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Variance Estimation for Richness Measures

Michal Brzezinski

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Variance estimation for richness measures

Michal Brzezinski

Faculty of Economic Sciences
University of Warsaw
Dluga 44/50, 00-241, Warsaw, Poland
E-mail: mbrzezinski@wne.uw.edu.pl

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Abstract

Richness indices are distributional statistics used to measure incomes, earnings or wealth of the rich. This paper uses a linearization method to derive the sampling variances for recently introduced two classes of distributionally-sensitive richness measures, when estimated from survey data. The results are derived for both absolute and relative richness lines. The approach used allows easily to take into account the effects of complex sampling design. The variance formulae are illustrated with a comparison of wealth richness in Canada, Sweden, the United Kingdom and the United States.

Keywords: richness, affluence, distributional indices, variance estimation, statistical inference

JEL Classification Numbers: C12, C46, D31

1. Introduction

In recent years, there has been growing interest in distributional analysis of the top part of income and wealth distributions. The measures most commonly applied in this literature are the richness headcount ratio (Medeiros, 2006), which is the proportion of individuals in the population above the richness line and top quantile shares (see, e.g., Atkinson *et al.*, 2011 for the case of top income shares and Jäntti *et al.*, 2008 for that of top wealth shares).

As noticed by Peich *et al.* (2010), using richness headcount ratio or top quantile shares as measures of distributional affluence is associated with serious limitations.[1] Assuming that there is no mobility between rich and non-rich individuals, the former measure is insensitive to the changes in both affluence of the rich (richness intensity) and inequality among the rich. The latter measures assume that the proportion of the rich is fixed and do not take take into account the distribution of well-being among the rich. In order to overcome these deficiencies, Peich *et al.* (2010) introduced a new class of distributionally-sensitive richness indices, analogous to the well-known family of additively separable poverty indices (Atkinson, 1987), which is sensitive to both richness intensity and inequality among the rich. Peichl *et al.* (2010) use their indices to provide a detailed analysis of income richness in Germany and a comparison of income richness among 26 European countries. Peichl and Pestel (2011) compare income and wealth affluence in Germany and the United States in 2007 as well as analyze changes in the US richness over the period 1989–2007. Moreover, their paper generalizes the framework of Peich *et al.* (2010) to the multidimensional one. In addition, the measures of Peich *et al.* (2010) have been recently applied to analyze changes in income richness over time in Poland (Brzezinski, 2010) and to study wealth affluence in Italy (Eisenhauer, 2011).

Although much of the recent literature on top quantile shares uses income tax data (see, in particular, twenty two country specific chapters in Atkinson and Piketty, 2007, 2010), many other works use rather household survey data. Survey data are used to measure the top of the distribution when tax return data are not available or only available for a limited time span (see, e.g., Leigh and Van der Eng, 2009; Piketty and Qian, 2009; Brzezinski, 2010) as well as when one wants to compare results from tax returns and survey data (Burkhauser *et al.*, 2011) or when one wants to compute measures requiring micro-data just as those introduced by Peichl *et al.* (2010) and Peichl and Pestel (2011).

Income tax data used in the recent literature on top income shares are usually based on very large samples, sometimes covering the whole population of taxpayers. On the other hand, household survey data often come from samples of moderate or rather small size. In this context, the question of sampling variability of richness statistics calculated using survey data arises naturally. This paper uses variance linearization approach of Deville (1999) to derive the sampling variances for the richness headcount radio and distributionally-sensitive intensity measures introduced by Peichl *et al.* (2010).[2] The approach used allows for estimating variance of richness measures with either absolute or relative richness line, the latter defined as a multiple of the mean, median or other

[1] Top quantile shares may be of course used not only as measures of richness, but also as distributional measures *per se* or as approximate inequality indices.

[2] For variance estimation of top quantile shares without sample weights, see Beach and Davidson (1983). Beach and Kaliski (1986) provide an extension to the case with weights. See also Binder and Kovacevic (1995) and Verma and Betti (2011).

quantile of the underlying distribution.

