables to the wealth generating model for this subset of countries.

4 Age-adjusted estimates of wealth inequality

4.1 Descriptive statistics

Table 1 demonstrates that there is considerable variation in the demographic structure of the six OECD countries examined in this study. First and foremost, the age structure differs substantially across the countries. For instance, Italy has on average older household heads, which may be because Italians move out from their parents' house later in life than what is typical in most OECD countries (see e.g. Manacorda and Moretti, 2006). By contrast, Sweden has relatively young household heads. The fact that the age structure differs means that the inequality ranking of countries may be affected by age adjustments, even if countries have the same age—wealth profile. Furthermore, Table 1 demonstrates significant cross-country differences in educational attainment. In particular, the educational level is substantially lower in Italy compared with the United States and Germany. We can also see that having a female household head is most common in the United

show, however, that age-adjusted inequality estimates are not substantially different for age groups of one, five, and 10 year intervals.

Table 1: Descriptive statistics by country

		Laure	1. Describe	table 1. Descriptive statistics by country	by country	
	Canada	Germany	Italy	Sweden	$\overline{\mathrm{UK}}$	$\overline{\mathrm{USA}}$
	SFS 1999	SOEP 2002	SHIW 2002	HINK 2002	BHPS 2000	SCF 2001
Mean age	46.96	51.68	55.32	45.81	53.15	48.97
Age comp. $(\%)$						
24 or less	5.95	3.64	89.0	99.2	3.81	5.63
25-34	19.53	15.28	9.40	19.62	14.26	17.11
35-44	24.66	20.77	21.47	20.57	19.41	22.30
45-54	19.57	17.19	18.80	20.32	17.37	20.60
55-64	11.97	16.87	16.90	19.22	14.94	13.25
65-74	10.44	14.70	18.21	12.61	13.99	10.74
75 and over	7.89	11.54	14.53	I	16.22	10.37
Education (%)						
Less high sch	26.95	1.68	36.19	22.54	30.10	8.67
Grad high sch	23.36	15.70	27.21	47.24	22.82	37.11
Post-secondary	28.35	53.00	28.59	6.36	44.48	15.50
Univ. degree	21.34	29.62	8.01	23.07	15.88	38.72
Female (%)	ı	41.77	36.62	37.11	49.77	54.44
Mean wealth	52795.02	77482.71	112518.8	41443.3	98561.12	162970.7
Max wealth	2927042	8241594	3160886	3.89e + 07	1830569	3.06e + 08
Min wealth	-98875.95	-4323347	-155909	-4131085	-453952.1	-1.52e+07

Note: National household wealth surveys included and harmonized in the LWS database. Household weights are used to ensure nationally representative results. Wealth levels expressed in USD (2000 exchange rates).

States. The United States stands out with the highest mean wealth, whereas Sweden has the lowest.

Figure 1 reveals that not only is there considerable variation in the age structure across the countries but the age—wealth relationship also differs substantially. This makes it even more likely that cross-country comparisons of inequality are affected by adjusting for age effects. In particular, the United States has a markedly more hump-shaped age—wealth profile than the rest of the countries.⁹

4.2 Empirical findings

Equation (3) is estimated separately by OLS for each of the six countries in the study. The estimation results presented in Table 2 show a standard hump-shaped age—wealth profile where wealth increases during the working lifespan and declines somewhat after retirement in most countries. Wealth generally increases with education; the increase is, however, larger in Canada and in the United Kingdom than in the other countries. As expected, there is a negative association between female household head and wealth.

The first row of Table 3 reports the estimated age-adjusted Gini coefficient for the six countries in study, based on the baseline specification. The estimated age effects reported in Table 2 are used to compute the equalizing wealth levels defined by equation (4) and the associated AG given by equation (1). We can see that Italy has the least unequal wealth distribution whereas the United States has the most unequal distribution. The high wealth inequality for Sweden contrast with its low income inequality but conforms to findings from other studies (see e.g. Sierminska et al, 2006). 10

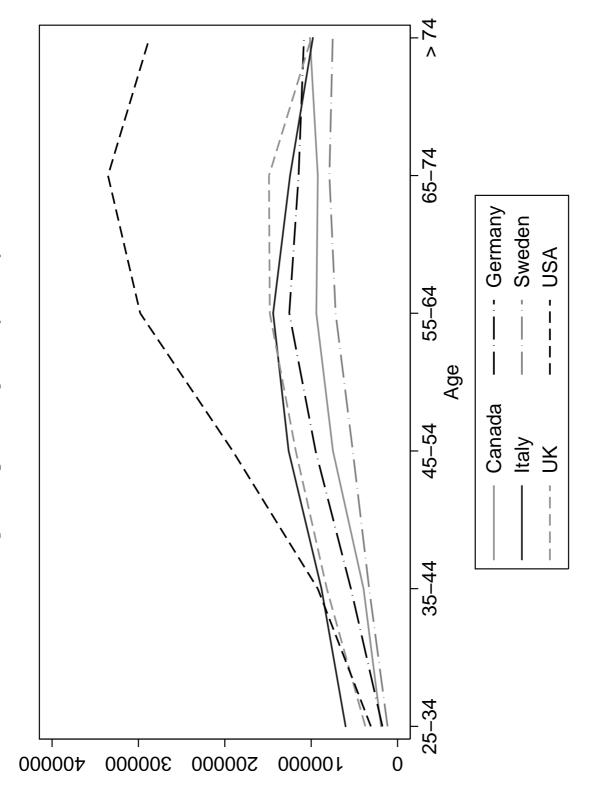
4.3 Robustness analysis

We run a battery of robustness checks to examine to what extent our results are sensitive to methodological choices. In some cases, the robustness analysis is performed only for a subset of the countries due to data availability. As shown

⁹The descriptive statistics are consistent with previous evidence showing substantial variation among OECD countries in the age structure (Burkhauser et al., 1997; Banks et al., 2003) as well as savings patterns (Borsch-Supan, 2003). We refer to Sierminska et al. (2006) for detailed descriptive statistics.

¹⁰Part of the explanation may be the relatively high proportion with zero or negative net wealth compared to the other countries.

Figure 1: Age–wealth profile by country



Note: National household wealth surveys included and harmonized in the LWS database. Household weights are used to ensure nationally representative results. Wealth levels expressed in USD (2000 exchange rates).

