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### Capital and Labor Income Pareto Exponents across Time and Space

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# Capital and Labor Income Pareto Exponents across Time and Space\*

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June 5, 2021

## Abstract

We estimate capital and labor income Pareto exponents across 475 country-year observations that span 52 countries over half a century (1967–2018). We document two stylized facts: (i) capital income is more unequally distributed than labor income in the tail; namely, the capital exponent (1–3, median 1.46) is smaller than labor (2–5, median 3.35), and (ii) capital and labor exponents are nearly uncorrelated. To explain these findings, we build an incomplete market model with job ladders and capital income risk that gives rise to a capital income Pareto exponent smaller than but nearly unrelated to the labor exponent. Our results suggest the importance of distinguishing income and wealth inequality.

**Keywords:** income fluctuation problem, inequality, power law.

**JEL codes:** C46, D15, D31, D52.

## 1 Introduction

The purpose of this paper is to estimate and document the Pareto exponents for capital and labor income separately for as many countries and years as possible. We say that a positive random variable  $X$  obeys a power law with Pareto exponent  $\alpha > 0$  if the tail probability decays like a power function:  $P(X > x) \sim x^{-\alpha}$  for

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large  $x$ .<sup>1</sup> In the context of the income distribution, the Pareto exponent characterizes the tail heaviness of high incomes and hence top tail inequality. We remain agnostic about the shape of the income distribution away from the tail. Our study is motivated by the following two observations. First, we are not aware of a comprehensive study that documents the capital and labor income Pareto exponents separately for many countries and years, despite their importance. Second, the Pareto exponent has desirable properties relative to other popular inequality measures such as the Gini coefficient or top income shares.

Consider the first point. Conceptually, capital and labor income are different entities. While the former is the return for providing capital (wealth), the latter is the return for providing labor services, and there is no particular reason to expect a relation between the two. Although these two forms of income are conceptually distinct, it is often put together as just “income” and discussed in the context of inequality and related policies. If capital and labor income are quantitatively different, a policy design based on total income may be misleading. To give one example, consider the theory of optimal taxation (Saez, 2001), where the income Pareto exponent plays an important role. Saez and Stantcheva (2018) carefully distinguish capital and labor income and apply the theory of optimal taxation in the United States. They find that with an income elasticity of  $e = 0.5$ , the optimal top marginal tax rate is about 50% for labor and 60% for capital (see their Figure 5). This difference directly comes from the fact that capital and labor income Pareto exponents are distinct. Thus, distinguishing capital and labor income inequality is potentially important for policy designs.

Consider the second point about the desirable properties of Pareto exponents. In the applied literature such as Piketty (2003) and Piketty and Saez (2003), top income shares (such as the top 1% income share) are more commonly reported than the income Pareto exponent, perhaps because top shares are summary statistics that can be computed without specifying functional forms or can be understood by non-experts without special knowledge of statistics. However, Atkinson (2005) documents methodological problems regarding the cross-country comparison of top income shares, citing the differences in tax units (e.g., individual or household) and legislation (e.g., whether social security benefits are taxable). One of the reasons such issues arise is because it is not always clear how to define the population

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<sup>1</sup>More precisely, we say that a random variable  $X$  has a Pareto upper tail with exponent  $\alpha > 0$  if  $P(X > x) = x^{-\alpha}\ell(x)$  for some slowly varying function  $\ell$ . A function  $\ell : (0, \infty) \rightarrow \mathbb{R}$  is said to be *slowly varying (at infinity)* if it is nonzero for sufficiently large  $x$  and  $\lim_{x \rightarrow \infty} \ell(tx)/\ell(x) = 1$  for each  $t > 0$ . See Bingham, Goldie, and Teugels (1987) for a comprehensive treatment of the theory of regular variation.

and measure small units.<sup>2</sup> Because common inequality measures such as the Gini coefficient and top income shares require the knowledge of the entire distribution, these quantities are greatly affected by the definition and measurement of small units. Using the Pareto exponents significantly alleviates these definition and measurement issues because the Pareto distribution is scale invariant (see [Jessen and Mikosch, 2006](#) for a summary) and its exponent depends only on the tail behavior, not the entire distribution. For example, doubling the income of all households in the top 1% of the income distribution makes the top 1% income share (roughly) twice as large, but the Pareto exponent is unaffected. A similar comment applies to any inequality measure that depends on the entire distribution, such as the Gini coefficient. While we do not claim that the Pareto exponent is the only interesting inequality measure, it is certainly a robust (detail-independent) measure for top tail inequality. See [Gabaix \(2009, 2016\)](#) for more discussion on the robustness of the Pareto distribution.

In this paper, we use the harmonized *Luxembourg Income Study* database (hereafter [LIS](#)) to document the capital and labor income Pareto exponents across all available 475 country-year observations that span 52 countries over half a century. We document two empirical findings. First, we find that the capital income Pareto exponent is roughly in the range 1–3 (with median 1.46) and is smaller than the labor income Pareto exponent, which ranges between 2–5 (with median 3.35). This implies that capital income is more unequally distributed than labor income. This fact is unsurprising and well known for a specific country or year (see, for example, the Lorenz curve in Figure 1 of [Saez and Stantcheva, 2018](#)). However, we are not aware of a comprehensive study that systematically analyzes datasets from many countries and years, and therefore our finding suggests that capital income is generally more unequal than labor income. More specifically, using a statistical test recently developed by [Hoga \(2018\)](#), we formally test the equality of capital and labor income Pareto exponents and the null is rejected in 86% of samples. In every single case of rejection, the capital exponent is smaller than the labor exponent. Second, we find that the capital income Pareto exponent is nearly unrelated to the labor exponent. In particular, the correlation between the two exponents across countries is close to zero.

To explain our empirical findings, we build a simple incomplete market model with job ladders and capital income risk. In the model, agents get randomly

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<sup>2</sup>Imagine how to formally distinguish cities, towns, villages, and settlements; continents, islands, and islets; and inland seas, lakes, and ponds. How to define units and how to measure small units matter for the size distribution of population, land mass, and water surface area.

promoted to the next job ladder. Because individual income follows a random multiplicative process, we obtain a Pareto-tailed labor income distribution. The agents also save assets and face idiosyncratic investment risk, which generates a Pareto-tailed wealth (hence capital income) distribution. Because the capital income Pareto exponent is mainly determined by the asset return distribution, while the labor income Pareto exponent is mainly determined by the income growth distribution, the relation between the two is weak. Furthermore, we analytically characterize the capital and labor income Pareto exponents and show that the former tends to be smaller than the latter for common parametrization. Our results suggest the importance of distinguishing income and wealth inequality.

**Related literature** The power law behavior of income was first recognized by Pareto (1895, 1896, 1897), who used tabulation data of tax returns in many European countries. More recent research that employs micro data include Reed (2001) for U.S., Reed (2003) for U.S., Canada, Sri Lanka, and Bohemia, Nirei and Souma (2007) for Japan, Toda (2011, 2012) for U.S., Jenkins (2017) for U.K., and Ibragimov and Ibragimov (2018) for Russia. These papers all concern specific countries and years. Bandourian, McDonald, and Turley (2003) estimate eleven parametric distributions (some of which exhibit Pareto tails) using 82 household labor income datasets from *Luxembourg Income Study (LIS)* as we do, though they neither focus on the Pareto exponent nor consider capital income. Gabaix (2009) mentions “[the] tail exponent of income seems to vary between 1.5 and 3”, citing Atkinson and Piketty (2007), though without providing specific details. Atkinson and Piketty (2010, Table 13A.23) document income Pareto exponents across many countries and years estimated from top income share data based on tax returns. However, these estimates are computed from total income, and since (as we document in Section 3.3) the capital income Pareto exponents tend to be smaller than labor exponents, their estimates are best understood as capital income (hence wealth) Pareto exponents. Benhabib, Bisin, and Luo (2017) make the point that wealth is more skewed than income, citing a few Pareto exponent estimates from Badel, Daly, Huggett, and Nybom (2018) for income and Vermeulen (2018) for wealth. Using the 2013-2014 individual income tax data from Romania, Oancea, Pirjol, and Andrei (2018) document that the capital income Pareto exponent (1.44) is smaller than the labor exponent (2.53). Using time series regressions for 21 countries, Bengtsson and Waldenström (2018) document a positive correlation between the gross capital share in national accounts and the top 1% income shares. Their finding can be explained if top income earners tend to have higher

capital income share, which is the case if the capital income Pareto exponent is smaller than the labor exponent as we document in this paper. As mentioned in the introduction, there seems to be no comprehensive studies that document the capital and labor income Pareto exponents separately for many countries and years.

## 2 Data

In this section we describe the dataset that we use and discuss its limitations.

### 2.1 The LIS database

We use the data from the *Luxembourg Income Study* (LIS), which is a large, harmonized database of micro-level income data that covers over 50 countries worldwide and many years since the late 1960s. In many countries, the data derive from government surveys (for example, the U.S. data is based on the *Current Population Survey*). The LIS data are available at both individual and household level. We focus on the household labor and capital income because (i) it is reasonable to assume that economic decisions such as financial planning are made at the household level, and (ii) incomes among couples are likely correlated due to assortative matching in the marriage market (Siow, 2015), which invalidates statistical estimation.<sup>3</sup> The LIS defines *labor income* as “cash payments and value of goods and services received from dependent employment, as well as profits/losses and value of goods from self-employment, including own consumption”. *Capital income* is defined as “cash payments from property and capital (including financial and non-financial assets), including interest and dividends, rental income and royalties, and other capital income from investment in self-employment activity”. Together these two categories make up total *factor income*. See the LIS 2019 USER GUIDE<sup>4</sup> for a detailed summary on how these data are retrieved and calculated.

### 2.2 Data limitations

Our analysis draws upon datasets from many different countries that are harmonized into a common framework by the LIS. However, many details about the collection of data in the different countries are omitted. For example, we find

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<sup>3</sup>In our data, we find an average correlation of 0.22 between labor income of husband and wife, which underpins the conjectured dependency.

<sup>4</sup><https://www.lisdatacenter.org/wp-content/uploads/files/data-lis-guide.pdf>

evidence of top-coding in some countries and years, as the largest income order statistic is equal to the second largest.<sup>5</sup> Top-coding induces an upward bias in the estimation of the Pareto exponent. This issue is not necessarily resolved if, instead, one relies on administrative tax income data, for similar biases arise such as rich households trying to understate their taxable income (Atkinson, Piketty, and Saez, 2011). Burkhauser, Feng, Jenkins, and Larrimore (2012) detail a method that can be used to overcome the bias due to top-coding, however at the end of their paper they show that the results are robust even if estimates are based on the top-coded series. For these reasons we treat the datasets as not being top-coded in our analysis.

Another limitation of the LIS database is that it is based on government surveys and the measurement error may be larger compared to administrative data based on tax returns. The fact that the income distribution in administrative data is often reported as tabulations, not micro data, causes no problem for estimating Pareto exponents, as Toda and Wang (2021) provide an efficient estimation method for such data. In fact, Atkinson and Piketty (2010, Table 13A.23) document income Pareto exponents across countries and years estimated from top income share data. However, their table is based on total income, and since (as we document in Section 3.3) the capital income Pareto exponents tend to be smaller than labor exponents, the estimates in Atkinson and Piketty (2010) are best understood as capital income (hence wealth) Pareto exponents. Since we are not aware of a comprehensive income database that distinguishes capital and labor income, we decided to use the LIS database.

### 3 Pareto exponents across countries and years

In this section we estimate the capital and labor income Pareto exponents for all countries and years that are available in the LIS database, which spans 52 countries over half a century (475 country-year observations in total). We then formally test for the equality of the Pareto exponents of capital and labor income.

#### 3.1 Estimation method

For each country and year, we suppose that the (capital or labor) income observations  $\{X_n\}_{n=1}^N$  are independent and identically distributed (IID) with cumulative

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<sup>5</sup>Among all 475 country-year observations, the first and second order statistics are equal in 6 cases for labor income and 11 cases for capital income. Therefore we conjecture that the top-coding issue is not severe.

distribution function (CDF)  $F(x) = P(X_n \leq x)$ . The assumption that the upper tail of income obeys a power law with Pareto exponent  $\alpha > 0$  translates into the regular variation condition

$$1 - F(x) = x^{-\alpha} \ell(x) \quad (3.1)$$

for some slowly varying function  $\ell$  (see Footnote 1). Note that the assumption on  $\ell$  involves only the limit as  $x \rightarrow \infty$ ; we are assuming a power law behavior in the upper tail without taking a stance on the shape of the entire distribution.

We are interested in estimating the Pareto exponent  $\alpha$  for each country and year. For this purpose, we employ the Hill (1975) (maximum likelihood) estimator

$$\frac{1}{\hat{\alpha}(k)} := \frac{1}{k} \sum_{n=1}^k \log \left( \frac{X_{(n)}}{X_{(k)}} \right). \quad (3.2)$$

Here  $X_{(n)}$  denotes the  $n$ -th largest order statistic from the sample  $\{X_n\}_{n=1}^N$  and  $k \in \{1, \dots, N\}$  denotes the number of tail observations used to estimate the Pareto exponent. Hall (1982) shows that the standard error of  $\hat{\alpha}(k)$  is  $\hat{\alpha}(k)/\sqrt{k}$  under flexible assumptions on the CDF (see (3.3) below). We use the Hill estimator because we are interested in formally testing the equality of the capital and labor income Pareto exponents using the method of Hoga (2018), for which the Hill estimator is required.<sup>6</sup>

When the population distribution is known to be exactly Pareto (so  $\ell$  in (3.1) is zero below the minimum size  $x_{\min}$  and constant above this threshold), it is well known that the Hill estimator for the full sample ( $k = N$ ) is consistent, asymptotically normal, and asymptotically efficient because it is a maximum likelihood estimator. In practice, the CDF is not exactly Pareto and the researcher needs to select an appropriate value of  $k$ . For instance, if  $F(x)$  satisfies

$$1 - F(x) = Cx^{-\alpha}(1 + Dx^{-\beta} + o(x^{-\beta})) \quad (3.3)$$

with some  $\beta > 0$ , then Hall (1982) shows that choosing  $k = o(N^{2\beta/(2\beta+\alpha)})$  together with  $k \rightarrow \infty$  as  $N \rightarrow \infty$  is sufficient for consistency and asymptotic normality (see also Embrechts, Klüppelberg, and Mikosch, 2013).<sup>7</sup> Notice that this choice

<sup>6</sup>If we are only interested in estimating the Pareto exponents, then there are many alternative methods available. Gomes and Guillou (2015) review 13 commonly used estimators. Fedotenkov (2020) reviews more than 100.

<sup>7</sup>Recall that we write  $f(x) = o(g(x))$  if for all  $\epsilon > 0$ , we have  $|f(x)| \leq \epsilon |g(x)|$  for large enough  $x$ .



puts a bound on the growth rate of  $k$ . The expansion (3.3) covers a wide range of distributions of interest, such as the  $t$ -distribution and the type II extreme value distribution (Danielsson and de Vries, 1997).