In order to deal with statistical inference on richness indices, Brzezinski (2010) and Peichl and Pestel (2011) use bootstrap methods to calculate confidence intervals and to test hypotheses on richness changes. However, bootstrap methods are computationally intensive and require special methods when applied to frequently found in practice complex sample surveys with such features as probability weighting, stratification and clustering. Wolter (2007, p. 365) suggests that bootstrap methods for complex surveys should be tested adequately before they can be recommended in a unqualified way. The proposed proposed linearlization methodology does not suffer from these two limitations – it is fast and can be used in a standard way under any complex survey design applied in real-world surveys.

2. Richness indices

Let U be the population of N individuals with incomes denoted by $x_1,...,x_k,...,x_N$. For a given richness line, ρ, an individual is rich if her income is above ρ. The richness line may be defined in absolute terms as well as in relative terms.[3] The most simple richness index is richness headcount ratio defined as a proportion of the population above the richness line and given by

$$R^{HC} = \frac{1}{N}\sum_{k\in U}\delta(x_k > \rho) = 1 - F(\rho), \tag{1}$$

where $\delta(\cdot)$ is an indicator function equal to 1 when its argument is true and 0 otherwise and $F(\cdot)$ is the cumulative distribution function of x_k. Richness indices introduced by Peichl et al. (2010) belong to the class of decomposable (additively separable) measures defined as follows

$$R = \frac{1}{N}\sum_{k\in U} h(x_k,\rho), \tag{2}$$

where $h(x_k,\rho)$ is the individual "richness function", which is 0 if $x_k \leqslant \rho$ and is continuous and strictly increasing when $x_k > \rho$. This class of measures satisfies several axioms (i.e. focus, continuity, monotonicity and subgroup decomposability) taken from the axiomatic literature on poverty (for reviews see, e.g., Zheng, 1997 and Chakravarty, 2009) and properly redefined for the purposes of measuring richness instead of poverty.

Peichl *et al.* (2010) notice that the familiar (minimal) transfer axiom, which requires poverty to increase after a transfer of income from a poor person to another poor person with higher income, can be translated to richness measurement framework in two normatively justifiable ways. Accordingly, they propose a concave (convex) transfer axiom T1 (T2), according to which a richness index decreases (increases) after a rank-preserving transfer from a rich person to another rich person with higher income.[4] Their preferred "concave" class of richness measures, which satisfies the concave transfer axiom, is anal-

[3] See Medeiros (2006) and Eisenhauer (2011) for attempts to justify richness lines defined in relation to the poverty line. Peichl *et al.* (2010) use relative richness lines equal to twice the median income, while Brzezinski (2010) employs three richness lines equal to twice, trice and four times the median income. Peichl and Pestel (2011) take the 80th percentile as the richness threshold.

[4] In order to fulfill T1 (T2), individual richness function $h(x_k,\rho)$ in eq. (2) has to be strictly concave (convex).

ogous to the family of poverty indices introduced by Chakravarty (1983) and defined as

$$R_{\beta}^{Cha} = \frac{1}{N}\sum_{k\in U}\left[1-\left(\frac{\rho}{x_k}\right)^{\beta}\right]\delta(x_k>\rho),\, \beta>0, \tag{3}$$

where β is a parameter describing a preference for richness.

The "convex" richness indices are defined by Peichl *et al.* (2010) in analogy to the popular Foster-Greer-Thorbecke (1984) (FGT) family of poverty indices satisfying axiom T2 as follows

$$R_{\alpha}^{FGT,T2} = \frac{1}{N}\sum_{k\in U}\left(\frac{x_k}{\rho}-1\right)^{\alpha}\delta(x_k>\rho),\, \alpha>1, \tag{4}$$

where α again expresses a degree of preference for richness. See Peichl *et al.* (2010) for a comparison of "concave" versus "convex" richness measures.