Table 2: Estimation results of the log-linear wealth regression: baseline specification

	Canada	Germany	Italy	Sweden	UK	USA
25-34 years	0.0262	-0.0012	-0.0369	0.000003	0.0450	-0.0018
	(0.0139)	(0.0004)	(0.0632)	(0.0005)	(0.0097)	(0.0006)
35-44 years	0.1736	0.0045	0.0456	0.0048	0.1194	0.0012
	(0.0140)	(0.0011)	(0.0617)	(0.0005)	(0.0108)	(0.0006)
45-54 years	0.3298	0.0141	0.1972	0.0096	0.1980	0.0067
	(0.0160)	(0.0007)	(0.0621)	(0.0006)	(0.0115)	(0.0006)
55-64 years	0.4501	0.0212	0.3152	0.0136	0.2234	0.0137
	(0.0173)	(0.0007)	(0.0624)	(0.0015)	(0.0363)	(0.0007)
65-74 years	0.5014	0.0211	0.3094	0.0182	0.2758	0.0163
	(0.0177)	(0.0007)	(0.0625)	(0.0008)	(0.0155)	(0.0008)
75 years and older	0.5324	0.0197	0.2565	-	0.2169	0.0161
	(0.0202)	(0.0008)	(0.0630)	(-)	(0.0134)	(0.0007)
High school graduate	0.1091	0.0063	0.1322	0.0040	0.0750	0.0051
	(0.0102)	(0.0015)	(0.0159)	(0.0005)	(0.0096)	(0.0003)
Post secondary	0.1114	0.0090	0.3211	0.0088	0.1270	0.0083
	(0.0096)	(0.0015)	(0.0188)	(0.0010)	(0.0161)	(0.0004)
University degree	0.2537	0.0182	0.4757	0.0094	0.2537	0.0180
	(0.0123)	(0.0015)	(0.0289)	(0.0016)	(0.0347)	(0.0004)
Female household head	-	-0.0051	-0.0382	-0.0029	-0.0111	-0.0048
	(-)	(0.0006)	(0.0127)	(0.0007)	(0.0097)	(0.0003)
Constant	11.862	15.361	12.140	17.533	12.556	16.558
	(0.0134)	(0.0015)	(0.0621)	(0.0005)	(0.0136)	(0.0006)
Number of observations	15,795	15,131	8,010	15,084	4,158	22,210
R-squared	0.196	0.025	0.150	0.007	0.059	0.064

Note: Based on national household wealth surveys included and harmonized in the LWS database. Household weights are used to ensure nationally representative results. Reference category: 24 years or younger, male, and high school dropout. Standard errors in parentheses. Canada does not provide data on sex of household head, and there are no household in the sample for Sweden with age of household head 75 years or older.

Table 3: Age adjusted wealth inequality ranking of countries

Measure	Italy	UK	Canada	Germany	Sweden	USA
AG	0.585(1)	0.696(2)	0.743(3)	0.770(4)	0.907(5)	0.922 (6)
AG(1)	0.585 (1)	0.696 (2)	0.743 (3)	0.770 (4)	0.907 (5)	0.922 (6)
AG(2)	0.524(1)	0.693(2)	0.742(3)	0.773(4)	0.894(5)	0.922(6)
AG(3)	0.524(1)	0.693(2)		0.773(3)	0.894(4)	
AG(4)	0.585(1)	0.696(2)		0.770(3)	0.907(4)	0.919(5)
AG(5)	0.665(1)	0.763(2)	0.831(3)	0.863(4)	0.983(5)	1.076(6)
AG(6)	0.590(1)	0.701(2)	0.742(3)	0.774(4)	0.911(5)	0.924(6)
AG(7)	0.584(1)	0.696(2)	0.744(3)	0.770(4)	0.908(5)	0.922(6)
AG(8)	0.585(1)	0.696(2)	0.743(3)	0.770(4)	0.907(5)	0.922(6)
AG(9)	0.585(1)	0.695(2)	0.742(3)	0.770(4)	0.907(5)	0.922(6)

Note:

AG: Estimation including sex of household head and education as controls.

AG(1): Estimation adding number of children as a control variable.

AG(2): Estimation adding number of children, occupation, industry and marital status as control variables.

AG(3): Estimation adding number of children, occupation, industry, marital status region and immigration status as control variables.

AG(4): Estimation adding spouses' characteristics (education and age) as control variables.

AG(5): Estimation based on the subsample of single households.

AG(6): Estimation based on the EU equivalence scaling.

AG(7): Estimation based on a growth rate of two percent.

AG(8): Estimation based on a growth rate of three percent.

AG(9): Estimation based on polynomials of continuous age variables.

Based on national household wealth surveys included and harmonized in the LWS database. Household weights are used to ensure nationally representative results. Country ranking in parentheses.

in Table 3, the main result is that the country ranking by wealth inequality does not change for any of the alternative specifications.

To be specific, the country ranking is unaffected by adding number of children to the set of controls for all countries (AG(1)). Moreover, adding covariates such as occupation, industry, marital status region, and immigration status (AG(1)-AG(2)), as well as the age and education of the spouse (AG(3)), does not alter the picture of inequality. Acknowledging the inherent arbitrariness in the choice of equivalence scale, we use an alternative equivalence scale (AG(6)) and we estimate the model on the subsample of singles (AG(5)); once again, the ranking is unchanged. On top of this, we make sure that the choice of economic growth rate does not affect our results by applying alternative growth rates (AG(7)-AG(8)). Finally, we make sure that using polynomials of continuous age variables instead of age-group dummies does not change the country ranking (AG(9)). The robustness analysis undertaken is described in more detail in Appendix A.

5 Evaluation of existing age adjusted inequality measures

There are two distinguishing aspects of age-adjusted inequality measures. First, they hold different views on how equalizing wealth should be defined. Second, the formula for calculating the differences between individuals' actual and equalizing wealth levels differ. Below, we consider two alternative age-adjusted inequality measures: the Paglin-Gini (PG) and the Wertz' Gini (WG). They both have the same objective, namely to purge the classical Gini coefficient (G) applied to snapshots of wealth inequality of its intra-age or life cycle component. In particular, the condition of a flat age-wealth profile is abandoned.

Below, we provide a theoretical and empirical evaluation of existing age adjusted inequality measures. Finally, we compare the inequality ranking stemming from the classical Gini coefficient to that of the age adjusted inequality measures.

5.1 Theoretical Evaluation

Both PG and WG define the equalizing wealth of an individual as the mean wealth level of the age groups he belongs to, but differs in the way they calculate the differences in actual and equalizing wealth. Although they both aim at

eliminating the age effects, we show that only WG does so in a way consistent with G. Unlike AG, however, both WG and PG fail to account for the fact that other wealth generating factors, such as education, are correlated with age. Consequently, they not only eliminate inequality due to age, but also inequality owing to these other factors. AG proves to encompass WG in the case where age is uncorrelated with all other wealth generating factors.¹¹

Using Conditions 1-4, we may assess the properties of PG and WG, and their relationship to G. First, consider the much used PG, which can be expressed as

$$PG(Y) = \frac{\sum_{j} \sum_{i} (|w_i - w_j| - |\mu_i - \mu_j|)}{2\mu n^2},$$
(5)

where μ_i and μ_j denote the mean wealth level of all individuals belonging to the age group of individual i and j, respectively.