Despite these asymptotic results, it is notoriously difficult to pick  $k$  optimally in finite samples (Hall, 1990; Resnick and Stărică, 1997; Danielsson, de Haan, Peng, and de Vries, 2001). In practice, researchers often plot the Hill estimator (3.2) over a range of  $k$  to find a flat region or plot the log rank  $\log 1, \dots, \log N$  against the log size  $\log X_{(1)}, \dots, \log X_{(N)}$  to find a region that exhibits a straight line pattern and choose a size threshold to run the log-rank regression.<sup>8</sup> Unfortunately, this graphical approach is not feasible in our setting because LIS does not allow researchers to download the micro data for confidentiality concerns (researchers are required to submit their execution files to conduct statistical analyses) and there is little scope for exploratory graphical data analysis. In this paper we simply use the largest 5% observations, so

$$k = \lfloor 0.05N \rfloor, \quad (3.4)$$

which is standard in the literature.<sup>9</sup> Unreported simulations show that our results are robust to using other thresholds such as the largest 1% and 10% observations, or using a data-driven procedure using the Kolmogorov-Smirnov distance as in Danielsson, Ergun, de Haan, and de Vries (2016).

### 3.2 Capital and labor income Pareto exponents

We estimate the capital and labor income Pareto exponents for all countries and years available in the LIS database. The database spans 52 countries across the years 1967–2018, with a total of 475 country-year observations. The point estimates of the capital and labor income Pareto exponents for each country and year as well as their standard errors can be found in Table 2 in Appendix C. To

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<sup>8</sup>Gabaix and Ibragimov (2011) study the asymptotic behavior of log rank regression and show that the standard error is larger by a factor of  $\sqrt{2}$  than the Hill estimator. However, they do not discuss how to select the threshold. In their empirical application, they consider the size distribution of population in U.S. metropolitan statistical areas, which are already far into the tail and hence the threshold selection is less of an issue. Ibragimov and Ibragimov (2018) apply the same methodology to Russian household income data and consider the top 5% and 10% thresholds.

<sup>9</sup>An alternative approach is to estimate a parametric distribution  $F$  that admits a Pareto upper tail by maximum likelihood using the entire sample. The double Pareto-lognormal distribution proposed by Reed (2003) and Reed and Jorgensen (2004) often performs best. See Toda (2012) for a horse race across several parametric distributions in the context of U.S. labor income.

avoid small sample issues, we restrict our analysis to countries with at least 1,000 positive observations for income, resulting in (all) 475 country-year pairs for labor income and 342 for capital income. For visibility, Figure 1 shows the histogram and scatter plot of the capital and labor income Pareto exponents.

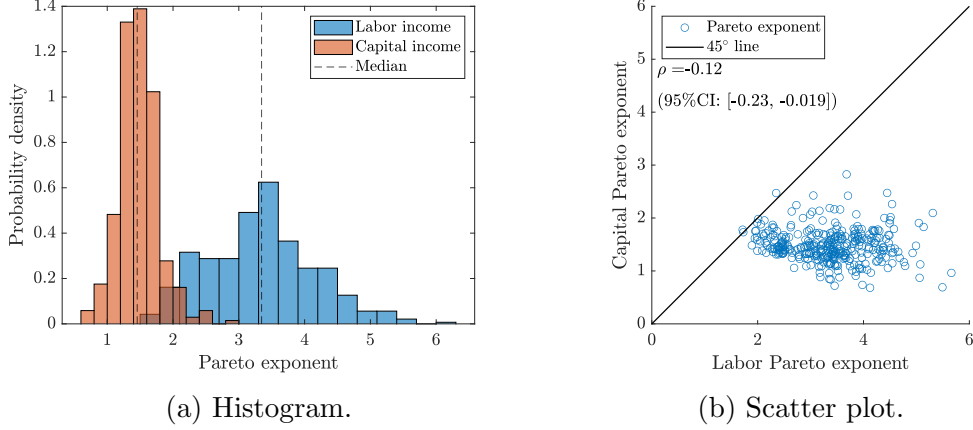


Figure 1: Histogram and scatter plot of capital and labor income Pareto exponents.

Figure 1a shows the histogram of the capital and labor income Pareto exponents pooled across all available countries and years. The capital and labor income Pareto exponents are generally in the range 1–3 and 2–5 with medians 1.46 and 3.35, respectively. This suggests that (i) capital income is generally more unequally distributed than labor income, but (ii) there is significant heterogeneity in both capital and labor income inequality across countries and years. Figure 1b shows the scatter plot of the Pareto exponents together with the 45 degree line. The confidence interval of the correlation coefficient  $\rho$  is computed assuming all country-year observations are independent and there is no sampling error in the point estimates of the Pareto exponents. Although it is not obvious how to account for these issues, doing so will only widen the confidence interval. Therefore the fact that the naïve confidence interval almost contains zero suggests that the correlation between the two Pareto exponents is weak. Furthermore, for the vast majority of countries and years, the capital income Pareto exponent is smaller than the labor exponent, again suggesting that capital income is more unequal than labor income.

How do the Pareto exponents evolve over time? Because many countries appear only sporadically in the LIS database, we only consider six countries for which the cross-sectional sample size is large and the time series is long, namely Canada, Germany, Switzerland, Taiwan, United Kingdom, and United States. Figure 2 plots the capital and labor Pareto exponents and their 95% confidence intervals

for these countries. The capital exponent tends to be stable at around 1–2 and is smaller than the labor exponent.<sup>10</sup> Again, capital income appears to be more unequally distributed than the labor income.

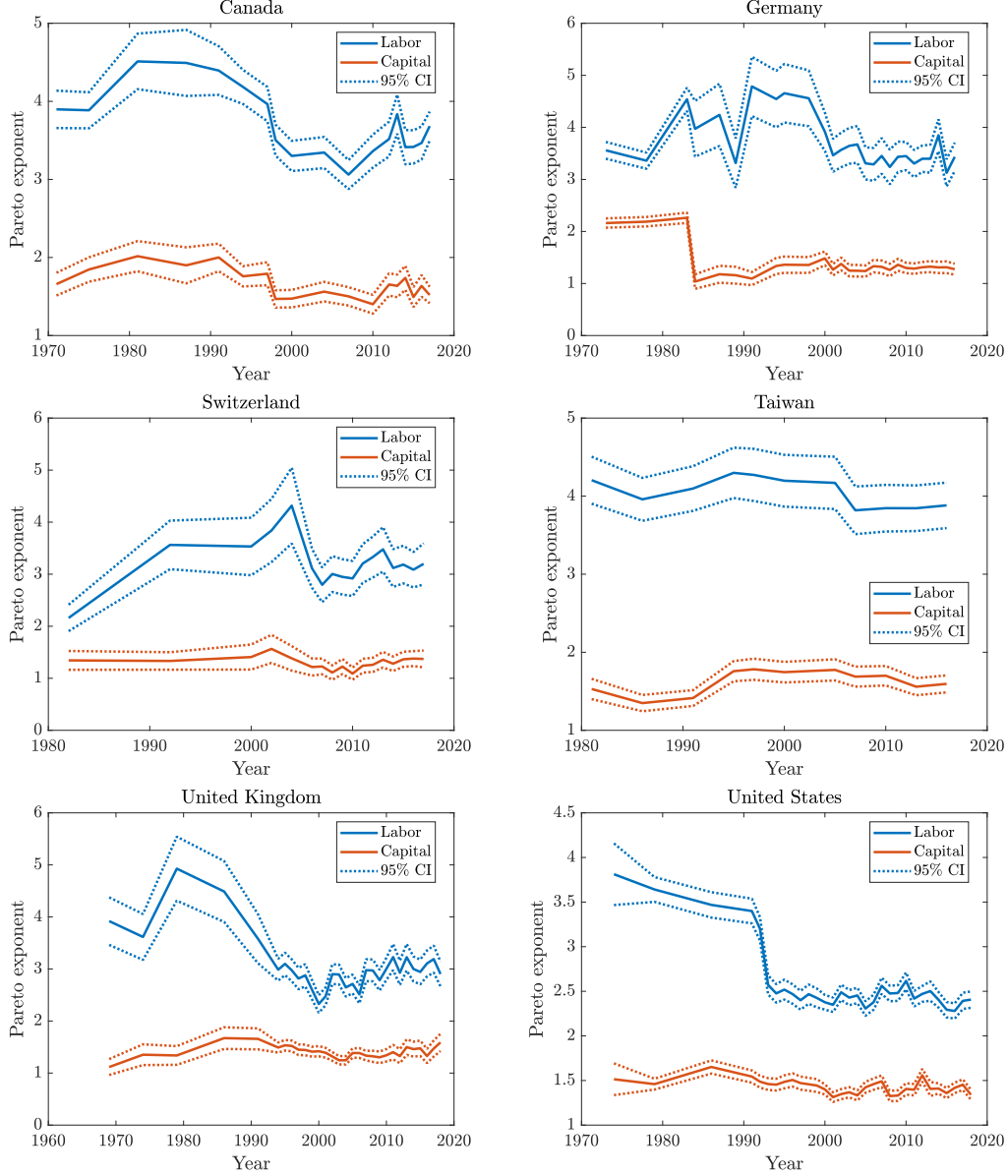


Figure 2: Time evolution of capital and labor income Pareto exponents.

<sup>10</sup>An exception may be Germany before 1983. However, this may be an artifact of the sudden change in sample size, which was over 40,000 until 1983 and about 5,000 since 1984. Thus the 5% rule for capital income may be including too many observations in the body of the distribution before 1983.

### 3.3 Testing equality of capital and labor Pareto exponents

We now formally test whether the capital and labor Pareto exponents are equal. In particular our test is

$$H_0 : \alpha_{\text{lab}} = \alpha_{\text{cap}} \quad \text{against} \quad H_1 : \alpha_{\text{lab}} \neq \alpha_{\text{cap}},$$

where  $\alpha_{\text{cap}}, \alpha_{\text{lab}}$  denote the capital and labor income Pareto exponents. Testing the null hypothesis  $H_0$  is complicated by the fact that there is dependency between labor income  $\{X_{\text{lab},n}\}_{n=1}^N$  and capital income  $\{X_{\text{cap},n}\}_{n=1}^N$ , because individuals who are rich (receive high labor income) tend to be wealthy and receive high capital income. Thus we cannot use the 95% confidence intervals in Figure 2 to test the equality of Pareto exponents. Instead, we apply the test recently developed by Hoga (2018), which allows for dependence in the data but assumes restrictions on the growth rate of the tail dependence (see Hoga, 2018, Assumption A2). The test is based upon the inverse of the Hill estimator (3.2), which we denote by  $\hat{\gamma} := 1/\hat{\alpha}$ . The test statistic is defined by

$$T_N = \frac{(\hat{\gamma}_{\text{lab}}(1) - \hat{\gamma}_{\text{cap}}(1))^2}{\int_{t_0}^1 t^2 [(\hat{\gamma}_{\text{lab}}(t) - \hat{\gamma}_{\text{cap}}(t)) - (\hat{\gamma}_{\text{lab}}(1) - \hat{\gamma}_{\text{cap}}(1))]^2 dt}, \quad (3.5)$$

where  $t_0 \in (0, 1)$  is a tuning parameter and  $\hat{\gamma}(t)$  is the inverse Hill estimator

$$\hat{\gamma}(t) := \frac{1}{[kt]} \sum_{n=1}^{\lfloor kt \rfloor} \log \left( \frac{X_{(n)}}{X_{(\lfloor kt \rfloor)}} \right). \quad (3.6)$$

Using the Hill estimator based on the subsample with only  $\lfloor kt \rfloor$  observations leads to self-normalization of the test statistic  $T_N$  and renders a test that is asymptotically pivotal. The limiting distribution is

$$T_N \xrightarrow{d} \frac{W(1)^2}{\int_{t_0}^1 [W(t) - tW(1)]^2 dt}, \quad (3.7)$$

where  $W(t)$  is a standard Brownian motion. Since the test statistic (3.5) can be computed using only the Hill estimator and conducting numerical integration, there is no need to estimate the (potentially difficult) tail covariance. The tuning parameter  $t_0$  affects the size of the test in finite samples: high values of  $t_0$  make the integral in (3.5) based on too few differences of  $\hat{\gamma}$ , and low values of  $t_0$  yield volatile  $\hat{\gamma}$  in (3.6) when  $t$  is close to  $t_0$ . Both of these effects may cause size distortions. Therefore we set  $t_0 = 0.2$  following the recommendation of Hoga (2018), who finds

that this choice leads to favorable size properties.<sup>11</sup> We reject the null  $H_0$  when the test statistic  $T_N$  is large. According to Table I of Hoga (2018), the 95 percentile of (3.7) for  $t_0 = 0.2$  is 55.44, which we use as the critical value for testing  $H_0$  at 5% significance level.

One issue with the test statistic (3.5) is that it requires the same number of tail observations  $k$  for both cross-sections of capital and labor income. Hence the 5% rule (3.4) discussed in Section 3.1 becomes problematic as the number of people with capital income in our data set is rather small. Many households do not hold liquid financial wealth and hence have no capital income. The resulting test is thus not feasible since  $k_{\text{lab}}$  based on our 5% rule could be wildly different from  $k_{\text{cap}}$ . To overcome this issue, we only test the equality of Pareto exponents for countries that have more than 1,000 positive capital income observations and set  $k = \lfloor 0.05N_{\text{cap}} \rfloor$ , where  $N_{\text{cap}}$  is the number of positive capital income observations (these households always have positive labor income), resulting in 342 country-year observations out of 475. In practice this means that we estimate the Pareto exponent of labor income further in the tail because the sample size of labor income  $N_{\text{lab}}$  tends to be larger than  $N_{\text{cap}}$ . However, this is acceptable since the Pareto approximation tends to fit better for smaller  $k$ . Figure 3 presents a scatter plot of the estimated labor income Pareto exponent  $\hat{\alpha}_{\text{lab}}$  using 5% of the full sample ( $N_{\text{lab}}$ ) and 5% of the sample with positive capital income ( $N_{\text{cap}}$ ). The fact that most points are close to the 45 degree supports our claim.

Table 3 in Appendix C shows the test results of the null hypothesis  $H_0 : \alpha_{\text{lab}} = \alpha_{\text{cap}}$ . We reject the null in 294 country-year observations out of 342 (86%) that meet our sample selection criterion. In every single case of rejection, we have  $\hat{\alpha}_{\text{lab}} > \hat{\alpha}_{\text{cap}}$ , and therefore we formally confirm the observation in Section 3.2 that capital income is more unequally distributed than labor income.

## 4 Model of capital and labor Pareto exponents

Our empirical analysis in Section 3 suggests that (i) the capital income Pareto exponent is smaller than the labor one (i.e., capital income is more unequally distributed than labor income), and (ii) the correlation between capital and labor income Pareto exponents is weak. To explain these empirical findings, we present a simple dynamic model of consumption and savings, which builds on one of the

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<sup>11</sup>The choice of  $t_0$  in Hoga (2018) comes out using an automated selection procedure to choose  $k$ , which is different than our 5% rule. An earlier version of our paper employs the same automated procedure, which leads to very similar results.