Given a random household sample of size N, denoted by S, which is a subset of U, and a set of survey weights w_k $(k\in S)$, the measures given in eq. (2) can be estimated by

$$\hat{R} = \frac{1}{\sum_{k\in S} w_k}\sum_{k\in S} w_k h(x_k,\rho), \tag{5}$$

if ρ is known, or by substituting an estimator $\hat{\rho}$ for ρ otherwise.

3. Variance estimation by linearization

When richness line is known, sampling variances of indices defined in eq. (2) can be estimated straightforward, analogously to the sampling variances of decomposable poverty indices (Kakwani, 1993; Bishop *et al.*, 1995). However, if richness lines are estimated from samples one needs to take into account additional sampling variability of richness lines and, therefore, other methods are needed.[5] Existing variance estimation approaches can be broadly divided into resampling techniques and linearization methods (for a review, see Wolter, 2007). In this paper, we use a linearization method introduced by Deville (1999), which is easy to use and provides a powerful variance-estimation tool for complex statistics estimated under any sampling design.[6] Deville's approach is based on a slightly modified concept of the influence function, which is heavily used in the field of robust statistics (Hampel *et al.*, 1986). According to Deville's approach, a population parameter of interest θ can be written as a functional $T(M)$, where M is a finite and discrete measure that allocates a unit mass to all $k\in U$. The influence function is defined by Deville as

$$z_k = I[T(M)]_k = \lim_{\epsilon\to 0}\frac{T(M+\epsilon\delta_k)-T(M)}{\epsilon}, \tag{6}$$

where δ_k is the unit mass for unit k. Under asymptotic assumptions provided in Deville (1999), the variance of the estimator of θ, $\hat{\theta}$, can be approximated by the variance of the total of z_k given by

$$\mathrm{var}(\hat{\theta}) \approx \mathrm{var}\left(\sum_{k\in S} z_k w_k\right). \tag{7}$$

[5]See, in particular, Zheng (2001), who derives sampling variances for decomposable poverty indices with relative poverty lines.

[6]Recently, Deville's linearization method has been applied by Osier (2009), Langel and Tille (2011) and Verma and Betti (2011) to derive sampling variances of various poverty and inequality measures.

The pseudovariable z is then called *linearized variable* and the sampling variance of a total $\sum_{k\in S} z_k w_k$ may be estimated by standard survey sampling methods allowing to take into account any actual sampling design used (see, e.g., Cochran, 1977; Deaton, 1997). Deville (1999) provides also several derivation rules, which simplify the task of obtaining linearized variables so that tedious limit calculations in eq. (6) can be avoided.

4. Linearization for richness indices

If richness line ρ does not have to be estimated from the sample, linearized variables for variance estimation of richness indices can be obtained by applying Deville's (1999, p. 197) derivation rule for the linearization of a ratio to eq. (2) and written as

$$z_k = I(R \mid \rho = \text{const.})_k = \frac{1}{N}\left[h(x_k,\rho) - R\right]. \tag{8}$$

However, if richness line is defined in relation to a quantile or the mean of the underlying distribution, than further linearization is required. This can be performed using Deville's (1999, p. 198) *rule 7* for the linearization of a functional with a parameter. The application of this rule gives the linearized variable defined by

$$z_k = I(R)_k = I(R \mid \rho = \text{const.})_k + \frac{\delta R}{\delta \rho} I(\rho)_k. \tag{9}$$

The first term of equation (9) is equivalent to equation (8) and gives the influence function of richness index R assuming that the richness line is fixed, while the second term accounts for the influence of the richness line. Using eq. (9) for variance linearization of the richness headcount as well as of a quantile of the distribution (see eq. 10 below) involves the derivative of the cumulative distribution function F. Since F is a discontinuous step function, Deville (1999) suggests that it should be replaced by its smoothed version $\tilde{F}$. The derivative of $\tilde{F}$ at x_k is then the density function corresponding to $F, f(x_k)$. Following Berger and Skinner (2003) and Osier (2009), we use Gaussian kernel smoothing to estimate $f(x_k)$. Table 1 reports formulae for partial derivatives of our richness indices with respect to the richness line.