Wertz (1979) claims that Paglin fails to adjust properly for age effects. This comment has been largely neglected, perhaps because Wertz does not put up conditions which allow a formal assessment of the properties of PG and WG. As an alternative Wertz proposes WG, which is given by

$$WG(Y) = \frac{\sum_{j} \sum_{i} |(w_i - \mu_i) - (w_j - \mu_j)|}{2un^2}.$$
(6)

Note that both G and WG are based on a comparison of the absolute values of the differences in actual and equalizing wealth levels between all pairs of individuals.¹² The distinguishing feature is that WG defines equalizing wealth of an individual i as μ_i , whereas G takes the equalizing wealth to be μ for everyone. By contrast, PG has no such analogue to G. Specifically, PG fails when it comes to the unequalism condition, since $|(w_i - \mu_i) - (w_j - \mu_j)| = 0$ does not necessarily imply that $|(w_i - w_j)| - |(\mu_i - \mu_j)| = 0$. It is also clear that WG is a special case

¹¹Even though Danziger et al. (1977) in an early comment to the Paglin-Gini (Paglin, 1975) pointed out that "no single indicator is sufficient to capture trends in normatively relevant inequality without a well-specified multivariate model," we are not aware of any previous studies that adjust for age effects while controlling for other determinants of individual wealth holdings. Minarik (1977), Kurien (1977), and Bishop et al. (1997) make similar criticisms.

 $^{^{12}}$ If everyone has nonnegative wealth, then PG is in the interval [0, G(Y)] and WG is in the interval [0, 2]. However, the numerical values of these ordinal inequality measures are only of interest as a way of comparing and ordering distributions by degree of inequality.

of AG where age is not correlated with any other wealth generating factors, that is, when \widetilde{w}_i equals μ_i for all i,.

A numerical example illustrates the deficiency of PG. Consider two countries A and B with two age groups, each consisting of two individuals. Suppose that country A's distribution of $(w_i(A), \mu_i(A))$ is given by

$$A = [(100, 60), (20, 60), (100, 80), (60, 80)].$$

Assume that country B's distribution of $(w_i(B), \mu_i(B))$ is given by

$$B = [(80, 40), (0, 40), (120, 100), (80, 100)].$$

In both countries, the distribution of differences between the actual and equalizing wealth $w_i - \mu_i$ is given by [{40, -40}, {20, -20}]. According to the condition of unequalism, age-adjusted inequality should therefore be the same in these two countries. It is clear that WG satisfies this condition, whereas PG violates it.¹³

As $|(w_i - w_j) - (\mu_i - \mu_j)|$ provides an upper bound for $|(w_i - w_j)| - |(\mu_i - \mu_j)|$, it follows that $WG \geq PG$. This begs the question: under which conditions will WG be equal to PG, and subsequently, can we be sure that the two measures produce the same inequality ranking? Wertz emphasizes that overlap in the wealth distributions across age groups is sufficient for PG to differ from WG. As shown in Appendix B, however, far stronger conditions than no overlap are required for PG to be equal to WG. In particular, PG will differ from WG if there is any age effect on wealth, provided that there is some within age group wealth variation. Consequently, empirical analyses using PG to adjust for age effects will generally be in danger of yielding a misleading inequality ranking of distributions. In comparison, studies applying WG risk to produce an erroneous inequality ranking only if age is correlated with other wealth generating factors.

¹³Specifically, WG(A) = WG(B) = 0.25, while $PG(A) = 0.179 \neq PG(B) = 0.107$.

¹⁴Overlap implies that the wealth holding of the richest person in an age group with a relatively low mean wealth level exceeds the wealth holding of the poorest person in an age group with a higher mean wealth level, that is $w_i < w_j$ and $\mu_i > \mu_j$ for at least one pair of individuals i and j.

Table 4: Wealth inequality ranking of countries according to different measures

	Italy	UK	Canada	Germany	Sweden	USA
G	0.587 (1)	0.708 (2)	0.769(3)	0.771 (4)	0.908 (5)	0.923 (6)
AG	0.585(1)	0.696(2)	0.743(3)	0.770(4)	0.907(5)	0.922(6)
WG	0.583(1)	0.697(2)	0.780(4)	0.779(3)	0.893(5)	1.101(6)
PG	0.495(3)	0.443(1)	0.452(2)	0.530(4)	0.616(6)	0.530(4)

Note: Based on national household wealth surveys included and harmonized in the LWS database. Household weights are used to ensure nationally representative results. Country ranking in parentheses.

5.2 Empirical evaluation

Table 4 shows the country ranking of wealth inequality according to the different age adjusted inequality measures, demonstrating that the existing approaches yield an erroneous wealth inequality ranking of countries. In particular, the country ranking given by PG is shown to be seriously distorted, whereas WG reports only a slightly different ranking. For example, according to PG the wealth inequality in Sweden is higher than that of the United States — a finding that runs counter to findings from the other age-adjusted inequality measures as well as from the classical Gini coefficient. Moreover, using PG alters the ranking of Italy from having clearly the most equal wealth distribution to being third out of the six countries.

As discussed above, there are two reasons why the country ranking may differ with the choice of age-adjusted inequality measure. First, PG fails to eliminate the age effects in a consistent way. Second, both PG and WG come short of accounting for the fact that other wealth generating factors are correlated with age, and thus confounding the age adjustment of inequality if not controlled for. For instance, younger cohorts have higher education than older cohorts and education is positively correlated with individual wealth holding. When employing the classical Gini coefficient, the age effects on inequality are offset by the impact of education. In comparison, when PG or WG are used to make age adjustments, no account is taken of the negative correlation between age and

education. Consequently, they overestimate the impact on inequality of making age adjustments.

5.3 What about the classical Gini coefficient?

The relationship between wealth and age implies that the classical Gini coefficient may suggest wealth inequality even if everyone is completely equal in all respects but age. To avoid confusing older with wealthier, it is necessary in principle to adjust for age effects in cross-section data. Nevertheless, it is interesting to investigate how G performs in practice, and whether it does perform better or worse than PG and WG.

Surprisingly, we observe from Table 4 that G produces the same ranking as AG. By contrast, PG and WG yield erroneous wealth inequality ranking. Although this may be reassuring for statistical offices and government agencies, which regularly rely on the classical Gini coefficient to evaluate cross-section wealth distributions, it remains to be seen whether this finding holds true for other applications.