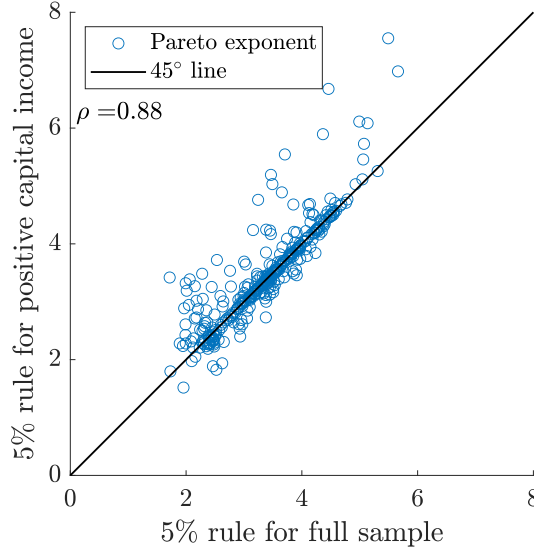


Figure 3: Labor Pareto exponent with full and positive capital income samples.

authors' prior works (Ma, Stachurski, and Toda, 2020; Ma and Toda, 2021). Our model is more specialized but the characterizations are sharper. The proofs of propositions in this section are deferred to Appendix A.

#### 4.1 Income fluctuation problem

Time is discrete and denoted by  $t = 0, 1, 2, \dots$ . Let  $a_t$  be the financial wealth of a typical agent at the beginning of period  $t$  including current income. The agent chooses consumption  $c_t \geq 0$  and saves the remaining wealth  $a_t - c_t$ . The period utility function is  $u : (0, \infty) \rightarrow \mathbb{R}$ , the discount factor is  $\beta > 0$ , the gross return on wealth between time  $t - 1$  and  $t$  is  $R_t > 0$ , and non-financial income at time  $t$  is  $Y_t > 0$ . Thus the agent solves

$$\text{maximize} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (4.1a)$$

$$\text{subject to} \quad a_{t+1} = R_{t+1}(a_t - c_t) + Y_{t+1}, \quad (4.1b)$$

$$0 \leq c_t \leq a_t, \quad (4.1c)$$

where the initial wealth  $a_0 = a > 0$  is given, (4.1b) is the budget constraint, and (4.1c) implies that the agent cannot borrow (which is without loss of generality according to the discussion in Chamberlain and Wilson, 2000). Throughout the rest of the paper we maintain the following assumptions.

**Assumption 1** (CRRA utility). *The utility function exhibits constant relative risk aversion (CRRA) with coefficient  $\gamma > 0$ , so  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$  if  $\gamma \neq 1$  and  $u(c) = \log c$  if  $\gamma = 1$ .*

**Assumption 2** (IID shocks). *Let  $G_{t+1} := Y_{t+1}/Y_t$  be gross growth rate of income. The sequence  $\{R_{t+1}, G_{t+1}\}_{t=0}^\infty$  is independent and identically distributed (IID).*

These assumptions are similar to [Carroll \(2020\)](#), except that we allow for stochastic returns on savings. Note that the asset return  $R_{t+1}$  and income growth  $G_{t+1}$  are potentially mutually dependent. Due to the IID assumption, the state variables of the income fluctuation problem (4.1) are financial wealth  $a_t > 0$  and current income  $Y_t > 0$ . Exploiting homotheticity (Assumption 1), we can reduce the number of state variables to just one, namely the wealth-income ratio (normalized wealth)  $\tilde{a}_t := a_t/Y_t$ . To see this, letting  $\tilde{c}_t := c_t/Y_t$  be the consumption-income ratio (normalized consumption), dividing the borrowing constraint (4.1c) by  $Y_t$ , we obtain  $0 \leq \tilde{c}_t \leq \tilde{a}_t$ . Similarly, dividing the budget constraint (4.1b) by  $Y_{t+1}$ , we obtain

$$\begin{aligned}\tilde{a}_{t+1} &= a_{t+1}/Y_{t+1} = (R_{t+1}Y_t/Y_{t+1})(a_t/Y_t - c_t/Y_t) + 1 \\ &= (R_{t+1}/G_{t+1})(\tilde{a}_t - \tilde{c}_t) + 1 \\ &= \tilde{R}_{t+1}(\tilde{a}_t - \tilde{c}_t) + 1,\end{aligned}\tag{4.2}$$

where  $\tilde{R}_{t+1} := R_{t+1}/G_{t+1}$  is the asset return relative to income growth. As for the utility function, since

$$c_t = Y_t \tilde{c}_t = Y_0 \left( \prod_{s=1}^t G_s \right) \tilde{c}_t$$

(here we interpret  $\prod_{s=1}^0 \bullet = 1$ ), assuming  $Y_0 = 1$  (which is without loss of generality) and  $\gamma \neq 1$ , it follows from (4.1a) that

$$\begin{aligned}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) &= \mathbb{E}_0 \sum_{t=0}^{\infty} \left( \prod_{s=1}^t \beta G_s^{1-\gamma} \right) \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} \\ &= \mathbb{E}_0 \sum_{t=0}^{\infty} \left( \prod_{s=1}^t \tilde{\beta}_s \right) \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma},\end{aligned}\tag{4.3}$$

where  $\tilde{\beta}_t := \beta G_t^{1-\gamma}$ . The discussion for  $\gamma = 1$  is similar. Therefore the problem reduces to an income fluctuation problem with CRRA utility, random discount factors  $\{\tilde{\beta}_t\}_{t=1}^\infty$ , stochastic returns  $\{\tilde{R}_t\}_{t=1}^\infty$  on wealth, and constant income ( $\tilde{Y}_t \equiv 1$ ). The general theory of income fluctuation problems with stochastic discounting,

returns, and income in a Markovian setting was developed by [Ma et al. \(2020\)](#). Therefore we immediately obtain the following result. In what follows, we drop the time subscript when no confusion arises.

**Proposition 1.** *Suppose Assumptions 1, 2 hold and*

$$\beta \mathbb{E}[G^{1-\gamma}] < 1 \quad \text{and} \quad \beta \mathbb{E}[RG^{-\gamma}] < 1. \quad (4.4)$$

*Then the income fluctuation problem (4.1) has a unique solution. The consumption function can be expressed as  $c(a, Y) = Y\tilde{c}(a/Y)$ , where  $\tilde{c} : (0, \infty) \rightarrow (0, \infty)$  is the consumption function of the detrended problem (maximizing (4.3) subject to (4.2)), which can be computed by policy function iteration.<sup>12</sup>*

## 4.2 Tail behavior of income and wealth

We now characterize the tail behavior of income and wealth in the context of the income fluctuation problem in Section 4.1.

To make the model stationary, suppose that agents survive to the next period with probability  $v \in (0, 1)$  (perpetual youth model as in [Yaari, 1965](#)). Whenever agents die, they are replaced by newborn agents. For simplicity, assume that the discount factor  $\beta$  in (4.1a) already accounts for survival probability and that there is no market for life insurance (allowing for life insurance only changes  $R$  to  $R/v$  and is thus mathematically equivalent after reparametrization). Without loss of generality, suppose that newborn agents start with income  $Y_0 = 1$ . Then the income of a randomly selected agent is  $Y_T$ , where  $T$  is a geometric random variable with mean  $\frac{1}{1-v}$ . By the assumption on income growth, the log income of a randomly selected agent

$$\log Y_T = \log(Y_T/Y_0) = \sum_{t=1}^T \log G_t$$

is a geometric sum of IID random variables, for which we can characterize the tail behavior as follows.

**Proposition 2** (Income Pareto exponent). *Suppose that  $\mathbb{P}(G > 1) > 0$  and  $1 < v \mathbb{E}[G^z] < \infty$  for some  $z > 0$ . Then the cross-sectional income distribution has a Pareto upper tail, whose exponent is the unique positive solution  $z = \alpha_Y$  to*

$$v \mathbb{E}[G^z] = 1. \quad (4.5)$$

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<sup>12</sup>See [Li and Stachurski \(2014\)](#) and [Ma et al. \(2020\)](#) for details on policy function iteration.



*Proof.* See [Beare and Toda \(2017, Theorem 3.4\)](#).  $\square$

To characterize the tail behavior of wealth, we first note that the normalized consumption function  $\tilde{c}$  in Proposition 1 is concave and asymptotically linear with a specific slope.<sup>13</sup>

**Proposition 3** (Concavity and asymptotic linearity). *Let everything be as in Proposition 1. Then  $\tilde{c}$  is concave and*

$$\lim_{a \rightarrow \infty} \frac{\tilde{c}(a)}{a} = \begin{cases} 1 - (\mathbb{E}[\beta R^{1-\gamma}])^{1/\gamma} & \text{if } \mathbb{E}[\beta R^{1-\gamma}] < 1, \\ 0 & \text{otherwise.} \end{cases} \quad (4.6)$$

Using Proposition 3 and setting  $\rho = \min \{(\mathbb{E}[\beta R^{1-\gamma}])^{1/\gamma}, 1\}$ , for high enough asset level, the detrended budget constraint (4.2) becomes approximately

$$\tilde{a}_{t+1} \approx \rho \tilde{R}_{t+1} \tilde{a}_t + 1,$$

which is a random multiplicative process ([Kesten \(1973\)](#) process). Under specific assumptions, [Ma et al. \(2020, Theorem 3.3\)](#) prove that the upper tail of the stationary distribution of normalized wealth  $\tilde{a}_t$  has a Pareto lower bound. Although a sharp characterization of the tail behavior is generally difficult, in our setting it is possible to obtain an exact characterization due to concavity and the IID assumption.

**Proposition 4.** *Let  $\rho = \min \{(\mathbb{E}[\beta R^{1-\gamma}])^{1/\gamma}, 1\}$  and  $H = \rho \tilde{R}$ . Suppose that (i)  $R$  is thin-tailed (meaning  $\mathbb{E}[R^z] < \infty$  for all  $z > 0$ ), (ii)  $\log H$  is non-lattice (not supported on an evenly spaced grid), and (iii)  $\mathbb{P}(H > 1) > 0$ , and  $1 < v \mathbb{E}[H^z] < \infty$  for some  $z > 0$ . Then the cross-sectional normalized wealth distribution is either bounded or has a Pareto upper tail, in which case the exponent is the unique positive solution  $z = \tilde{\alpha}$  of*

$$v \mathbb{E}[H^z] = 1. \quad (4.7)$$

*The Pareto exponent for wealth and capital income is then  $\alpha = \min \{\tilde{\alpha}, \alpha_Y\}$ .*

Proposition 4 is significant despite its simplicity. According to the model, we always have  $\alpha \leq \alpha_Y$ , typically with a strict inequality as we see in the numerical example below. This is in sharp contrast to canonical incomplete market general equilibrium models such as [Aiyagari \(1994\)](#), where agents can save using only a

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<sup>13</sup>[Carroll and Kimball \(1996\)](#) showed the sufficiency of hyperbolic absolute risk aversion (HARA, which includes CRRA) for the concavity of the consumption function. [Toda \(2021\)](#) proved the necessity.

risk-free asset. In such models, the impossibility theorem of [Stachurski and Toda \(2019\)](#) implies that the tail behavior of income and wealth is the same, implying  $\alpha = \alpha_Y$  in our setting.<sup>14</sup> Therefore, unlike canonical incomplete market models, our model can explain the empirical fact that capital income is more unequal than labor income. The key assumption leading to this conclusion is the presence of stochastic returns.

We discuss an analytically solvable example to build intuition.

**Example 1.** Let  $\Delta > 0$  be the length of time of one period and the discount factor be  $\beta = e^{-\delta\Delta}$ , where  $\delta > 0$  is the discount rate. Suppose income grows at a constant rate  $g > 0$ , so  $G = e^{g\Delta}$ . Suppose asset return is risk-free, so  $R = e^{r\Delta}$  with  $r > 0$ . Finally, let the survival probability be  $v = e^{-\eta\Delta}$ , where  $\eta$  is the death rate. Then (4.5) becomes

$$1 = e^{-\eta\Delta} e^{zg\Delta} \iff z = \eta/g,$$

so the income Pareto exponent is  $\alpha_Y = \eta/g$ . (This is the classical result of [Wold and Whittle \(1957\)](#) in discrete-time.) Suppose in addition that  $-\eta + r(1 - \gamma) < 0$  so that  $\beta R^{1-\gamma} < 1$ . Since

$$H = (E[\beta R^{1-\gamma}])^{1/\gamma} \tilde{R} = (\beta R)^{1/\gamma} / G = e^{(\frac{r-\eta}{\gamma}-g)\Delta},$$

solving (4.7) the normalized wealth Pareto exponent is

$$\tilde{\alpha} = \frac{\eta\gamma}{r - \eta - g\gamma}$$

assuming  $r - \eta - g\gamma > 0$ . Therefore

$$\tilde{\alpha} < \alpha_Y \iff \frac{\eta\gamma}{r - \eta - g\gamma} < \frac{\eta}{g} \iff r > \eta + 2g\gamma,$$

so the wealth (hence capital income) Pareto exponent is smaller than the labor income Pareto exponent if the return on wealth  $r$  is sufficiently large. In summary, we obtain the following result: suppose  $-\eta + r(1 - \gamma) < 0$  and let  $\alpha_{\text{cap}}, \alpha_{\text{lab}}$  be the capital and labor income Pareto exponents. Then

$$\begin{cases} \alpha_{\text{cap}} = \alpha_{\text{lab}} = \frac{\eta}{g} & \text{if } r \leq \eta + 2g\gamma, \\ \alpha_{\text{cap}} = \frac{\eta\gamma}{r - \eta - g\gamma} < \frac{\eta}{g} = \alpha_{\text{lab}} & \text{if } r > \eta + 2g\gamma. \end{cases} \quad (4.8)$$

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<sup>14</sup>The original proof in [Stachurski and Toda \(2019\)](#) contained an error; it has been corrected in [Stachurski and Toda \(2020\)](#).

Note that the labor income Pareto exponent  $\alpha_{\text{lab}} = \eta/g$  is highly sensitive to the income growth rate  $g$ . However, provided that  $r > \eta + 2g\gamma$ , the capital income Pareto exponent  $\alpha_{\text{cap}}$  is not very sensitive to the value of  $g$  because the denominator is  $r - \eta - g\gamma$ . This example is consistent with our result in Section 3.2 that the capital Pareto exponent is smaller than the labor Pareto exponent but the two values are only weakly related.