Table 1. Linearization component $\delta R/\delta\rho$ for richness indices

Richness index	$\delta R/\delta\rho$
$R^{HC} = \frac{1}{N}\sum_{k\in U}\delta(x_i > \rho) = 1 - F(\rho)$	$-f(\rho)$
$R^{Cha}_{\beta} = \frac{1}{N}\sum_{k\in U}\left[1 - \left(\frac{\rho}{x_k}\right)^{\beta}\right]\delta(x_k > \rho),\ \beta > 0$	$\frac{1}{N}\sum_{k\in U} -\frac{\beta}{\rho}\left(\frac{\rho}{x_k}\right)^{\beta}\delta(x_k > \rho)$
$R^{FGT,T2}_{\alpha} = \frac{1}{N}\sum_{k\in U}\left(\frac{x_k}{\rho} - 1\right)^{\alpha}\delta(x_k > \rho),\ \alpha > 1$	$\frac{1}{N}\sum_{k\in U} -\frac{\alpha x_k}{\rho(x_k - \rho)}\left(\frac{x_k}{\rho} - 1\right)^{\alpha}\delta(x_k > \rho)$

The linearized variable for the richness line $\rho = \gamma\xi_q$, where ξ_q is a quantile of order q (i.e. $\xi_q = \sup\{x_k | F(x_k) \leqslant q\}$) and $\gamma \geqslant 1$ is given by Deville (1999) as

$$z_k = I(\gamma\xi_q)_k = \gamma I(\xi_q)_k = \frac{-\gamma[\delta(x_k < \xi_q) - q]}{f(\xi_q)N}, \tag{10}$$

while for richness line defined as a multiple of the mean income (μ) it can be written as

$$z_k = I(\gamma\mu)_k = \gamma I(\mu)_k = \frac{\gamma}{N}(x_k - \mu). \tag{11}$$

A linearized variable specific for a given choice of richness index R and type of richness line can be obtained by substituting the appropriate partial derivative from Table 1 and either formula (10) or (11) into eq. (9). For example, the linearized variable for the $R_{0.5}^{Cha}$ index and richness line equal to thrice the median income, $\rho = 3\xi_{0.5}$, can be written as follows

$$\begin{aligned} z_k &= \frac{1}{N}\left(\left[1-\left(\frac{3\xi_{0.5}}{x_k}\right)^{0.5}\right]\delta(x_k > 3\xi_{0.5}) - R_{0.5}^{Cha}\right) + \\ &\quad \frac{1}{N}\left(\frac{-3[\delta(x_k < \xi_{0.5}) - 0.5]}{f(\xi_{0.5})}\sum_{k\in U} -\frac{0.5}{3\xi_{0.5}}\left(\frac{3\xi_{0.5}}{x_k}\right)^{0.5}\delta(x_k > 3\xi_{0.5})\right). \end{aligned} \tag{12}$$

Finally, a linearization variance estimator may be determined by replacing $N, \rho, R, f(\cdot)$ and either ξ_q or μ by $\hat{N} = \sum_{k\in S} w_k, \hat{\rho}, \hat{R}, \hat{f}(\cdot)$ and either $\hat{\xi}_q$ or $\hat{\mu} = 1/\hat{N}\sum_{k\in S} w_k x_k$.

5. Empirical illustration

We illustrate the methods proposed in this paper with a comparison of household wealth richness in Canada, Sweden, the United Kingdom and the United States around year 2000 using data drawn from the Luxembourg Wealth Study (LWS, 2011). The LWS is a cross-country comparable database for household wealth research (see Sierminska *et al*., 2006). Although the LWS did not achieve perfect data comparability, the constructed aggregate wealth categories are broadly comparable across countries. As the main wealth variable we use household net worth defined as the sum of financial and non-financial assets (excluding business equity) net of total debt (*nw1* variable of the LWS). For details on the construction of the net worth variable and sampling designs of the particular surveys used, see Sierminska *et al*. (2006) and Jäntti *et al*. (2008).[7]

Using the same sample of the LWS data and robust modeling of the upper tail of wealth distribution, Cowell (2011) has recently shown that the Gini index of inequality for the rich population (defined as top 5% or 10% of households) is the highest in the US and the lowest in the UK, while the ranking of Canada and Sweden depends on the definition of the rich. Moreover, as shown by Jäntti *et al*. (2008), the ranking in ascending order of 10%, 5% and 1% top wealth shares is always as follows: the UK, Canada, Sweden, the US. Neither study provided statistical inference on achieved results.