6 Concluding remarks

A strong relationship between age and wealth implies that inequality of wealth at a given point in time is likely to exist even in a society where everyone is completely equal in all respects other than age. It has therefore been argued that age adjustments of inequality measures based on cross-section data are necessary.

This paper proposes a method to adjust for age effects in cross-section data, which eliminates transitory inequality, yet preserves inequality arising from other factors. Applying a cross-country comparable wealth database, we find that the existing approaches lead to an erroneous wealth inequality ranking of countries. Interestingly, our new age-adjusted Gini coefficient provides a wealth inequality ranking of countries that is identical to the ranking based on the classical Gini coefficient, which disregards age effects. A possible interpretation is that age adjustments are less important than previous studies have suggested, albeit this conclusion may not necessarily hold true for other applications.

There are a number of other applications where life cycle effects matter. For example, theoretical models and empirical results suggest a strong relationship between age and earnings. This raises several interesting questions. Is the

substantial increase in earnings inequality in developed countries over the last decades an artifact of the baby boomers growing older? Can reported divergence in global income inequality be explained by increased differences in the age structure of rich and poor countries? Our age-adjusted inequality measure can be used to investigate these questions.

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Appendix A Robustness analysis

A.1 Controlling for number of children—AG(1)

The first robustness check (AG(1)) includes the number of children in the household in the set of controls. As we can see from Table 5, wealth holdings increase with the number of children in the household in all countries, although the estimated coefficient is insignificant for Italy. Table 3 summarizes the wealth inequality ranking of countries by inequality measure for the robustness checks. We can see that the country ranking is unaffected by adding the number of children to the set of controls.

A.2 Controlling for number of children, occupation, industry and marital status—AG(2)

Table 6 presents the regression results from the second robustness check (AG(2)), which extends the set of controls with dummy variable for occupation, industry and marital status. This robustness analysis is performed only for Germany, Italy, Sweden, the United Kingdom, and the United States, due to data availability. For brevity, the coefficients for occupation and industry are excluded from the table. ¹⁵ Table 3 shows that controlling for number of children, occupation, industry, and marital status has no effect on country ranking by wealth inequality.

A.3 Controlling for number of children, occupation, industry, marital status, region and immigrant status— AG(3)

Table 7 shows the regression results from the third robustness check (AG(3)), where we add number of children, occupation, industry, marital status, region, and immigrant status to the set of controls. This robustness check is only carried out for Germany, Italy, Sweden, and the United Kingdom, as we lack information about some of these variables for the other countries. For brevity, the estimated coefficients for occupation and industry are excluded from the table. Table 3

¹⁵The dummy variables for some of the educational categories are dropped from the sample because of perfect collinearity between education and occupation.

Table 5: Number of children added as control variable - AG(1)

	Canada	Germany	Italy	Sweden	UK	USA
25-34 years	0.0118	-0.0021	-0.0395	-0.0008	0.0410	-0.0024
	(0.0141)	(0.0004)	(0.0636)	(0.0005)	(0.0100)	(0.0006)
35-44 years	0.1508	0.0025	0.0407	0.0030	0.1114	0.0003
	(0.0146)	(0.0008)	(0.0626)	(0.0006)	(0.0111)	(0.0006)
45-54 years	0.3203	0.0133	0.1951	0.0088	0.1967	0.0065
	(0.0161)	(0.0006)	(0.0621)	(0.0006)	(0.0114)	(0.0006)
55-64 years	0.4553	0.0212	0.3154	0.0136	0.2262	0.0142
	(0.0173)	(0.0007)	(0.0624)	(0.0015)	(0.0364)	(0.0007)
65-74 years	0.5092	0.0214	0.3100	0.0183	0.2792	0.0169
	(0.0177)	(0.0008)	(0.0625)	(0.0008)	(0.0156)	(0.0008)
75 years and older	0.5409	0.0199	0.2570	-	0.2206	0.0168
	(0.0202)	(0.0008)	(0.0629)	(-)	(0.0135)	(0.0007)
High school graduate	0.1103	0.0067	0.1321	0.0039	0.0754	0.0052
	(0.0102)	(0.0015)	(0.0159)	(0.0005)	(0.0096)	(0.0003)
Post secondary	0.1125	0.0097	0.3212	0.0088	0.1285	0.0085
	(0.0096)	(0.0015)	(0.0188)	(0.0010)	(0.0160)	(0.0004)
University degree	0.2558	0.0188	0.4760	0.0094	0.2555	0.0182
	(0.0123)	(0.0016)	(0.0289)	(0.0016)	(0.0346)	(0.0004)
Female household head	-	-0.0050	-0.0376	-0.0028	-0.0112	-0.0050
	(-)	(0.0006)	(0.0127)	(0.0007)	(0.0096)	(0.0003)
Number of children	0.0515	0.0021	0.0047	0.0016	0.0084	0.0012
	(0.0087)	(0.0004)	(0.0100)	(0.0002)	(0.0041)	(0.0001)
Constant	11.853	15.35985	12.13889	17.533	12.552	16.557
	(0.0134)	(0.0016)	(0.0621)	(0.0005)	(0.0136)	(0.0006)
Number of observations	15,778	15,131	8,010	15,084	4,158	22,210
R-squared	0.198	0.025	0.150	0.007	0.059	0.065

Table 6: Children, occupation, industry, and marital status added as control variables—AG(2)