### 4.3 Numerical example

We further examine the tail behavior of income and wealth using a numerical example of the income fluctuation problem (4.1). Suppose that asset return is IID lognormal, so  $\log R \sim N((\mu - \sigma^2/2)\Delta, \sigma^2\Delta)$ , where  $\Delta > 0$  is the length of one period,  $\mu$  is the expected return, and  $\sigma$  is volatility. Suppose every period the agent is “promoted” with some probability, so the income growth rate is

$$G_{t+1} = Y_{t+1}/Y_t = \begin{cases} 1 & \text{with probability } 1 - p, \\ e^g & \text{with probability } p, \end{cases}$$

where  $p \in (0, 1)$  is the promotion probability and  $g$  is the log income growth rate conditional on promotion. We parametrize the promotion probability as  $p = 1 - e^{-\Delta/L}$ , where  $L$  is the expected length of time until a promotion. Using (4.5), the labor income Pareto exponent is determined such that

$$1 = v \mathbb{E}[G^{\alpha_Y}] = v(1 - p + pe^{g\alpha_Y}) \iff \alpha_Y = \frac{1}{g} \log \frac{1 - v + vp}{vp}. \quad (4.9)$$

We set the parameter values as in Table 1. One unit of time corresponds to a year and one period is a quarter, so  $\Delta = 1/4$ . The preference parameters (discount rate and risk aversion) are standard. The death rate of  $\eta = 0.025$  implies an average (economically active) age of  $1/\eta = 40$  years. The expected return and volatility roughly correspond to the stock market. We set the labor income Pareto exponent to  $\alpha_Y = 3$ , which is roughly the median value in Figure 1. Using the survival probability  $v = e^{-\eta\Delta}$  and (4.9), the implied value of income growth upon promotion is  $g = 0.0403$ . The wealth Pareto exponent determined by (4.7) is then  $\alpha = 1.201$ .

To numerically solve the income fluctuation problem (4.1), we discretize the log asset return  $\log R$  using a 7-point Gauss-Hermite quadrature and apply policy function iteration (see Appendix B). After solving the individual problem, we apply the Pareto extrapolation algorithm developed in Gouin-Bonenfant and Toda

Table 1: Parameter values

Parameter	Symbol	Value
Length of one period	$\Delta$	1/4
Discount rate	$\delta$	0.04
Relative risk aversion	$\gamma$	2
Death rate	$\eta$	0.025
Expected return	$\mu$	0.07
Volatility	$\sigma$	0.15
Expected time to promotion	$L$	5
Labor income Pareto exponent	$\alpha_Y$	3

(2018) to accurately compute the stationary (normalized) wealth distribution. Finally, we also simulate an economy with  $10^5$  agents. Figure 4 shows the results.

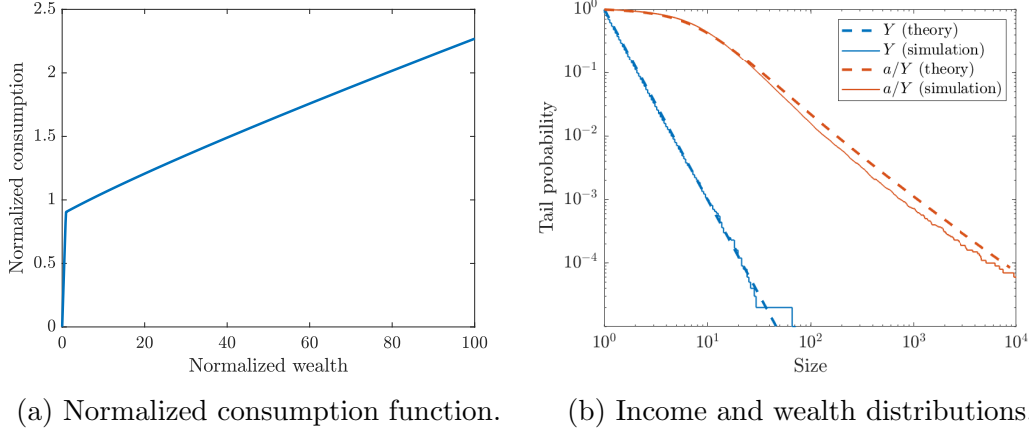


Figure 4: Solution to income fluctuation problem.

Figure 4a shows the normalized consumption function  $\tilde{c}(\tilde{a})$  in the range  $\tilde{a} \in [0, 100]$ . Consistent with Proposition 3, the consumption function is roughly linear for high asset level. Figure 4b shows the size distributions of income  $Y$  normalized wealth  $\tilde{a} = a/Y$  in a log-log plot, both from the theoretical model and the simulation. The fact that the tail probability  $P(X > x)$  exhibits a straight line pattern in a log-log plot suggests that the size distributions have Pareto upper tails, consistent with theory. Furthermore, the slope for income is steeper than that of normalized wealth, so wealth (hence capital income) is more unequally distributed than labor income.

Finally, Figure 5 shows the income and wealth Pareto exponents when we change the income growth rate  $g$  in the range  $g \in [0.02, 0.1]$ , fixing other parameters. Because the income Pareto exponent is inversely proportional to income

growth by (4.9), the labor income Pareto exponent is highly sensitive to income growth. On the other hand, the wealth (capital income) Pareto exponent does not depend much on income growth by the same intuition as in Example 1. Thus our model is consistent with our empirical findings in Section 3.2 that capital and labor income Pareto exponents are only weakly related.

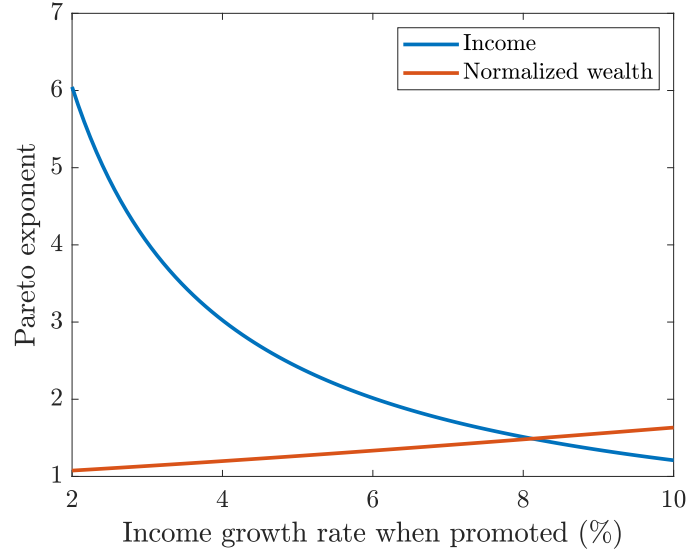


Figure 5: Dependence of income and wealth Pareto exponents on  $g$ .

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## A Proofs

*Proof of Proposition 1.* Applying Theorem 2.2 of [Ma et al. \(2020\)](#) to the IID case, a sufficient condition for the existence of a solution to the detrended problem is  $E[\tilde{\beta}] < 1$  and  $E[\tilde{\beta}\tilde{R}] < 1$ , which is equivalent to (4.4).  $\square$

*Proof of Proposition 3.* The concavity of  $\tilde{c}$  follows from Proposition 2.5 and Remark 2.1 of [Ma et al. \(2020\)](#). The asymptotic linearity of  $\tilde{c}$  follows from [Ma and Toda \(2021, Theorem 3\)](#). Noting that

$$E[\tilde{\beta}\tilde{R}^{1-\gamma}] = E[\beta G^{1-\gamma}(R/G)^{1-\gamma}] = E[\beta R^{1-\gamma}],$$

the limit (4.6) follows from their Example 2.  $\square$

*Proof of Proposition 4.* Since by Proposition 3  $\tilde{c}$  is concave, it is in particular Lipschitz continuous. Under the maintained assumptions, we can apply Theorem 1.8 of [Mirek \(2011\)](#) to deduce that the normalized wealth  $\tilde{a}$  is either bounded or has a Pareto upper tail with exponent characterized as the solution to (4.7), where we have used the asymptotic linearity of  $\tilde{c}$  established in Proposition 3.

By accounting, capital income (excluding capital loss) is

$$Y_{\text{cap}} := \max\{R - 1, 0\}(a - c(a)) = \max\{R - 1, 0\}Y(\tilde{a} - \tilde{c}(\tilde{a})).$$

Using Proposition 3, this quantity is approximately equal to  $\rho \max\{R - 1, 0\}Y\tilde{a}$ . The claim  $\alpha = \min\{\tilde{\alpha}, \alpha_Y\}$  then follows because asset return  $R$  is thin-tailed and  $a_t = Y_t\tilde{a}_t$  is the product of two (potentially dependent) random variables with Pareto upper tails, which inherits the smallest Pareto exponent by the result in [Jessen and Mikosch \(2006\)](#).  $\square$

## B Solving the income fluctuation problem

In this appendix we discuss how to solve the detrended income fluctuation problem. After detrending, the problem becomes

$$\begin{aligned} &\text{maximize} && E_0 \sum_{t=0}^{\infty} \left( \prod_{s=1}^t \beta_s \right) \frac{c_t^{1-\gamma}}{1-\gamma} \\ &\text{subject to} && a_{t+1} = R_{t+1}(a_t - c_t) + 1, \\ &&& 0 \leq c_t \leq a_t, \end{aligned}$$

where  $\{R_t, \beta_t\}_{t=1}^{\infty}$  is IID (though  $R_t$  and  $\beta_t$  are generally correlated.) According to [Ma et al. \(2020\)](#), the Euler equation is

$$c_t^{-\gamma} = \max \left\{ E_t[\beta_{t+1} R_{t+1} c_{t+1}^{-\gamma}], a_t^{-\gamma} \right\}. \quad (\text{B.1})$$

Let  $c(a)$  be the consumption function. Taking the  $-1/\gamma$ -th power of [\(B.1\)](#), we obtain

$$c(a) = \min \left\{ (E[\beta R c(a')^{-\gamma}])^{-1/\gamma}, a \right\}, \quad (\text{B.2})$$

where  $a' = R(a - c(a)) + 1$ . Therefore we can compute the consumption function using the following variant of the policy function iteration algorithm:

- (i) Initialize the consumption function  $c(a)$ . For example, we can set  $c(a) = \min \{a, 1 + ma\}$ , where  $m = \max \{1 - (E[\beta R^{1-\gamma}])^{1/\gamma}, 0\}$  is the theoretical asymptotic marginal propensity to consume according to [\(4.6\)](#).
- (ii) Update  $c(a)$  by the right-hand side of [\(B.2\)](#), where  $a' = R(a - c(a)) + 1$ .
- (iii) Iterate the above step until  $c(a)$  converges.

While the above algorithm has no guarantee to converge unlike the “true” policy function iteration algorithm discussed in [Ma et al. \(2020\)](#), it has the advantage of avoiding root-finding and hence it is fast.

In [Section 4](#), we use this algorithm on a 100-point exponential grid for normalized wealth  $\tilde{a}$  that spans  $[0, 10^4]$ , with a median grid point of 10. The details on the exponential grid are discussed in [Gouin-Bonenfant and Toda \(2018\)](#).

## C Tables (not for publication)

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Australia	1981	5.31	0.23	528	10568	2.1	0.1	474	9488
Australia	1985	4.28	0.26	275	5492	1.41	0.09	262	5232
Australia	1989	3.97	0.17	531	10629	1.47	0.06	549	10973
Australia	1995	3.1	0.2	229	4574	1.71	0.13	168	3364
Australia	2001	3.06	0.2	226	4510	1.47	0.12	155	3105
Australia	2003	3.45	0.19	337	6741	1.52	0.1	224	4473
Australia	2004	3.44	0.17	395	7893	1.42	0.08	307	6145

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Australia	2008	3.07	0.17	333	6666	1.28	0.08	237	4749
Australia	2010	3.39	0.15	547	10949	1.31	0.06	452	9030
Australia	2014	3.15	0.14	489	9788	1.2	0.05	478	9569
Austria	1994	3.38	0.32	109	2183	1.59	0.17	85	1693
Austria	1997	3.59	0.36	100	1998	1.76	0.19	89	1787
Austria	2000	3.17	0.36	79	1584	2.14	0.24	77	1547
Austria	2004	3.35	0.25	187	3749	0.96	0.07	203	4064
Austria	2007	3.24	0.23	205	4101	1.16	0.08	217	4334
Austria	2010	3.52	0.24	216	4323	1	0.06	238	4752
Austria	2013	3.37	0.24	201	4022	1.13	0.08	224	4471
Austria	2016	3.35	0.23	204	4082	1.12	0.08	191	3823
Belgium	1985	4.95	0.34	218	4357	—	—	—	—
Belgium	1988	5.36	0.48	126	2518	—	—	—	—
Belgium	1992	5.45	0.48	129	2581	—	—	—	—
Belgium	1995	3.49	0.37	90	1790	—	—	—	—
Belgium	1997	4.31	0.36	145	2906	1.62	0.11	206	4112
Belgium	2000	3	0.35	72	1431	—	—	—	—
Belgium	2003	3.79	0.3	164	3288	1.62	0.12	183	3664
Belgium	2004	3.39	0.27	157	3135	1.42	0.11	159	3185
Belgium	2005	3.69	0.27	189	3776	1.48	0.11	190	3799
Belgium	2006	4.39	0.31	203	4069	1.39	0.09	220	4398
Belgium	2007	3.36	0.24	202	4046	1.48	0.1	216	4328
Belgium	2008	4.1	0.29	197	3944	1.44	0.1	197	3942
Belgium	2009	4.47	0.32	196	3911	1.68	0.12	213	4259
Belgium	2010	4.07	0.3	186	3723	1.61	0.11	204	4072
Belgium	2011	3.95	0.29	181	3612	1.7	0.12	193	3859
Belgium	2012	4.26	0.31	190	3793	1.62	0.11	205	4102
Belgium	2013	4.22	0.31	183	3659	1.53	0.11	195	3908
Belgium	2014	3.9	0.29	183	3663	1.64	0.12	192	3847
Belgium	2015	4.24	0.32	178	3562	1.33	0.1	185	3695
Belgium	2016	3.81	0.28	183	3669	1.8	0.13	185	3703
Belgium	2017	4.22	0.32	179	3586	1.76	0.14	164	3276
Brazil	2006	1.99	0.03	4978	99559	1.58	0.1	239	4780
Brazil	2009	2.01	0.03	5000	100002	1.98	0.13	233	4667
Brazil	2011	1.97	0.03	4453	89066	1.76	0.15	142	2845
Brazil	2013	2.05	0.03	4583	91657	1.56	0.13	140	2804
Brazil	2016	2.06	0.03	5756	115110	1.88	0.11	315	6296
Canada	1971	3.9	0.12	1019	20386	1.66	0.07	498	9970
Canada	1975	3.89	0.12	1088	21750	1.85	0.08	536	10717
Canada	1981	4.51	0.18	617	12331	2.02	0.1	417	8342
Canada	1987	4.49	0.22	433	8667	1.9	0.12	260	5190