We complement these distributional analyses by providing point estimates of wealth richness indices together with their standard errors, calculated using methods proposed in Section 4, and with conventional 95% confidence intervals (see Table 2). The richness line is three times the median wealth. Point estimates of richness indices as well as estimates of their standard errors are weighted by the LWS household survey weights. For

[7]We use Canadian Survey of Financial Security (SFS) with wealth data for 1999 and actual sample size (N) of 15,333 households, Swedish Wealth Survey (HINK) with data for 2002 (N = 17,953), British Household Panel Survey (BHPS) with data for 2000 (N = 4,185) and the US Survey of Consumer Finances (SCF) with wealth data for 2001 (N = 4,442). Surveys for Canada and the US implement over-sampling of the wealthy and therefore provide a better coverage of the upper tail of wealth distribution, which is especially attractive for the purposes of estimating richness indices and their sampling variances.

Table 2. Estimates of wealth richness indices with standard errors and 95% confidence intervals for Canada, Sweden, the UK and the US, circa 2000

Index	Parameter	Country			
		Canada	Sweden	UK	US
R^{HC}	–	0.2842	0.3315	0.2006	0.2886
		(0.0598)	(0.0593)	(0.0787)	(0.0825)
		[0.1670, 0.4014]	[0.2153, 0.4477]	[0.0464, 0.3548]	[0.1269, 0.4503]
R^{Cha}	0.5	0.1069	0.1631	0.0572	0.1361
		(0.0398)	(0.0432)	(0.0468)	(0.0607)
		[0.0288, 0.1850]	[0.0784, 0.2479]	[–0.0346, 0.1490]	[0.0171, 0.2551]
	1	0.1349	0.1815	0.0832	0.1636
		(0.0447)	(0.0468)	(0.0552)	(0.0665)
		[0.0473, 0.2224]	[0.0897, 0.2732]	[–0.0250, 0,1915]	[0.0332, 0.2940]
	2	0.1899	0.2475	0.1227	0.2165
		(0.0514)	(0.0533)	(0.0652)	(0.0744)
		[0.0891, 0.2908]	[0.1431, 0.3518]	[–0.0052, 0.2506]	[0.0707, 0.3623]
	3	0.2184	0.2783	0.1448	0.2411
		(0.0543)	(0.0557)	(0.0698)	(0.0775)
		[0.1119, 0.3249]	[0.1691, 0.3876]	[0.0080, 0.2816]	[0.0893, 0.3930]
$R^{FGT,T2}$	1.5	1.0690	1.9820	0.3127	7.1068
		(0.2356)	(0.6999)	(0.1565)	(1.8857)
		[0.6072, 1.5307]	[0.6103, 3.3536]	[0.0059, 0.6195]	[3.4005, 10.813]
	2	3.1397	11.234	0.5583	85.299
		(0.5272)	(3.9171)	(0.2511)	(12.360)
		[2.1064, 4.1730]	[3.5564, 18.911]	[0.0662, 1.0504]	[59.981, 110.62]
	2.5	11.594	167.27	1.1447	1796.5
		(1.2346)	(22.387)	(0.4206)	(139.87)
		[9.1740 14.014]	[123.39, 211.15]	[0.3203, 1.9692]	[1435.3, 2157.7]
	3	50.217	4310.6	2.6103	53413
		(2.9671)	(128.28)	(0.7227)	(4893.8)
		[44.402, 56.033]	[4059.2, 4562.0]	[1.1938, 4.0268]	[39859, 66968]