	Germany	Italy	Sweden	UK	USA
25-34 years	-0.0035 (0.0005)	-0.0251 (0.1011)	-0.0012 (0.0008)	0.0201 (0.0128)	-0.0029 (0.0006)
35-44 years	0.0006 (0.0008)	0.0478 (0.0992)	0.0032 (0.0008)	0.0945 (0.0159)	-0.0006 (0.0006)
45-54 years	0.0108 (0.0007)	0.2284 (0.1001)	0.0092 (0.0009)	0.1772 (0.0162)	0.0056 (0.0007)
55-64 years	0.0200 (0.0008)	0.3493 (0.1016)	0.0140 (0.0015)	0.2006 (0.0353)	0.0128 (0.0008)
65-74 years	0.0219 (0.0010)	0.3850 (0.1065)	0.0202 (0.0020)	0.2648 (0.0206)	0.0150 (0.0009)
75 years and older	0.0225 (0.0009)	0.3120 (0.1157)	- (-)	0.2115 (0.0200)	0.0152 (0.0009)
High school graduate	0.0050 (0.0017)	(dropped) (-)	0.0039 (0.0010)	0.0723 (0.0095)	0.0050 (0.0003)
Post secondary	0.0074 (0.0017)	-0.3068 (0.1099)	0.0079 (0.0016)	0.1052 (0.0189)	0.0079 (0.0004)
University degree	0.0137 (0.0017)	(dropped)	0.0090 (0.0029)	0.2104 (0.0375)	0.0159 (0.0005)
Female household head	-0.0023 (0.0007)	0.0184 (0.0355)	-0.0001 (0.0008)	-0.0053 (0.0081)	-0.0044 (0.0003)
Number of children	-0.0001 (0.0005)	-0.0213 (0.0214)	0.0007 (0.0004)	0.0113 (0.0094)	0.0008 (0.0002)
Single parent	0.0046 (0.0012)	0.0045	-0.0002 (0.0012)	0.0230 (0.0215)	0.0023 (0.0005)
Couple without children	0.0079 (0.0006)	0.0303 (0.0424)	0.0029 (0.0012)	0.0443 (0.0117)	0.0038 (0.0004)
Couple with children	0.0100 (0.0008)	0.0615 (0.0413)	0.0036 (0.0008)	0.0175 (0.0282)	0.0030 (0.0005)
Constant	15.356 (0.0019)	12.671 (0.1586)	17.531 (0.0016)	12.549 (0.0232)	16.558 (0.0009)
Number of observations R-squared	14,760 0.028	2,352 0.147	11,838 0.007	4,001 0.081	22,210 0.071

shows that controlling for number of children, occupation, industry, and marital status has no effect on country ranking by wealth inequality.

A.4 Controlling for characteristics of the spouse—AG(4)

Table 8 shows the regression results from the fourth robustness check (AG(4)), which includes the age and education of the spouse. This robustness check is only carried out for Germany, Italy, Sweden, the United Kingdom, and the United States, as we lack information about the characteristics of spouses for Canada. In this case, we have a small multicollinearity problem, as the characteristics of the head of the household and the spouse are significantly correlated. However, most coefficients remain significant when we include the age and education of the spouse. Table 3 shows that the country ranking by wealth inequality is robust to adding the characteristics of the spouse to the regression model.

A.5 Estimating on the subsample of singles—AG(5)

There are a couple of reasons for estimating our model on the subsample of singles. First, it can be agued that the use of equivalence scaling is a crude way to capture pooling of wealth and economics of scale within the household. Second, in the main specification, we have followed common practice and used information about the head of the household to determine the age groups. However, the age of the spouse may also be relevant for determining the age or life cycle effects on household wealth holding. Table 9 presents regression results from the fifth robustness check (AG(5)), where we estimate the model on the subsample of singles. From table 3 we can see that the country ranking by inequality is robust to whether we estimate the model on the entire sample or on the subsample of singles.

A.6 The EU equivalence scale—AG(6)

Acknowledging the inherent arbitrariness in the choice of equivalence scale, we perform another robustness check (AG(6)), where we use the EU equivalence scale instead of the square root equivalence scale. Table 10 shows the corresponding regression results. As demonstrated by Table 3, the country ranking by wealth inequality is robust to the choice of equivalence scale.

Table 7: Children, occupation, industry, marital status, region, and immigrant status added as control variables—AG(3)

	Germany	Italy	Sweden	UK
25-34 years	-0.0034 (0.0006)	-0.0142 (0.0955)	-0.0017 (0.0008)	0.0143 (0.0127)
35-44 years	0.0009 (0.0008)	0.0579 (0.0942)	0.0030 (0.0009)	0.0974 (0.0174)
45-54 years	0.0110 (0.0008)	$0.2206 \\ (0.0956)$	0.0089 (0.0009)	0.1644 (0.0159)
55-64 years	0.0199 (0.0008)	0.3719 (0.0963)	0.0136 (0.0015)	0.1892 (0.0381)
65-74 years	0.0207 (0.0010)	0.3845 (0.1016)	0.0195 (0.0020)	0.2515 (0.0216)
75 years and older	$0.0205 \\ (0.0010)$	0.3090 (0.1125)	- (-)	0.1972 (0.0236)
High school graduate	0.0017 (0.0018)	(dropped) $(-)$	0.0033 (0.0010)	0.0781 (0.0090)
Post secondary	$0.0051 \\ (0.0017)$	(dropped) $(-)$	$0.0069 \\ (0.0016)$	0.1099 (0.0174)
University degree	0.0125 (0.0017)	0.3086 (0.1260)	0.0075 (0.0031)	0.2263 (0.0344)
Female household head	-0.0018 (0.0007)	$0.0105 \\ (0.0351)$	-0.0001 (0.0008)	-0.0028 (0.0081)
Single parent	0.0044 (0.0010)	-0.0382 (0.0735)	0.0003 (0.0010)	0.0254 (0.0200)
Couple no children	0.0072 (0.0007)	0.0107 (0.0410)	0.0035 (0.0011)	0.0379 (0.0099)
Couple with children	$0.0098 \\ (0.0008)$	0.0528 (0.0411)	0.0041 (0.0008)	0.0169 (0.0313)
Number of children	-0.0004 (0.0005)	-0.0075 (0.0214)	0.0007 (0.0004)	0.0068 (0.0074)
Immigration status	-0.0115 (0.0007)	-0.3277 (0.0520)	-0.0049 (0.0005)	-0.0173 (0.0287)
Constant	$ \begin{array}{c} 15.344 \\ (0.0047) \end{array} $	$ \begin{array}{c} 12.345 \\ (0.1272) \end{array} $	$ \begin{array}{c} 17.538 \\ (0.0020) \end{array} $	$ \begin{array}{c} 12.524 \\ (0.0978) \end{array} $
Number of observations R-squared	14,589 0.041	2,352 0.209	11,838 0.009	3,940 0.106

Note: Based on national household wealth surveys included and harmonized in the LWS database. Household weights are used to ensure nationally representative results. Reference category: 24 years or younger, male, and high school dropout. Standard errors in parentheses. The indicators for industry, occupation, and region are not shown in the table.