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Canada	1991	4.39	0.16	763	15257	2	0.09	492	9847
Canada	1994	4.18	0.11	1408	28155	1.76	0.07	704	14077
Canada	1997	3.96	0.11	1251	25018	1.79	0.08	564	11273
Canada	1998	3.5	0.1	1198	23969	1.47	0.06	674	13486
Canada	2000	3.3	0.1	1115	22298	1.47	0.06	663	13264
Canada	2004	3.34	0.1	1080	21594	1.56	0.06	593	11861
Canada	2007	3.06	0.09	1046	20918	1.5	0.06	627	12533
Canada	2010	3.36	0.11	975	19500	1.4	0.06	517	10341
Canada	2012	3.52	0.11	953	19053	1.65	0.07	518	10370
Canada	2013	3.84	0.13	903	18069	1.64	0.07	487	9741
Canada	2014	3.41	0.11	924	18474	1.74	0.08	492	9836
Canada	2015	3.42	0.11	989	19785	1.49	0.06	528	10552
Canada	2016	3.47	0.11	1026	20526	1.64	0.07	532	10641
Canada	2017	3.68	0.09	1516	30313	1.52	0.05	799	15982
Chile	1990	1.95	0.06	1138	22755	1.62	0.21	62	1242
Chile	1992	1.72	0.04	1597	31934	1.78	0.19	84	1678
Chile	1994	1.99	0.04	1993	39851	1.91	0.19	104	2071
Chile	1996	2.15	0.06	1509	30177	1.53	0.12	172	3437
Chile	1998	1.99	0.04	2145	42903	1.64	0.13	167	3335
Chile	2000	1.99	0.04	2886	57716	1.38	0.09	222	4432
Chile	2003	1.89	0.03	3003	60055	1.48	0.1	232	4637
Chile	2006	2.09	0.04	3256	65111	1.38	0.08	269	5374
Chile	2009	2.18	0.04	2983	59668	1.48	0.11	181	3619
Chile	2011	2.28	0.05	2544	50887	1.82	0.14	166	3312
Chile	2013	2.21	0.04	2854	57079	1.76	0.13	182	3635
Chile	2015	2.15	0.04	3574	71477	1.71	0.11	266	5314
Chile	2017	2.18	0.04	2986	59724	1.56	0.1	241	4829
China	2002	3.16	0.11	837	16745	1.72	0.17	108	2156
China	2013	2.99	0.11	794	15878	2.06	0.09	505	10098
Colombia	2004	2.13	0.1	413	8255	1.95	0.25	60	1196
Colombia	2007	2.1	0.02	9734	194688	1.7	0.05	1189	23787
Colombia	2010	2.24	0.02	9944	198885	1.63	0.05	1247	24944
Colombia	2013	2.37	0.02	10039	200779	1.75	0.05	1283	25654
Colombia	2016	2.45	0.02	10090	201793	1.64	0.05	1198	23957
Czech Rep.	1992	3.84	0.15	664	13287	—	—	—	—
Czech Rep.	1996	3.38	0.1	1179	23587	1.14	0.09	154	3077
Czech Rep.	2002	3.31	0.19	304	6082	—	—	—	—
Czech Rep.	2004	3.19	0.26	156	3125	—	—	—	—
Czech Rep.	2007	3.49	0.17	413	8254	1.11	0.12	87	1735
Czech Rep.	2010	3.74	0.21	328	6551	1.23	0.15	71	1422
Czech Rep.	2013	3.45	0.2	300	5996	1.59	0.21	60	1198

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Czech Rep.	2016	3.97	0.22	326	6526	1.25	0.17	54	1083
Denmark	2007	3.82	0.07	3063	61255	1.12	0.02	3938	78755
Denmark	2010	3.37	0.06	2974	59488	1.14	0.02	3464	69287
Denmark	2013	3.47	0.06	3044	60880	1.09	0.02	3404	68088
Denmark	2016	3.42	0.06	3098	61970	1.23	0.02	2949	58989
Denmark	1987	4.48	0.21	460	9199	1.53	0.06	579	11575
Denmark	1992	4.59	0.21	458	9164	1.42	0.06	593	11853
Denmark	1995	4.68	0.09	2865	57306	1.24	0.02	3510	70200
Dominican Rep.	2000	4.26	0.08	2977	59549	1.18	0.02	3486	69714
Dominican Rep.	2004	4.04	0.07	2991	59824	1.13	0.02	3394	67886
Dominican Rep.	2007	2.12	0.11	374	7471	—	—	—	—
Egypt	2012	1.97	0.09	505	10095	—	—	—	—
Estonia	2000	3.06	0.19	260	5196	—	—	—	—
Estonia	2004	3.23	0.26	156	3116	—	—	—	—
Estonia	2007	4.12	0.28	210	4201	0.68	0.06	112	2250
Estonia	2010	3.85	0.26	212	4241	0.91	0.09	97	1948
Estonia	2013	3.44	0.22	248	4951	1.22	0.14	72	1450
Estonia	2016	4.46	0.27	271	5422	1.05	0.13	62	1240
Finland	1987	4.37	0.18	561	11225	1.45	0.07	395	7906
Finland	1991	4.52	0.2	519	10380	1.43	0.07	417	8332
Finland	1995	4.42	0.22	393	7867	1.25	0.09	212	4236
Finland	2000	3.52	0.17	449	8977	1.1	0.06	296	5923
Finland	2004	4.07	0.19	468	9368	1.02	0.05	403	8061
Finland	2007	3.79	0.18	433	8664	1.01	0.05	425	8502
Finland	2010	3.92	0.2	377	7537	1.06	0.05	379	7571
Finland	2013	3.93	0.19	445	8899	1.04	0.05	457	9145
Finland	2016	3.72	0.18	412	8237	1.19	0.06	416	8325
France	1978	2.93	0.15	383	7664	1.75	0.21	71	1426
France	1984	1.96	0.1	394	7875	1.75	0.18	99	1974
France	1989	3.28	0.19	297	5942	1.68	0.18	84	1677
France	1994	3.13	0.16	394	7880	1.68	0.08	437	8734
France	2000	3.52	0.19	356	7112	1.58	0.08	426	8530
France	2005	3.34	0.17	399	7987	1.51	0.07	416	8330
France	2010	3.03	0.12	591	11818	1.49	0.06	583	11658
Germany	1973	3.56	0.08	1881	37614	2.16	0.05	2245	44891
Germany	1978	3.36	0.08	1787	35731	2.19	0.05	2233	44654
Germany	1981	4.66	0.46	101	2024	—	—	—	—
Germany	1983	4.54	0.11	1614	32288	2.26	0.05	2006	40130
Germany	1984	3.97	0.27	212	4240	1.04	0.07	213	4259
Germany	1987	4.24	0.31	193	3863	1.18	0.08	202	4050
Germany	1989	3.31	0.24	187	3733	1.16	0.08	197	3940

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Germany	1991	4.79	0.29	272	5448	1.09	0.06	289	5789
Germany	1994	4.54	0.28	269	5384	1.34	0.08	288	5758
Germany	1995	4.66	0.29	264	5275	1.36	0.08	290	5802
Germany	1998	4.56	0.27	277	5531	1.36	0.08	307	6136
Germany	2000	3.92	0.19	439	8784	1.48	0.07	491	9829
Germany	2001	3.46	0.16	464	9273	1.26	0.06	509	10174
Germany	2002	3.57	0.17	442	8850	1.38	0.06	490	9798
Germany	2003	3.64	0.18	430	8600	1.25	0.06	494	9875
Germany	2004	3.67	0.18	415	8300	1.24	0.06	477	9544
Germany	2005	3.31	0.16	443	8869	1.24	0.05	517	10338
Germany	2006	3.29	0.16	414	8276	1.33	0.06	478	9562
Germany	2007	3.45	0.18	389	7778	1.32	0.06	453	9055
Germany	2008	3.24	0.17	365	7295	1.26	0.06	425	8497
Germany	2009	3.43	0.15	531	10611	1.36	0.06	555	11102
Germany	2010	3.45	0.14	626	12512	1.3	0.05	646	12913
Germany	2011	3.3	0.13	615	12291	1.29	0.05	633	12664
Germany	2012	3.4	0.13	696	13912	1.31	0.05	653	13051
Germany	2013	3.4	0.14	610	12199	1.32	0.05	591	11824
Germany	2014	3.85	0.16	610	12193	1.31	0.06	559	11176
Germany	2015	3.13	0.13	550	11000	1.31	0.06	531	10625
Germany	2016	3.43	0.14	611	12215	1.27	0.05	560	11203
Georgia	2010	2.76	0.18	243	4867	—	—	—	—
Georgia	2013	3.14	0.28	126	2512	—	—	—	—
Georgia	2016	2.93	0.27	120	2390	—	—	—	—
Greece	1995	3.68	0.28	169	3375	2.83	0.39	52	1033
Greece	2000	3.56	0.31	129	2580	—	—	—	—
Greece	2004	3.03	0.21	199	3987	2.12	0.3	50	1006
Greece	2007	2.86	0.19	227	4533	2	0.25	63	1253
Greece	2010	2.64	0.19	193	3858	2.12	0.28	58	1170
Greece	2013	2.73	0.17	251	5015	2.24	0.26	76	1517
Greece	2016	3.05	0.12	643	12867	2.43	0.19	165	3299
Guatemala	2006	2.01	0.08	672	13448	—	—	—	—
Guatemala	2011	2.2	0.09	631	12622	—	—	—	—
Guatemala	2014	2.28	0.1	560	11203	—	—	—	—
Hungary	1991	3.23	0.35	85	1693	—	—	—	—
Hungary	1994	3.34	0.38	77	1532	—	—	—	—
Hungary	1999	3.04	0.35	77	1531	—	—	—	—
Hungary	2005	3.41	0.41	69	1384	—	—	—	—
Hungary	2007	3.29	0.4	66	1328	—	—	—	—
Hungary	2009	3.59	0.46	62	1250	—	—	—	—
Hungary	2012	2.91	0.36	67	1343	—	—	—	—



Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Hungary	2015	3.18	0.33	90	1809	—	—	—	—
Iceland	2004	4.76	0.41	132	2632	1.17	0.12	92	1836
Iceland	2007	4.34	0.38	129	2583	1.17	0.11	106	2112
Iceland	2010	4.54	0.39	134	2675	1.14	0.09	150	2994
India	2004	2.35	0.05	1992	39832	2.47	0.25	94	1874
India	2011	2.12	0.05	1998	39958	1.66	0.17	101	2011
Ireland	1987	3.46	0.31	124	2482	1.33	0.13	100	1998
Ireland	1994	2.93	0.27	121	2426	1.33	0.18	56	1113
Ireland	1995	2.65	0.26	107	2146	1.34	0.19	51	1025
Ireland	1996	2.95	0.29	101	2021	1.56	0.22	51	1022
Ireland	2000	3.23	0.33	95	1893	1.95	0.27	51	1017
Ireland	2002	3.96	0.4	100	2000	—	—	—	—
Ireland	2003	3.03	0.23	174	3471	—	—	—	—
Ireland	2004	3	0.22	187	3731	—	—	—	—
Ireland	2005	2.62	0.2	176	3522	1.83	0.25	52	1049
Ireland	2006	2.49	0.19	171	3411	1.49	0.15	99	1984
Ireland	2007	2.91	0.23	157	3143	1.48	0.17	75	1500
Ireland	2008	2.73	0.22	153	3069	—	—	—	—
Ireland	2009	3.27	0.28	134	2675	—	—	—	—
Ireland	2010	3.38	0.3	126	2520	—	—	—	—
Ireland	2011	3.78	0.32	136	2720	—	—	—	—
Ireland	2012	3.35	0.28	145	2894	—	—	—	—
Ireland	2013	3.2	0.25	164	3288	—	—	—	—
Ireland	2014	3.06	0.24	164	3285	—	—	—	—
Ireland	2015	2.91	0.23	158	3159	—	—	—	—
Ireland	2016	2.76	0.22	161	3214	—	—	—	—
Ireland	2017	2.46	0.21	142	2835	—	—	—	—
Israel	1979	3.95	0.4	97	1941	—	—	—	—
Israel	1986	3.25	0.23	197	3942	—	—	—	—
Israel	1992	3.75	0.26	203	4057	—	—	—	—
Israel	1997	3.01	0.21	204	4072	—	—	—	—
Israel	2001	3.13	0.21	219	4382	—	—	—	—
Israel	2002	3	0.2	227	4543	—	—	—	—
Israel	2003	3.27	0.21	232	4639	—	—	—	—
Israel	2004	3.75	0.25	228	4569	—	—	—	—
Israel	2005	3.62	0.23	238	4752	1.29	0.15	74	1475
Israel	2006	3.7	0.24	239	4780	1.38	0.17	63	1254
Israel	2007	3.46	0.23	234	4682	—	—	—	—
Israel	2008	3.05	0.2	230	4594	—	—	—	—
Israel	2009	3.49	0.23	240	4792	—	—	—	—
Israel	2010	2.96	0.19	237	4747	—	—	—	—

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Israel	2011	3.46	0.23	234	4683	—	—	—	—
Israel	2012	3.54	0.19	343	6854	1.33	0.16	65	1305
Israel	2013	3.27	0.17	372	7438	1.25	0.15	73	1451
Israel	2014	3.24	0.18	330	6607	1.69	0.21	66	1330
Israel	2015	3.46	0.19	341	6818	1.38	0.17	65	1301
Israel	2016	3.45	0.18	348	6968	2.16	0.28	61	1215
Israel	2017	3.6	0.19	358	7150	1.61	0.21	59	1173
Israel	2018	3.38	0.18	347	6944	1.59	0.2	64	1271
Italy	1986	3.5	0.2	301	6016	1.85	0.14	174	3474
Italy	1987	3.55	0.2	302	6045	1.61	0.09	342	6844
Italy	1989	4.15	0.24	308	6154	1.57	0.09	293	5865
Italy	1991	4.43	0.26	291	5827	1.6	0.09	329	6587
Italy	1993	3.76	0.23	272	5444	1.42	0.08	336	6724
Italy	1995	3.24	0.2	274	5487	1.52	0.08	341	6811
Italy	1998	3.37	0.22	244	4876	1.49	0.08	310	6191
Italy	2000	3.1	0.19	264	5279	1.29	0.07	326	6516
Italy	2004	2.39	0.15	246	4913	1.31	0.07	332	6641
Italy	2008	3.19	0.21	239	4778	1.41	0.08	328	6552
Italy	2010	3.35	0.22	236	4719	1.33	0.07	324	6490
Italy	2014	2.76	0.19	220	4402	1.41	0.08	337	6748
Italy	2016	2.69	0.19	193	3853	1.06	0.06	310	6197
Ivory Coast	2002	1.83	0.08	491	9811	—	—	—	—
Ivory Coast	2008	1.76	0.07	580	11594	—	—	—	—
Ivory Coast	2015	1.9	0.08	565	11293	—	—	—	—
Japan	2008	3.9	0.31	163	3256	—	—	—	—
Japan	2010	4.4	0.39	127	2540	—	—	—	—
Japan	2013	4.22	0.43	95	1891	—	—	—	—
Lithuania	2009	3.42	0.24	201	4026	—	—	—	—
Lithuania	2010	4.72	0.35	184	3680	—	—	—	—
Lithuania	2011	4.36	0.31	198	3951	—	—	—	—
Lithuania	2012	3.15	0.23	187	3746	—	—	—	—
Lithuania	2013	3.61	0.26	190	3792	—	—	—	—
Lithuania	2014	2.82	0.21	176	3510	—	—	—	—
Lithuania	2015	3.13	0.24	176	3526	—	—	—	—
Lithuania	2016	3.81	0.28	184	3683	—	—	—	—
Lithuania	2017	3.25	0.24	188	3750	—	—	—	—
Luxembourg	1985	6.01	0.68	78	1552	—	—	—	—
Luxembourg	1991	5.1	0.58	77	1535	—	—	—	—
Luxembourg	1994	4.75	0.58	68	1362	—	—	—	—
Luxembourg	1997	5.23	0.54	95	1906	—	—	—	—
Luxembourg	2000	4.37	0.46	92	1840	—	—	—	—