Notes: Richness line is equal to three times the median wealth. Wealth is defined as household net worth excluding business equity. Estimates are weighted by household weights provided by the LWS. Standard errors appear in parentheses, while 95% confidence intervals in brackets. The survey for the US uses multiple imputation technique to approximate the distribution of missing values and provides five replicates of each data record. Accounting for this, point estimates and standard errors are calculated independently for every replicate data set and combined using rules outlined by Rubin (1987).

the assumed richness line, the richness headcount and all "concave" indices (R^{Cha}) rank the countries in ascending order of wealth richness as follows: the UK, Canada, the US, Sweden. The standard errors for the estimates are, however, rather large and the differences in richness indices for every pair of countries are not statistically significant (95% confidence intervals are always overlapping). If a higher richness threshold were used, which would not be unreasonable, standard errors would be even larger, probably making statistical inference still less conclusive. The fact that the estimates of standard errors are not small is not surprising since there is relatively little sample information above any reasonable richness line. Moreover, it is well-known that sampling variability is in practice greater in the upper-tail of income or wealth survey data. It is likely that these problems will be often encountered in statistical inference for differences in "concave" richness indices unless observed richness differences or sample sizes are sufficiently large.

The ranking of the countries according to the "convex" richness measures ($R^{FGT,T2}$) is a little different. For these indices, independently of the value of sensitivity parameter α, Sweden and the US switch their places with the rest of the ranking unchanged. This is a consequence of the fact that inequality among the rich in the US is usually greater than in Sweden (see Cowell, 2011). For larger values of α, point estimates of these richness indices are very sensitive to extreme wealth observations and take rather large values. In case of measuring wealth richness with $R^{FGT,T2}$ measures, it may be therefore safer to set the range of admissible values of α to $1 < \alpha \leqslant 2$.

Turning to statistical inference, we can observe that for the $R^{FGT,T2}$ indices almost all pairwise differences between wealth richness for compared countries are statistically significant at the 95% confidence level.[8] Therefore, despite the moderate sample sizes of our surveys, it is possible to conclude that according to the "convex" indices and chosen threshold between the rich and the non-rich, wealth richness is significantly higher in the US than in Sweden and higher in Canada than in the UK. In most of the cases, we can also establish statistical significance of higher wealth richness in Sweden than in Canada.

6. Conclusions

This paper used a linearization approach to provide variance estimators for recently introduced distributionally-sensitive income, earnings or wealth richness indices (Peichl *et al.*, 2010), when estimated from survey data. The proposed methods can be used both with absolute and relative richness lines as well as under a variety of survey designs that are applied in real-world surveys. We have used micro-data from the LWS database to illustrate our methods with a comparison of wealth richness in Canada, Sweden, the UK and the US around year 2000. Results suggest that the ranking of the countries changes slightly depending on whether "concave" or "convex" richness indices are used. However, only in case of the "convex" measures statistical significance of the results can be established. For these measures, the US emerges at top of the ranking, while the UK is the least rich.

A final remark concerns the nature of the upper tail survey data from which rich-

[8]The only exception is Canada versus Sweden for $\alpha = 1.5$. P-values for other instances of overlapping confidence intervals, i.e., Canada versus the UK ($\alpha = 1.5$), Sweden versus the UK ($\alpha = 1.5$) and Canada versus Sweden ($\alpha = 2$) are, respectively, 0.007, 0.020 and 0.041.

ness indices and their variances are estimated. As is well known, top income or wealth survey data may be less reliable than data from other parts of the distribution due to the higher rate of non-response of the rich and higher under-reporting of some income or wealth items. This problem is more relevant to richness measures than to most of other distributional statistics, such as inequality indices, as the former are calculated exclusively on the basis of the more or less broadly defined upper tail data. If over-sampling of the rich is unavailable or insufficient, than one could possibly use (semi-)parametric (robust) modelling of the upper tail to construct more appropriate methods of estimation and inference (see Cowell and Flachaire, 2007 and Cowell and Victoria-Feser, 2007).

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