Table 8: The characteristics of the spouse added as control variables—AG(4)

	Germany	Italy	Sweden	UK	USA
25-34 years	-0.0021 (0.0021)	-0.0196 (0.0646)	-0.0001 (0.0005)	0.0430 (0.0141)	-0.0044 (0.0108)
35-44 years	0.0015 (0.0021)	0.0272 (0.0633)	0.0035 (0.0007)	$0.1001 \\ (0.0140)$	-0.0009 (0.0107)
45-54 years	0.0143 (0.0022)	0.1305 (0.0642)	0.0075 (0.0013)	0.1687 (0.0145)	0.0117 (0.0107)
55-64 years	0.0205 (0.0022)	0.2367 (0.0651)	0.0102 (0.0016)	0.1982 (0.0153)	$0.0500 \\ (0.0112)$
65-74 years	0.0191 (0.0022)	0.2398 (0.0645)	0.0153 (0.0010)	0.2164 (0.0155)	0.0994 (0.0118)
75 years and older	0.0221 (0.0022)	0.2053 (0.0646)	- (-)	0.1963 (0.0149)	0.0991 (0.0122)
High school graduate	0.0078 (0.0028)	$0.1072 \\ (0.0171)$	0.0035 (0.0005)	0.0594 (0.0082)	0.0272 (0.0084)
Post secondary	0.0103 (0.0027)	0.2586 (0.0199)	0.0080 (0.0009)	0.1062 (0.0076)	0.0458 (0.0093)
University degree	0.0213 (0.0028)	0.3671 (0.0321)	0.0078 (0.0012)	0.1937 (0.0197)	$0.0905 \\ (0.0085)$
Spouse 25-34 years	-0.0018 (0.0021)	-0.1518 (0.0341)	-0.0026 (0.0010)	-0.0678 (0.0128)	-0.0359 (0.0103)
Spouse 35-44 years	0.0018 (0.0020)	-0.0971 (0.0276)	0.0011 (0.0008)	-0.0294 (0.0126)	-0.0074 (0.0097)
Spouse 45-54 years	0.0003 (0.0020)	0.0026 (0.0258)	0.0014 (0.0026)	-0.0321 (0.0128)	0.0369 (0.0099)
Spouse 55-64 years	0.0048 (0.0020)	0.0319 (0.0245)	0.0055 (0.0018)	0.0324 (0.0138)	0.0420 (0.0107)
Spouse 65-74 years	0.0012 (0.0021)	0.0213 (0.0236)	0.0055 (0.0016)	0.0283 (0.0148)	0.0145 (0.0117)
Spouse 75 years or older	0.0067 (0.0013)	-0.0090 (0.0270)	- (-)	0.0174 (0.0099)	0.0888 (0.0150)
Spouse high school graduate	0.0006 (0.0019)	0.0652 (0.0205)	0.0031 (0.0007)	0.0626 (0.0104)	-0.0055 (0.0087)
Spouse post secondary	0.0073 (0.0017)	0.1584 (0.0242)	0.0046 (0.0014)	0.0905 (0.0096)	0.0177 (0.0104)
Spouse university degree	0.0108 (0.0018)	0.2403 (0.0446)	0.0058 (0.0022)	0.1254 (0.0287)	0.0741 (0.0090)
Female household head	-0.0048 (0.0008)	-0.0287 (0.0143)	-0.0014 (0.0005)	0.0068 (0.0062)	-0.0569 (0.0038)
Constant	$ \begin{array}{c} 15.357 \\ (0.0032) \end{array} $	$ \begin{array}{c} 12.187 \\ (0.0642) \end{array} $	$ \begin{array}{c} 17.532 \\ (0.0005) \end{array} $	$ \begin{array}{c} 12.539 \\ (0.0142) \end{array} $	$ \begin{array}{c} 16.547 \\ (0.0119) \end{array} $
Number of observations	15,603 0.026	8,010 0.169	15,084 0.008	7,331 0.107	22,210 0.101
R-squared	0.020	0.109	0.008	0.107	0.101

Table 9: Estimation results based on the subsample of singles—AG(5)

	Canada	Germany	Italy	Sweden	UK	USA
25-34 years	-0.0057	0.0039	0.0292	-0.0021	0.1554	-0.0026
	(0.0121)	(0.0039)	(0.2594)	(0.0015)	(0.0469)	(0.0009)
35-44 years	0.1182	0.0253	-0.1428	0.0071	0.3211	-0.0002
	(0.0140)	(0.0038)	(0.2505)	(0.0020)	(0.0494)	(0.0009)
45-54 years	0.2411	0.0990	0.3807	0.0206	0.5460	0.0042
	(0.0216)	(0.0079)	(0.2481)	(0.0023)	(0.0536)	(0.0011)
55-64 years	0.3486	0.1468	0.8814	0.0282	0.6467	0.0087
	(0.0242)	(0.0078)	(0.2477)	(0.0075)	(0.0654)	(0.0011)
65-74 years	0.4560	0.1874	0.7929	0.0466	0.7837	0.0097
	(0.0233)	(0.0089)	(0.2488)	(0.0038)	(0.0597)	(0.0011)
75 years or older	0.5369	0.1490	0.7012	-	0.7065	0.0130
	(0.0250)	(0.0069)	(0.2469)	(-)	(0.0487)	(0.0011)
High school graduate	0.1026	0.0137	0.2707	0.0078	0.2841	0.0049
	(0.0151)	(0.0276)	(0.0902)	(0.0034)	(0.0385)	(0.0004)
Post secondary	0.1079	0.0326	0.9263	0.0174	0.4373	0.0087
	(0.0149)	(0.0273)	(0.0883)	(0.0037)	(0.0386)	(0.0006)
University degree	0.2201	0.0580	1.0063	0.0253	0.6965	0.0136
	(0.0211)	(0.0278)	(0.1845)	(0.0030)	(0.1603)	(0.0007)
Female household head	-	-0.0113	-0.1856	-0.0023	-0.0522	-0.0031
	(-)	(0.0050)	(0.0671)	(0.0027)	(0.0309)	(0.0005)
Constant	11.829	12.881	10.398	16.296	10.823	16.559
	(0.0131)	(0.0270)	(0.2492)	(0.0022)	(0.0538)	(0.0010)
Number of observations	$5,\!555$	4,546	2,634	5,528	1,363	7,485
R-squared	0.220	0.102	0.118	0.012	0.188	0.039

Table 10: The EU equivalence scale—AG(6)