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Luxembourg	2004	4.09	0.35	138	2753	—	—	—	—
Luxembourg	2007	3.02	0.25	151	3028	1.68	0.17	103	2066
Luxembourg	2010	3.37	0.23	206	4111	1.71	0.13	175	3494
Luxembourg	2013	3.59	0.3	141	2828	1.6	0.14	127	2534
Mexico	1984	2.67	0.18	223	4461	—	—	—	—
Mexico	1989	2.25	0.1	544	10876	—	—	—	—
Mexico	1992	2.12	0.09	498	9959	—	—	—	—
Mexico	1994	2.12	0.09	603	12068	—	—	—	—
Mexico	1996	2.23	0.09	656	13128	—	—	—	—
Mexico	1998	2.23	0.1	505	10092	—	—	—	—
Mexico	2000	2.25	0.1	463	9262	—	—	—	—
Mexico	2002	2.54	0.09	795	15905	—	—	—	—
Mexico	2004	2.32	0.07	1046	20928	1.58	0.22	53	1055
Mexico	2008	2.24	0.06	1359	27171	1.43	0.16	82	1635
Mexico	2010	2.33	0.07	1252	25038	1.66	0.2	66	1323
Mexico	2012	2.33	0.11	412	8247	—	—	—	—
Mexico	2014	2.38	0.08	894	17887	—	—	—	—
Mexico	2016	2.46	0.04	3227	64534	1.53	0.12	159	3186
Mexico	2018	2.52	0.04	3406	68114	1.6	0.12	169	3389
Netherlands	1983	5.16	0.42	154	3090	—	—	—	—
Netherlands	1987	4.14	0.35	140	2792	—	—	—	—
Netherlands	1990	3.79	0.31	149	2971	1.38	0.12	128	2570
Netherlands	1993	4.3	0.32	178	3565	1.23	0.11	121	2423
Netherlands	1999	4.34	0.35	153	3061	1.32	0.13	97	1942
Netherlands	2004	3.43	0.18	358	7166	1.39	0.07	409	8171
Netherlands	2007	3.17	0.16	401	8011	0.97	0.05	457	9136
Netherlands	2010	3.65	0.18	400	8001	1.29	0.06	477	9539
Netherlands	2013	3.62	0.18	390	7808	1.3	0.06	474	9486
Norway	1979	1.73	0.08	414	8276	1.73	0.1	302	6048
Norway	1986	5.05	0.35	213	4254	1.91	0.17	129	2588
Norway	1991	3.83	0.2	368	7361	1.76	0.09	372	7448
Norway	1995	3.46	0.17	401	8027	1.12	0.05	490	9800
Norway	2000	3.19	0.13	574	11474	0.88	0.03	640	12804
Norway	2004	3.98	0.17	547	10947	0.74	0.03	649	12979
Norway	2007	3.74	0.04	8485	169708	1.09	0.01	10768	215363
Norway	2010	3.55	0.04	8861	177229	0.88	0.01	11279	225584
Norway	2013	3.35	0.04	9157	183146	0.85	0.01	11660	233192
Norway	2016	3.45	0.04	9371	187424	0.72	0.01	11974	239485
Palestine	2017	2.82	0.22	161	3214	1.43	0.2	50	1000
Panama	2007	2.17	0.09	579	11571	—	—	—	—
Panama	2010	2.24	0.09	588	11754	—	—	—	—

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Panama	2013	2.22	0.1	528	10551	—	—	—	—
Panama	2016	2.4	0.11	511	10229	—	—	—	—
Paraguay	2000	1.87	0.1	384	7685	—	—	—	—
Paraguay	2004	1.89	0.1	372	7434	—	—	—	—
Paraguay	2007	1.66	0.11	226	4528	—	—	—	—
Paraguay	2010	1.77	0.12	235	4697	—	—	—	—
Paraguay	2013	2.04	0.13	254	5073	—	—	—	—
Paraguay	2016	1.87	0.09	474	9470	—	—	—	—
Peru	2004	2.28	0.08	864	17289	1.87	0.18	110	2193
Peru	2007	2.29	0.07	1023	20464	1.68	0.15	129	2571
Peru	2010	2.49	0.08	986	19720	1.68	0.13	157	3137
Peru	2013	2.53	0.07	1384	27670	1.64	0.12	191	3827
Peru	2016	2.63	0.07	1633	32651	1.87	0.12	232	4649
Poland	1986	3.79	0.18	461	9213	—	—	—	—
Poland	1992	3.55	0.21	297	5944	—	—	—	—
Poland	1995	2.67	0.08	1055	21091	—	—	—	—
Poland	1999	3.31	0.1	1211	24216	—	—	—	—
Poland	2004	2.92	0.09	1178	23557	—	—	—	—
Poland	2007	2.84	0.08	1394	27872	—	—	—	—
Poland	2010	3.03	0.08	1382	27640	—	—	—	—
Poland	2013	2.91	0.08	1324	26475	—	—	—	—
Poland	2016	3.16	0.09	1299	25983	—	—	—	—
Romania	1995	3.55	0.09	1576	31519	—	—	—	—
Romania	1997	3.51	0.09	1606	32122	—	—	—	—
Russia	2000	2.39	0.21	132	2631	—	—	—	—
Russia	2004	2.91	0.25	135	2705	—	—	—	—
Russia	2007	3.69	0.3	149	2986	—	—	—	—
Russia	2010	3.21	0.2	251	5025	—	—	—	—
Russia	2011	3.4	0.16	435	8697	—	—	—	—
Russia	2013	3.38	0.08	1915	38297	1.67	0.11	218	4360
Russia	2014	3.49	0.08	1922	38433	1.79	0.12	221	4412
Russia	2015	3.65	0.07	2539	50779	1.65	0.1	288	5752
Russia	2016	3.47	0.04	6739	134781	1.91	0.07	786	15716
Russia	2017	3.71	0.07	2539	50774	2.42	0.14	284	5671
Serbia	2006	3.74	0.28	175	3491	—	—	—	—
Serbia	2010	3.84	0.3	164	3270	—	—	—	—
Serbia	2013	2.48	0.2	160	3201	—	—	—	—
Serbia	2016	3.42	0.23	228	4566	—	—	—	—
Slovakia	1992	4.31	0.18	572	11439	—	—	—	—
Slovakia	1996	4.65	0.18	659	13176	—	—	—	—
Slovakia	2004	3.14	0.22	199	3988	—	—	—	—

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Slovakia	2007	4.24	0.28	227	4541	—	—	—	—
Slovakia	2010	3.86	0.26	219	4373	0.98	0.14	51	1024
Slovakia	2013	3.38	0.22	226	4520	0.84	0.1	64	1289
Slovakia	2014	4.18	0.28	229	4576	1.4	0.18	64	1285
Slovakia	2015	4.14	0.27	228	4557	0.97	0.12	66	1317
Slovakia	2016	5.06	0.34	218	4367	0.87	0.12	57	1139
Slovakia	2017	5.66	0.38	218	4368	0.96	0.13	55	1105
Slovakia	2018	5.49	0.38	212	4246	0.69	0.09	54	1079
Slovenia	1997	5.24	0.48	119	2378	—	—	—	—
Slovenia	1999	4.91	0.37	175	3492	—	—	—	—
Slovenia	2004	5.27	0.41	164	3275	—	—	—	—
Slovenia	2007	4.78	0.38	161	3219	—	—	—	—
Slovenia	2010	4.9	0.37	173	3468	—	—	—	—
Slovenia	2012	4.99	0.39	163	3252	1.48	0.15	92	1841
Slovenia	2015	5.07	0.39	170	3399	1.12	0.12	84	1673
South Africa	2008	1.76	0.11	251	5011	—	—	—	—
South Africa	2010	1.89	0.14	194	3878	—	—	—	—
South Africa	2012	2.08	0.13	258	5150	—	—	—	—
South Africa	2015	2.2	0.12	336	6715	—	—	—	—
South Africa	2017	2.27	0.12	364	7281	—	—	—	—
South Korea	2006	4.11	0.15	720	14390	1.69	0.11	218	4353
South Korea	2008	3.91	0.16	627	12542	1.79	0.13	181	3616
South Korea	2010	4.15	0.17	614	12276	1.62	0.13	150	3003
South Korea	2012	4.09	0.17	596	11911	1.58	0.12	166	3329
South Korea	2014	4.36	0.18	561	11216	1.39	0.1	176	3526
South Korea	2016	3.67	0.16	511	10214	1.34	0.1	164	3282
Spain	1980	3.86	0.12	983	19651	2.07	0.24	74	1470
Spain	1985	3.76	0.32	135	2707	—	—	—	—
Spain	1990	4.23	0.15	809	16178	1.58	0.11	202	4036
Spain	1995	3.91	0.27	204	4088	1.6	0.15	112	2249
Spain	2000	3.21	0.25	169	3379	1.09	0.08	210	4206
Spain	2004	4.07	0.19	459	9175	1.32	0.09	222	4433
Spain	2007	4.37	0.2	485	9701	1.44	0.09	266	5327
Spain	2010	5.14	0.24	441	8822	1.83	0.15	152	3043
Spain	2013	3.56	0.17	441	8823	1.7	0.09	334	6681
Spain	2016	3.33	0.15	508	10165	1.52	0.08	373	7451
Sweden	1967	3.16	0.2	246	4912	—	—	—	—
Sweden	1975	3.93	0.18	477	9535	1.55	0.08	392	7845
Sweden	1981	4.45	0.21	432	8635	2.47	0.14	305	6092
Sweden	1987	4.57	0.22	433	8658	1.79	0.09	398	7953
Sweden	1992	4.5	0.19	541	10824	1.96	0.08	561	11215

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
Sweden	1995	3.98	0.16	583	11655	1.61	0.06	650	12993
Sweden	2000	3.62	0.16	516	10319	1.23	0.05	564	11273
Sweden	2005	3.16	0.13	598	11951	1.08	0.04	606	12120
Switzerland	1982	2.16	0.13	278	5570	1.34	0.09	212	4233
Switzerland	1992	3.56	0.24	224	4486	1.33	0.09	242	4846
Switzerland	2000	3.53	0.28	157	3140	1.41	0.12	132	2646
Switzerland	2002	3.84	0.31	154	3074	1.56	0.14	129	2578
Switzerland	2004	4.32	0.37	135	2697	1.38	0.12	132	2648
Switzerland	2006	3.11	0.19	266	5329	1.22	0.08	210	4206
Switzerland	2007	2.8	0.17	271	5427	1.23	0.08	253	5057
Switzerland	2008	3	0.18	291	5822	1.11	0.07	250	5004
Switzerland	2009	2.95	0.17	290	5805	1.23	0.07	270	5396
Switzerland	2010	2.92	0.17	288	5759	1.09	0.06	337	6749
Switzerland	2011	3.21	0.19	286	5714	1.24	0.07	337	6746
Switzerland	2012	3.33	0.2	274	5471	1.26	0.07	329	6586
Switzerland	2013	3.48	0.22	255	5097	1.36	0.08	310	6190
Switzerland	2014	3.12	0.19	284	5672	1.28	0.07	328	6562
Switzerland	2015	3.18	0.18	297	5947	1.36	0.07	340	6794
Switzerland	2016	3.09	0.17	316	6310	1.38	0.07	353	7068
Switzerland	2017	3.2	0.2	252	5048	1.37	0.08	278	5568
Taiwan	1981	4.2	0.15	750	14997	1.53	0.07	532	10635
Taiwan	1986	3.96	0.14	796	15911	1.35	0.05	643	12864
Taiwan	1991	4.1	0.15	788	15756	1.41	0.05	749	14978
Taiwan	1995	4.3	0.16	681	13619	1.76	0.07	709	14184
Taiwan	1997	4.27	0.17	629	12572	1.78	0.07	666	13311
Taiwan	2000	4.2	0.17	615	12301	1.75	0.07	670	13396
Taiwan	2005	4.17	0.17	595	11903	1.77	0.07	656	13127
Taiwan	2007	3.82	0.16	602	12042	1.69	0.07	664	13290
Taiwan	2010	3.85	0.15	637	12748	1.7	0.06	714	14287
Taiwan	2013	3.85	0.15	665	13300	1.56	0.06	781	15615
Taiwan	2016	3.88	0.15	682	13648	1.59	0.06	822	16449
United Kingdom	1969	3.92	0.23	283	5669	1.12	0.08	205	4102
United Kingdom	1974	3.62	0.22	259	5181	1.35	0.1	177	3547
United Kingdom	1979	4.93	0.31	247	4937	1.34	0.09	214	4280
United Kingdom	1986	4.49	0.3	226	4515	1.67	0.11	247	4942
United Kingdom	1991	3.58	0.24	222	4445	1.66	0.1	259	5174
United Kingdom	1994	2.99	0.11	778	15567	1.49	0.05	905	18098
United Kingdom	1995	3.1	0.11	789	15777	1.53	0.05	879	17586
United Kingdom	1996	2.98	0.11	770	15401	1.52	0.05	843	16869
United Kingdom	1997	2.82	0.1	728	14554	1.45	0.05	764	15287
United Kingdom	1998	2.88	0.11	699	13980	1.44	0.05	762	15237