	Canada	Germany	Italy	Sweden	UK	USA
25–34 years	0.0377 (0.0154)	-0.0014 (0.0005)	-0.0176 (0.0751)	-0.00008 (0.0005)	0.0551 (0.0113)	-0.0018 (0.0006)
35–44 years	0.1911 (0.0153)	0.0045 (0.0011)	0.0590 (0.0733)	0.0035 (0.0005)	0.1248 (0.0118)	0.00008 (0.0006)
45–54 years	0.3657 (0.0180)	0.0155 (0.0008)	0.1874 (0.0734)	0.0079 (0.0006)	0.2107 (0.0127)	0.0049 (0.0006)
55–64 years	0.5584 (0.0196)	0.0279 (0.0009)	0.3404 (0.0739)	0.0130 (0.0015)	0.2673 (0.0365)	0.0127 (0.0007)
65–74 years	0.6516 (0.0205)	0.0291 (0.0010)	0.3910 (0.0742)	0.0183 (0.0007)	0.3397 (0.0177)	0.0153 (0.0008)
75 years or older	0.6970 (0.0236)	0.0280 (0.0010)	0.3506 (0.0748)	- (-)	0.2764 (0.0154)	0.0155 (0.0007)
High school graduate	0.1246 (0.0118)	0.0076 (0.0022)	0.1467 (0.0173)	0.0038 (0.0005)	0.0907 (0.0109)	0.0049 (0.0003)
Post secondary	0.1260 (0.0110)	0.0119 (0.0022)	0.3529 (0.0202)	0.0079 (0.0010)	0.1548 (0.0166)	0.0080 (0.0004)
University degree	0.2735 (0.0144)	0.0228 (0.0022)	0.5299 (0.0310)	0.0083 (0.0016)	0.2947 (0.0390)	0.0162 (0.0004)
Female household head	- (-)	-0.0059 (0.0007)	-0.0338 (0.0137)	-0.0023 (0.0007)	-0.0103 (0.0103)	-0.0043 (0.0003)
Constant	11.41 (0.0149)	14.96 (0.0022)	11.73 (0.0734)	17.47 (0.0005)	12.19 (0.0151)	16.56 (0.0006)
Number of observations R-squared	15,778 0.219	15,131 0.038	8,010 0.152	15,084 0.006	4,158 0.079	22,210 0.063

Note: Based on national household wealth surveys included and harmonized in the LWS database. Household weights are used to ensure nationally representative results. Reference category: 24 years or younger, male, and high school dropout. Standard errors in parentheses. The EU equivalence scale is given by: $EU_EQ = 1 + 0.5*(adults - 1) + 0.3*children.$

A.7 Alternative growth rates—AG(7) and AG(8)

This paper attempts to separate age effects from cohort/time-specific effects by adjusting for economic growth. In the main specification, we use an annual (real) growth rate of 2.5 percent. As a robustness check, we also experiment with annual growth rates of two percent (AG(7)) and three percent (AG(8)). Tables 11 and 12 show the estimation results based on these alternative assumptions about the economic growth rate. It is evident from Table 3 that the country ranking by wealth inequality is robust to the choice of growth rate.

A.8 Polynomials of continuous age variables—AG9

The last robustness check performed in this paper is to replace the age-group dummies with polynomials of continuous age variables (AG9). Table presents the estimation results with continuous age variables. It is clear from Table 13 that the country ranking is robust, whether we represent the age effects by age-group dummies or polynomials of continuous age variables.

Table 11: Two percent annual growth rate—AG(7)

	Canada	Germany	Italy	Sweden	UK	USA
25–34 years	0.0262	-0.0012	-0.0369	0.000003	0.0450	-0.0018
	(0.0139)	(0.0004)	(0.0632)	(0.0005)	(0.0097)	(0.0006)
35–44 years	0.1736	0.0045	0.0456	0.0048	0.1194	0.0012
	(0.0140)	(0.0011)	(0.0617)	(0.0005)	(0.0108)	(0.0006)
45–54 years	0.3298	0.0141	0.1972	0.0096	0.1980	0.0067
	(0.0160)	(0.0007)	(0.0621)	(0.0006)	(0.0115)	(0.0006)
55–64 years	0.4501	0.0212	0.3152	0.0136	0.2234	0.0137
	(0.0173)	(0.0007)	(0.0624)	(0.0015)	(0.0363)	(0.00073)
65–74 years	0.5014	0.0211	0.3094	0.0182	0.2758	0.0163
	(0.0177)	(0.0007)	(0.0625)	(0.0008)	(0.0155)	(0.0008)
75 years or older	0.5324	0.0197	0.2565	-	0.2169	0.0161
	(0.0202)	(0.0008)	(0.0630)	(-)	(0.0134)	(0.0007)
High school graduate	0.1091	0.0063	0.1322	0.0040	0.0750	0.0051
	(0.0102)	(0.0015)	(0.0159)	(0.0005)	(0.0096)	(0.0003)
Post secondary	0.1114	0.0090	0.3211	0.0088	0.1270	0.0083
	(0.0096)	(0.0015)	(0.0188)	(0.0010)	(0.0161)	(0.0004)
University degree	0.2537	0.0182	0.4757	0.0094	0.2537	0.0180
	(0.0123)	(0.0015)	(0.0289)	(0.0016)	(0.0347)	(0.0004)
Female household head	-	-0.0051	-0.0382	-0.0029	-0.0111	-0.0048
	(-)	(0.0006)	(0.0127)	(0.0007)	(0.0096)	(0.0003)
Constant	11.86	15.36	12.13	17.53	12.55	16.55
	(0.0134)	(0.0015)	(0.0621)	(0.0005)	(0.0136)	(0.0006)
Number of observations	15,795	15,131	8,010	15,084	4,158	22,210
R-squared	0.196	0.025	0.150	0.007	0.059	0.064

Table 12: Three percent annual growth rate—AG8

	Canada	Germany	Italy	Sweden	UK	USA
25–34 years	0.0262 (0.0139)	-0.0012 (0.0004)	-0.0369 (0.0632)	0.000003 (0.0005)	0.0450 (0.0097)	-0.0018 (0.0006)
35–44 years	0.1736 (0.0140)	0.0045 (0.0011)	0.0456 (0.0617)	0.0048 (0.0005)	0.1194 (0.0108)	0.0012 (0.0006)
45–54 years	0.32984 (0.0160)	0.01406 (0.0007)	0.1972 (0.0621)	0.0096 (0.0006)	0.1980 (0.0115)	0.0067 (0.0006)
55–64 years	0.4501 (0.0173)	0.0212 (0.0007)	0.3152 (0.0624)	0.0136 (0.0015)	0.2234 (0.0363)	0.0137 (0.0007)
65–74 years	0.5014 (0.0177)	0.0211 (0.0007)	0.3094 (0.0625)	0.0182 (0.0008)	0.2758 (0.0155)	0.0163 (0.0008)
75 years or older	0.5324 (0.0202)	0.0197 (0.0008)	0.2565 (0.0630)	- (-)	0.2169 (0.0134)	0.0161 (0.0007)
High school graduate	0.1091 (0.0102)	0.0063 (0.0015)	0.1322 (0.0159)	0.0040 (0.0005)	0.0750 (0.0096)	0.0051 (0.0003)
Post secondary	0.1114 (0.0097)	0.0090 (0.0015)	0.3211 (0.0188)	0.0088 (0.0010)	0.1270 (0.0161)	0.0083 (0.0004)
University degree	0.2537 (0.0123)	0.0182 (0.0015)	0.4757 (0.0289)	0.0094 (0.0016)	0.2537 (0.0347)	0.0180 (0.0004)
Female household head	- (-)	-0.0051 (0.0006)	-0.0382 (0.0127)	-0.0029 (0.0007)	-0.0111 (0.0097)	-0.0048 (0.0003)
Constant	11.87 (0.01334)	15.37 (0.0015)	12.14 (0.0621)	17.54 (0.0005)	12.56 (0.0136)	16.56 (0.0006)
Number of observations R-squared	15,795 0.196	15,131 0.025	8,010 0.150	15,084 0.007	4,158 0.059	22,210 0.064

Table 13: Polynomials of continuous age variables—AG9

	Canada	Germany	Italy	Sweden	UK	USA
Age	-0.0208 (0.0055)	-0.0017 (0.0004)	-0.0040 (0.0106)	-0.0012 (0.0006)	-0.0011 (0.0039)	-0.0020 (0.0002)
Age^2	0.0008 (0.0001)	0.00006 (0.000007)	0.0005 (0.0002)	0.00004 (0.00002)	0.0002 (0.00008)	0.00005 (0.000005)
$\mathrm{Age^3}$	0.000006 (0.0000008)	0.0000004 (0.00000004)	0.000004 (0.000001)	0.0000002 (0.0000001)	0.000002 (0.0000005)	0.0000003
High school graduate	0.1132 _(0.0101)	0.0064 (0.0015)	0.1358 (0.0160)	0.0042 (0.0005)	0.0742 (0.0094)	0.0054 (0.0003)
Post secondary	0.1155 (0.0096)	0.0090 (0.0015)	0.3254 (0.0188)	0.0092 (0.0010)	0.1276 (0.0155)	0.0086 (0.0004)
University degree	0.2577 (0.0123)	0.0181 (0.0015)	0.4790 (0.0291)	0.0096 (0.0016)	0.2575 (0.0347)	0.0182 (0.0004)
Female household head	- (-)	-0.0049 (0.0007)	-0.0368 (0.0128)	-0.0029 (0.0007)	-0.0098 (0.0096)	-0.0047 (0.0003)
Constant	12.15 (0.0777)	15.57 (0.0062)	12.10 (0.1822)	17.74 (0.0079)	12.68 (0.0629)	16.78 (0.0028)
Number of observations Adjusted R-squared	15,795 0.204	15,131 0.025	8,010 0.151	15,084 0.007	4,158 0.062	22,210 0.064

Appendix B Is it all about overlap?

As stated in Proposition 1, PG will differ from WG if there is any age effect on wealth, provided that there is some within age group wealth variation.

Proposition 1. For any distribution Y, $WG(Y) \ge PG(Y)$, with strong inequality whenever $\mu_i \ne \mu_j$ for at least one pair of individuals and $w_i \ne \mu_i$ or $w_j \ne \mu_j$ for at least one of these individuals.

Proof of Proposition 1

Proof. The triangle inequality theorem states that $|x - y| \ge |x| - |y|$, and inequality holds if and only if one of the following conditions is satisfied:

- (i) x > 0 and y < 0
- (ii) x < 0 and y > 0
- (iii) x > y and y < 0
- (iv) x < y and y > 0

Let $x = (w_i - w_j)$ and $y = (\mu_i - \mu_j)$. It follows that WG > PG if and only if one of the above conditions hold for at least one pair of individuals i and j.

Let min_i denote minimum wealth in the age group of individual i, and let max_j denote the maximum wealth in the age group of individual j.

Suppose that $\mu_i > \mu_j$, and that $min_i < \mu_i$ or $max_j > \mu_j$. ¹⁶

Assume that $min_i \geq max_j$. Then, $min_i - \mu_i < max_j - (\mu_j)$ and condition (iv) holds.

Assume that $min_i < max_i$. Then, condition (ii) holds.

Consequently, $\mu_i \neq \mu_j$ for at least one pair of individuals and $w_i \neq \mu_i$ or $w_j \neq \mu_j$ for at least one of these individuals are sufficient conditions for WG > PG.

Note that overlap requires that $w_i < w_j$ and $\mu_i > \mu_j$ for at least one pair of individuals i and j, which is a special case of Proposition 1.

¹⁶Note that $min_i = \mu_i$ if and only if $w_i = \mu_i$ for every individual in the age group of individual i, whereas $max_j = \mu_j$ if and only if $w_j = \mu_j$ for every individual in the age group of individual j.

Proposition 1 also has implications for the ongoing controversy about using the standard Gini decomposition to adjust for differences attributable to age. Applying the standard Gini decomposition, it is well known that PG can be expressed as:

$$PG(Y) = G(Y) - G_b(Y) = \sum_{i} \theta_i G_i(Y) + R(Y), \tag{7}$$

where G_b represents the Gini coefficient that would be obtained if the wealth holding of each individual in every age group were replaced by the relevant age group mean μ_i , G_i is the Gini coefficient of wealth within the age group of individual i, θ_i is the weight given by the product of this group's wealth share $\frac{n_i \mu_i}{\mu n}$ and population share $\frac{n_i}{n}$ (n_i being the number of individuals in the age group of individual i), and R is the overlap term (see e.g. Lambert and Aronson, 1993).

Nelson (1977) and others argue that R is part of between-group inequality and should thus be netted out when constructing age-adjusted inequality measures. Paglin (1977), however, maintains that R is capturing within-group inequality and that PG is accurately defined. Until recently, the issue was unsettled simply because little was known about the overlap term; Shorrocks and Wan (2005), for example, refer to R as a "poorly specified" element of the Gini decomposition. However, Lambert and Decoster (2005) provide a novel characterization of the properties of R, showing that it unambiguously falls as a result of a within-group progressive transfer, and increases by scaling up the incomes in the group by the lower mean, reaching a maximum when the two means become the same. This makes Lambert and Decoster (2005, p. 378) conclude that "The overlap term in R is at once a between-groups and a within-groups effect: it measures a betweengroups phenomenon, overlapping that is generated by inequality within groups". Thus, it appears that neither Paglin nor Nelson was right, and that no overlap is necessary for equation (7) to eliminate wealth differences attributable to age in a consistent way.

A corollary to Proposition 1 is that we may very well have PG(Y) < WG(Y) even when R(Y) = 0 in the PG expression given by (4). Hence, the absence of overlap is far from a sufficient condition. Indeed, the standard Gini decomposition approach will, in general, come short of correctly netting out the age effects.