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		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
United Kingdom	1999	2.6	0.09	764	15289	1.41	0.05	862	17241
United Kingdom	2000	2.33	0.09	732	14636	1.42	0.05	799	15978
United Kingdom	2001	2.47	0.09	794	15887	1.39	0.05	840	16804
United Kingdom	2002	2.9	0.1	898	17957	1.32	0.04	944	18872
United Kingdom	2003	2.89	0.1	886	17714	1.25	0.04	936	18728
United Kingdom	2004	2.65	0.09	867	17337	1.25	0.04	927	18535
United Kingdom	2005	2.71	0.09	872	17447	1.38	0.05	914	18276
United Kingdom	2006	2.51	0.09	796	15913	1.39	0.05	837	16738
United Kingdom	2007	2.98	0.11	766	15327	1.33	0.05	819	16371
United Kingdom	2008	2.97	0.11	771	15411	1.32	0.05	586	11718
United Kingdom	2009	2.78	0.1	760	15197	1.3	0.06	526	10528
United Kingdom	2010	3	0.11	766	15318	1.34	0.06	492	9839
United Kingdom	2011	3.23	0.13	638	12768	1.4	0.07	391	7823
United Kingdom	2012	2.93	0.12	609	12180	1.33	0.07	374	7479
United Kingdom	2013	3.23	0.13	616	12322	1.5	0.08	386	7715
United Kingdom	2014	3	0.12	600	12009	1.46	0.07	396	7922
United Kingdom	2015	2.94	0.12	586	11724	1.48	0.08	383	7663
United Kingdom	2016	3.11	0.13	595	11902	1.33	0.07	367	7340
United Kingdom	2017	3.19	0.13	581	11614	1.47	0.08	348	6965
United Kingdom	2018	2.9	0.12	579	11580	1.59	0.08	353	7062
United States	1974	3.81	0.18	468	9366	1.51	0.09	278	5558
United States	1979	3.64	0.07	2639	52783	1.46	0.03	2237	44749
United States	1986	3.47	0.07	2283	45660	1.65	0.04	1914	38274
United States	1991	3.4	0.07	2310	46195	1.54	0.04	1944	38871
United States	1992	3.2	0.07	2296	45921	1.49	0.03	1907	38137
United States	1993	2.57	0.05	2211	44224	1.46	0.03	1863	37252
United States	1994	2.48	0.05	2213	44261	1.45	0.03	1868	37355
United States	1995	2.52	0.06	1948	38958	1.48	0.04	1604	32076
United States	1996	2.47	0.06	1978	39567	1.51	0.04	1558	31165
United States	1997	2.4	0.05	1991	39812	1.47	0.04	1546	30913
United States	1998	2.47	0.06	2013	40267	1.46	0.04	1557	31144
United States	1999	2.43	0.05	2044	40886	1.44	0.04	1566	31320
United States	2000	2.38	0.04	3229	64575	1.4	0.03	2348	46961
United States	2001	2.35	0.04	3216	64322	1.31	0.03	2308	46168
United States	2002	2.49	0.04	3200	64000	1.35	0.03	2198	43963
United States	2003	2.43	0.04	3147	62939	1.37	0.03	2170	43410
United States	2004	2.45	0.04	3117	62334	1.33	0.03	2118	42354
United States	2005	2.31	0.04	3095	61899	1.42	0.03	2081	41624
United States	2006	2.38	0.04	3085	61707	1.46	0.03	2013	40264
United States	2007	2.56	0.05	3100	62002	1.49	0.03	1966	39312
United States	2008	2.48	0.04	3103	62055	1.33	0.03	2026	40523

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\hat{\alpha}$	s.e.	$k$	$N$	$\hat{\alpha}$	s.e.	$k$	$N$
United States	2009	2.48	0.04	3064	61288	1.33	0.03	1922	38446
United States	2010	2.62	0.05	2997	59932	1.41	0.03	1869	37383
United States	2011	2.42	0.04	2946	58921	1.4	0.03	1836	36728
United States	2012	2.47	0.05	2970	59404	1.56	0.04	1843	36865
United States	2013	2.5	0.06	2036	40721	1.41	0.04	1225	24494
United States	2014	2.41	0.04	2905	58099	1.41	0.03	2316	46312
United States	2015	2.29	0.04	2726	54510	1.36	0.03	2213	44260
United States	2016	2.28	0.04	2736	54727	1.42	0.03	2270	45401
United States	2017	2.39	0.05	2641	52816	1.45	0.03	2255	45102
United States	2018	2.41	0.05	2658	53170	1.34	0.03	2265	45307
Uruguay	2004	2.31	0.09	684	13671	1.82	0.23	62	1241
Uruguay	2007	2.44	0.06	1855	37097	1.53	0.11	212	4236
Uruguay	2010	2.59	0.06	1771	35428	1.64	0.11	204	4085
Uruguay	2013	3	0.07	1763	35264	1.8	0.14	175	3497
Uruguay	2016	2.76	0.07	1674	33484	1.69	0.13	182	3632
Vietnam	2011	2.77	0.13	464	9279	1.26	0.15	68	1362
Vietnam	2013	2.97	0.14	464	9276	1.77	0.21	69	1384

Note:  $\hat{\alpha}$ : point estimate of Pareto exponent; s.e.: standard error ( $\hat{\alpha}(k)/\sqrt{k}$ );  $k$ : the number of order statistics used for the Hill-estimator;  $N$ : the number of positive labor (resp. capital) income observations. “—” indicates fewer than 1,000 positive observations so estimates are omitted.

Table 3: Hypothesis testing of  $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$ .

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	$k$	$N$	Reject $H_0$ ?
Australia	1981	5.26	0.24	2.1	0.1	474	9488	Yes
Australia	1985	4.22	0.26	1.4	0.09	262	5232	Yes
Australia	1989	3.95	0.17	1.46	0.06	531	10629	Yes
Australia	1995	2.91	0.22	1.7	0.13	168	3364	Yes
Australia	2001	3.24	0.26	1.45	0.12	155	3105	Yes
Australia	2003	3.25	0.22	1.51	0.1	224	4473	Yes
Australia	2004	3.38	0.19	1.41	0.08	307	6145	Yes
Australia	2008	3.3	0.21	1.27	0.08	237	4749	Yes
Australia	2010	3.31	0.16	1.29	0.06	455	9093	Yes
Australia	2014	3.14	0.14	1.19	0.05	481	9616	Yes
Austria	1994	3.12	0.34	1.59	0.17	85	1695	Yes
Austria	1997	3.55	0.37	1.78	0.19	90	1800	No
Austria	2000	3.1	0.35	2.1	0.24	79	1584	No
Austria	2004	3.34	0.24	0.99	0.07	188	3750	Yes
Austria	2007	3.23	0.23	1.17	0.08	205	4101	Yes
Austria	2010	3.51	0.24	1.05	0.07	216	4323	Yes



Table 3: Hypothesis testing of  $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$ .

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	$k$	$N$	Reject $H_0$ ?
Austria	2013	3.36	0.24	1.12	0.08	201	4022	Yes
Austria	2016	3.31	0.24	1.11	0.08	192	3835	Yes
Belgium	1997	4.25	0.35	1.82	0.15	145	2906	Yes
Belgium	2003	3.79	0.29	1.6	0.12	173	3451	Yes
Belgium	2004	3.38	0.27	1.41	0.11	159	3185	Yes
Belgium	2005	3.68	0.27	1.47	0.11	189	3777	Yes
Belgium	2006	4.38	0.31	1.5	0.11	203	4069	Yes
Belgium	2007	3.35	0.24	1.48	0.1	202	4046	Yes
Belgium	2008	4.1	0.29	1.44	0.1	197	3942	Yes
Belgium	2009	4.46	0.32	1.75	0.12	196	3911	Yes
Belgium	2010	4.05	0.3	1.7	0.12	186	3723	Yes
Belgium	2011	3.91	0.29	1.6	0.12	181	3612	Yes
Belgium	2012	4.25	0.31	1.69	0.12	190	3793	Yes
Belgium	2013	4.18	0.31	1.64	0.12	183	3659	Yes
Belgium	2014	3.89	0.29	1.73	0.13	183	3663	Yes
Belgium	2015	4.24	0.32	1.3	0.1	178	3562	Yes
Belgium	2016	3.77	0.28	1.81	0.13	183	3669	Yes
Belgium	2017	4.19	0.33	1.72	0.13	164	3276	Yes
Brazil	2006	3.32	0.21	1.53	0.1	240	4804	No
Brazil	2009	3.19	0.21	1.98	0.13	236	4724	No
Brazil	2011	2.89	0.24	1.7	0.14	145	2897	No
Brazil	2013	2.94	0.25	1.48	0.12	143	2869	Yes
Brazil	2016	3.39	0.19	1.88	0.11	315	6296	Yes
Canada	1971	3.56	0.16	1.66	0.07	498	9970	Yes
Canada	1975	4.02	0.17	1.84	0.08	536	10717	Yes
Canada	1981	4.59	0.22	2.01	0.1	417	8342	Yes
Canada	1987	4.78	0.3	1.88	0.12	260	5190	Yes
Canada	1991	4.34	0.2	2	0.09	492	9847	Yes
Canada	1994	4.07	0.15	1.75	0.07	704	14077	Yes
Canada	1997	4.01	0.17	1.79	0.08	564	11273	Yes
Canada	1998	3.22	0.12	1.46	0.06	674	13486	Yes
Canada	2000	3.17	0.12	1.47	0.06	663	13264	Yes
Canada	2004	3.15	0.13	1.56	0.06	593	11861	Yes
Canada	2007	2.85	0.11	1.49	0.06	627	12533	Yes
Canada	2010	3.06	0.13	1.4	0.06	517	10341	Yes
Canada	2012	3.35	0.15	1.65	0.07	518	10370	Yes
Canada	2013	3.93	0.18	1.56	0.07	487	9741	Yes
Canada	2014	3.23	0.15	1.71	0.08	492	9836	Yes
Canada	2015	3.33	0.15	1.49	0.06	528	10552	Yes
Canada	2016	3.38	0.15	1.64	0.07	532	10641	Yes
Canada	2017	3.53	0.12	1.51	0.05	799	15982	Yes
Chile	1990	2.24	0.28	1.62	0.21	62	1242	No

Table 3: Hypothesis testing of  $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$ .

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	$k$	$N$	Reject $H_0$ ?
Chile	1992	3.42	0.37	1.76	0.19	84	1678	Yes
Chile	1994	2.61	0.26	1.91	0.19	104	2071	No
Chile	1996	2.71	0.21	1.52	0.12	172	3437	Yes
Chile	1998	2.43	0.19	1.64	0.13	167	3335	No
Chile	2000	2.28	0.15	1.38	0.09	222	4432	Yes
Chile	2003	2.28	0.15	1.45	0.1	232	4637	No
Chile	2006	2.31	0.14	1.38	0.08	269	5374	Yes
Chile	2009	3.02	0.22	1.48	0.11	181	3619	Yes
Chile	2011	3.48	0.27	1.82	0.14	166	3312	Yes
Chile	2013	2.87	0.21	1.76	0.13	182	3635	No
Chile	2015	3.27	0.2	1.71	0.11	266	5314	Yes
Chile	2017	2.73	0.18	1.56	0.1	241	4829	No
China	2002	4.24	0.41	1.72	0.17	108	2167	Yes
China	2013	3.1	0.14	2.06	0.09	505	10098	No
Colombia	2004	2.37	0.28	1.97	0.23	71	1411	No
Colombia	2007	1.97	0.06	1.7	0.05	1189	23787	No
Colombia	2010	2.52	0.07	1.63	0.05	1247	24944	No
Colombia	2013	2.64	0.07	1.75	0.05	1283	25654	Yes
Colombia	2016	2.55	0.07	1.64	0.05	1198	23957	Yes
Czech Rep	1996	3	0.24	1.14	0.09	154	3077	Yes
Czech Rep	2007	3.62	0.39	1.11	0.12	87	1735	Yes
Czech Rep	2010	4.09	0.49	1.23	0.15	71	1422	Yes
Czech Rep	2013	3.12	0.4	1.54	0.2	60	1198	No
Czech Rep	2016	3.74	0.51	1.24	0.17	54	1083	Yes
Denmark	2007	3.82	0.07	1.17	0.02	3063	61255	Yes
Denmark	2010	3.37	0.06	1.2	0.02	2974	59488	Yes
Denmark	2013	3.47	0.06	1.13	0.02	3044	60880	Yes
Denmark	2016	3.41	0.06	1.23	0.02	2949	58989	Yes
Denmark	1987	4.46	0.21	1.64	0.08	460	9199	Yes
Denmark	1992	4.57	0.21	1.46	0.07	458	9164	Yes
Denmark	1995	4.68	0.09	1.26	0.02	2865	57306	Yes
Dominican Rep.	2000	4.26	0.08	1.19	0.02	2977	59549	Yes
Dominican Rep.	2004	4.03	0.07	1.17	0.02	2991	59824	Yes
Estonia	2007	4.53	0.43	0.67	0.06	112	2250	Yes
Estonia	2010	4.68	0.48	0.91	0.09	97	1948	Yes
Estonia	2013	3.65	0.42	1.16	0.13	77	1549	Yes
Estonia	2016	6.68	0.85	1.05	0.13	62	1240	Yes
Finland	1987	4.35	0.22	1.45	0.07	395	7906	Yes
Finland	1991	4.48	0.22	1.43	0.07	417	8332	Yes
Finland	1995	4.5	0.31	1.25	0.09	212	4236	Yes
Finland	2000	3.53	0.21	1.09	0.06	296	5923	Yes
Finland	2004	3.91	0.19	1.02	0.05	403	8061	Yes

Table 3: Hypothesis testing of  $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$ .

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	$k$	$N$	Reject $H_0$ ?
Finland	2007	3.82	0.19	1.01	0.05	425	8502	Yes
Finland	2010	3.91	0.2	1.05	0.05	377	7537	Yes
Finland	2013	3.9	0.19	1.04	0.05	445	8899	Yes
Finland	2016	3.71	0.18	1.19	0.06	412	8237	Yes
France	1978	2.4	0.28	1.75	0.21	71	1426	No
France	1984	1.52	0.15	1.73	0.17	99	1974	No
France	1989	3.43	0.37	1.65	0.18	84	1677	Yes
France	1994	3.13	0.16	1.68	0.08	394	7880	Yes
France	2000	3.51	0.19	1.72	0.09	356	7112	Yes
France	2005	3.33	0.17	1.53	0.08	399	7987	Yes
France	2010	3.01	0.12	1.49	0.06	583	11658	Yes
Germany	1973	3.56	0.08	2.13	0.05	1881	37614	No
Germany	1978	3.36	0.08	2.18	0.05	1787	35731	No
Germany	1983	4.54	0.11	2.23	0.06	1614	32288	Yes
Germany	1984	3.96	0.27	1.03	0.07	212	4240	Yes
Germany	1987	4.23	0.3	1.17	0.08	193	3863	Yes
Germany	1989	3.29	0.24	1.18	0.09	187	3733	Yes
Germany	1991	4.77	0.29	1.1	0.07	272	5448	Yes
Germany	1994	4.54	0.28	1.32	0.08	269	5384	Yes
Germany	1995	4.65	0.29	1.33	0.08	264	5275	Yes
Germany	1998	4.56	0.27	1.31	0.08	277	5531	Yes
Germany	2000	3.91	0.19	1.51	0.07	439	8784	Yes
Germany	2001	3.46	0.16	1.31	0.06	464	9273	Yes
Germany	2002	3.57	0.17	1.35	0.06	442	8850	Yes
Germany	2003	3.64	0.18	1.28	0.06	430	8600	Yes
Germany	2004	3.67	0.18	1.29	0.06	415	8300	Yes
Germany	2005	3.29	0.16	1.27	0.06	443	8869	Yes
Germany	2006	3.28	0.16	1.32	0.06	414	8276	Yes
Germany	2007	3.44	0.17	1.35	0.07	389	7778	Yes
Germany	2008	3.22	0.17	1.29	0.07	365	7295	Yes
Germany	2009	3.43	0.15	1.37	0.06	531	10611	Yes
Germany	2010	3.44	0.14	1.29	0.05	626	12512	Yes
Germany	2011	3.3	0.13	1.28	0.05	615	12291	Yes
Germany	2012	3.43	0.13	1.31	0.05	653	13051	Yes
Germany	2013	3.4	0.14	1.32	0.05	591	11824	Yes
Germany	2014	3.78	0.16	1.3	0.06	559	11176	Yes
Germany	2015	3.09	0.13	1.31	0.06	531	10625	Yes
Germany	2016	3.48	0.15	1.27	0.05	560	11203	Yes
Greece	1995	3.7	0.51	2.83	0.39	52	1033	No
Greece	2004	3.64	0.52	2.12	0.3	50	1006	No
Greece	2007	2.81	0.35	1.97	0.25	63	1253	No
Greece	2010	3.01	0.39	2.09	0.27	58	1170	No

Table 3: Hypothesis testing of  $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$ .

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	$k$	$N$	Reject $H_0$ ?
Greece	2013	2.68	0.31	2.22	0.25	76	1517	No
Greece	2016	2.72	0.21	2.46	0.19	174	3480	No
Iceland	2004	4.71	0.49	1.15	0.12	92	1840	Yes
Iceland	2007	4.4	0.43	1.16	0.11	106	2113	Yes
Iceland	2010	4.51	0.39	1.13	0.1	134	2675	Yes
India	2004	2.85	0.29	2.47	0.25	94	1876	No
India	2011	2.21	0.22	1.43	0.14	101	2027	No
Ireland	1987	3.41	0.34	1.29	0.13	100	1998	Yes
Ireland	1994	2.54	0.34	1.26	0.17	56	1114	Yes
Ireland	1995	2.31	0.32	1.3	0.18	51	1026	Yes
Ireland	1996	2.68	0.38	1.56	0.22	51	1023	No
Ireland	2000	3.44	0.41	1.7	0.2	69	1371	Yes
Ireland	2005	1.94	0.27	1.8	0.25	52	1049	No
Ireland	2006	2.23	0.22	1.48	0.15	99	1984	Yes
Ireland	2007	2.61	0.3	1.47	0.17	75	1500	No
Israel	2005	3.82	0.44	1.25	0.15	74	1475	Yes
Israel	2006	3.42	0.43	1.34	0.17	63	1254	Yes
Israel	2012	3.49	0.43	1.3	0.16	65	1305	Yes
Israel	2013	3.15	0.37	1.24	0.14	73	1451	Yes
Israel	2014	4.76	0.59	1.69	0.21	66	1330	Yes
Israel	2015	4.17	0.52	1.36	0.17	65	1301	Yes
Israel	2016	4.23	0.54	2.14	0.27	61	1215	Yes
Israel	2017	3.39	0.44	1.56	0.2	59	1173	Yes
Israel	2018	3.85	0.48	1.58	0.2	64	1271	Yes
Italy	1986	3.17	0.24	1.85	0.14	174	3474	Yes
Italy	1987	3.51	0.2	1.66	0.1	302	6045	Yes
Italy	1989	3.95	0.23	1.56	0.09	293	5865	Yes
Italy	1991	4.43	0.26	1.64	0.1	291	5827	Yes
Italy	1993	3.76	0.23	1.52	0.09	272	5444	Yes
Italy	1995	3.24	0.2	1.62	0.1	274	5487	Yes
Italy	1998	3.37	0.22	1.46	0.09	244	4876	Yes
Italy	2000	3.05	0.19	1.33	0.08	264	5280	Yes
Italy	2004	2.39	0.15	1.52	0.1	246	4913	Yes
Italy	2008	3.13	0.2	1.44	0.09	239	4778	Yes
Italy	2010	3.35	0.22	1.34	0.09	236	4719	Yes
Italy	2014	2.75	0.19	1.61	0.11	220	4402	Yes
Italy	2016	2.68	0.19	1.5	0.11	193	3853	Yes
Luxembourg	2007	3.26	0.32	1.61	0.16	104	2085	Yes
Luxembourg	2010	3.43	0.26	1.71	0.13	175	3494	Yes
Luxembourg	2013	3.68	0.33	1.59	0.14	127	2534	No
Mexico	2004	2.58	0.35	1.58	0.22	53	1055	Yes
Mexico	2008	2.21	0.24	1.43	0.16	82	1635	Yes

Table 3: Hypothesis testing of  $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$ .

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	$k$	$N$	Reject $H_0$ ?
Mexico	2010	2.64	0.32	1.64	0.2	66	1323	Yes
Mexico	2016	1.89	0.15	1.51	0.12	159	3186	No
Mexico	2018	1.82	0.14	1.59	0.12	169	3389	No
Netherlands	1990	3.75	0.33	1.38	0.12	128	2570	Yes
Netherlands	1993	4.16	0.38	1.23	0.11	121	2423	Yes
Netherlands	1999	4.21	0.37	1.27	0.11	128	2563	Yes
Netherlands	2004	3.41	0.18	1.36	0.07	358	7166	Yes
Netherlands	2007	3.17	0.16	0.98	0.05	401	8011	Yes
Netherlands	2010	3.63	0.18	1.37	0.07	400	8001	Yes
Netherlands	2013	3.62	0.18	1.3	0.07	390	7808	Yes
Norway	1979	1.8	0.1	1.71	0.1	302	6048	No
Norway	1986	5.11	0.45	1.89	0.17	129	2588	Yes
Norway	1991	3.83	0.2	1.76	0.09	368	7361	Yes
Norway	1995	3.45	0.17	1.14	0.06	401	8027	Yes
Norway	2000	3.18	0.13	0.88	0.04	574	11474	Yes
Norway	2004	3.98	0.17	0.77	0.03	547	10947	Yes
Norway	2007	3.74	0.04	1.11	0.01	8485	169708	Yes
Norway	2010	3.55	0.04	0.86	0.01	8861	177229	Yes
Norway	2013	3.35	0.04	0.83	0.01	9157	183146	Yes
Norway	2016	3.45	0.04	0.72	0.01	9371	187424	Yes
Palestine	2017	2.64	0.37	1.43	0.2	50	1000	Yes
Peru	2004	2.5	0.23	1.75	0.16	123	2463	Yes
Peru	2007	3.05	0.26	1.69	0.15	133	2667	Yes
Peru	2010	2.78	0.22	1.66	0.13	162	3239	Yes
Peru	2013	3.72	0.27	1.68	0.12	196	3930	Yes
Peru	2016	2.92	0.19	1.94	0.12	241	4813	Yes
Russia	2013	4.25	0.29	1.67	0.11	218	4360	Yes
Russia	2014	5.03	0.34	1.79	0.12	221	4412	Yes
Russia	2015	4.89	0.29	1.65	0.1	288	5752	No
Russia	2016	5.19	0.19	1.91	0.07	786	15716	No
Russia	2017	5.54	0.33	2.42	0.14	284	5671	No
Slovakia	2010	3.49	0.49	0.96	0.13	51	1024	Yes
Slovakia	2013	2.73	0.34	0.84	0.1	64	1289	Yes
Slovakia	2014	4.51	0.56	1.4	0.18	64	1285	No
Slovakia	2015	4.69	0.58	0.93	0.11	66	1317	Yes
Slovakia	2016	5.46	0.72	0.87	0.12	57	1139	No
Slovakia	2017	6.98	0.94	0.93	0.13	55	1105	No
Slovakia	2018	7.55	1.03	0.69	0.09	54	1079	Yes
Slovenia	2012	6.11	0.64	1.43	0.15	92	1841	Yes
Slovenia	2015	5.73	0.62	1.09	0.12	84	1673	Yes
South Korea	2006	4.67	0.32	1.69	0.11	218	4353	No
South Korea	2008	4.2	0.31	1.7	0.13	181	3616	No

Table 3: Hypothesis testing of  $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$ .

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	$k$	$N$	Reject $H_0$ ?
South Korea	2010	4.38	0.36	1.62	0.13	150	3003	No
South Korea	2012	3.79	0.29	1.58	0.12	166	3329	No
South Korea	2014	5.89	0.44	1.39	0.1	176	3526	Yes
South Korea	2016	3.95	0.31	1.33	0.1	164	3282	Yes
Spain	1980	4.21	0.49	2.07	0.24	75	1503	No
Spain	1990	3.84	0.27	1.58	0.11	202	4036	Yes
Spain	1995	3.46	0.33	1.57	0.15	112	2250	Yes
Spain	2000	3.19	0.24	1.09	0.08	177	3549	Yes
Spain	2004	4.22	0.28	1.32	0.09	222	4433	Yes
Spain	2007	4.3	0.26	1.41	0.09	266	5327	Yes
Spain	2010	6.08	0.49	1.79	0.15	152	3043	Yes
Spain	2013	3.59	0.2	1.69	0.09	334	6681	Yes
Spain	2016	3.44	0.18	1.51	0.08	373	7451	Yes
Sweden	1975	3.93	0.2	1.54	0.08	392	7845	Yes
Sweden	1981	4.56	0.26	2.47	0.14	305	6092	Yes
Sweden	1987	4.71	0.24	1.77	0.09	398	7953	Yes
Sweden	1992	4.49	0.19	1.98	0.08	541	10824	Yes
Sweden	1995	3.97	0.16	1.63	0.07	583	11655	Yes
Sweden	2000	3.6	0.16	1.26	0.06	516	10319	Yes
Sweden	2005	3.15	0.13	1.07	0.04	598	11951	Yes
Switzerland	1982	2.05	0.14	1.34	0.09	212	4233	Yes
Switzerland	1992	3.55	0.24	1.3	0.09	224	4486	Yes
Switzerland	2000	3.29	0.29	1.4	0.12	132	2646	Yes
Switzerland	2002	3.53	0.31	1.56	0.14	129	2578	Yes
Switzerland	2004	4.41	0.38	1.38	0.12	132	2648	Yes
Switzerland	2006	3.07	0.21	1.21	0.08	210	4206	Yes
Switzerland	2007	2.74	0.17	1.22	0.08	253	5057	Yes
Switzerland	2008	3.01	0.19	1.11	0.07	250	5004	Yes
Switzerland	2009	2.96	0.18	1.22	0.07	270	5396	Yes
Switzerland	2010	2.92	0.17	1.14	0.07	288	5759	Yes
Switzerland	2011	3.2	0.19	1.25	0.07	286	5714	Yes
Switzerland	2012	3.28	0.2	1.25	0.08	274	5471	Yes
Switzerland	2013	3.45	0.22	1.4	0.09	255	5097	Yes
Switzerland	2014	3.09	0.18	1.31	0.08	284	5672	Yes
Switzerland	2015	3.18	0.18	1.4	0.08	297	5947	Yes
Switzerland	2016	3.07	0.17	1.43	0.08	316	6310	Yes
Switzerland	2017	3.19	0.2	1.46	0.09	252	5048	Yes
Taiwan	1981	4.5	0.19	1.53	0.07	532	10635	Yes
Taiwan	1986	3.88	0.15	1.34	0.05	643	12864	Yes
Taiwan	1991	4.11	0.15	1.41	0.05	749	14978	Yes
Taiwan	1995	4.29	0.16	1.77	0.07	681	13619	Yes
Taiwan	1997	4.27	0.17	1.83	0.07	629	12572	Yes

Table 3: Hypothesis testing of  $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$ .

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	$k$	$N$	Reject $H_0$ ?
Taiwan	2000	4.19	0.17	1.76	0.07	615	12301	Yes
Taiwan	2005	4.17	0.17	1.8	0.07	595	11903	Yes
Taiwan	2007	3.82	0.16	1.75	0.07	602	12042	Yes
Taiwan	2010	3.85	0.15	1.74	0.07	637	12748	Yes
Taiwan	2013	3.85	0.15	1.55	0.06	665	13300	Yes
Taiwan	2016	3.88	0.15	1.66	0.06	682	13648	Yes
United Kingdom	1969	3.73	0.26	1.11	0.08	205	4102	Yes
United Kingdom	1974	3.56	0.27	1.35	0.1	177	3547	Yes
United Kingdom	1979	5.03	0.34	1.34	0.09	214	4280	Yes
United Kingdom	1986	4.48	0.3	1.79	0.12	226	4515	Yes
United Kingdom	1991	3.56	0.24	1.69	0.11	222	4445	Yes
United Kingdom	1994	2.99	0.11	1.56	0.06	781	15625	Yes
United Kingdom	1995	3.09	0.11	1.53	0.05	789	15781	Yes
United Kingdom	1996	2.97	0.11	1.53	0.06	770	15401	Yes
United Kingdom	1997	2.81	0.1	1.48	0.05	728	14554	Yes
United Kingdom	1998	2.87	0.11	1.47	0.06	699	13980	Yes
United Kingdom	1999	2.6	0.09	1.41	0.05	764	15289	Yes
United Kingdom	2000	2.33	0.09	1.42	0.05	732	14636	Yes
United Kingdom	2001	2.47	0.09	1.42	0.05	794	15887	Yes
United Kingdom	2002	2.9	0.1	1.3	0.04	898	17957	Yes
United Kingdom	2003	2.89	0.1	1.28	0.04	886	17714	Yes
United Kingdom	2004	2.64	0.09	1.26	0.04	867	17337	Yes
United Kingdom	2005	2.71	0.09	1.4	0.05	872	17447	Yes
United Kingdom	2006	2.51	0.09	1.42	0.05	796	15913	Yes
United Kingdom	2007	2.97	0.11	1.34	0.05	766	15327	Yes
United Kingdom	2008	2.92	0.12	1.32	0.05	586	11718	Yes
United Kingdom	2009	2.77	0.12	1.3	0.06	526	10528	Yes
United Kingdom	2010	3.15	0.14	1.34	0.06	492	9839	Yes
United Kingdom	2011	3.37	0.17	1.4	0.07	391	7823	Yes
United Kingdom	2012	2.97	0.15	1.32	0.07	374	7479	Yes
United Kingdom	2013	3.48	0.18	1.5	0.08	386	7715	Yes
United Kingdom	2014	3.03	0.15	1.46	0.07	396	7922	Yes
United Kingdom	2015	3.07	0.16	1.48	0.08	383	7663	Yes
United Kingdom	2016	3.03	0.16	1.32	0.07	367	7340	Yes
United Kingdom	2017	3.26	0.17	1.47	0.08	348	6965	Yes
United Kingdom	2018	3.03	0.16	1.59	0.08	353	7062	Yes
United States	1974	3.86	0.23	1.51	0.09	278	5558	Yes
United States	1979	3.7	0.08	1.46	0.03	2237	44749	Yes
United States	1986	3.36	0.08	1.64	0.04	1914	38274	Yes
United States	1991	3.45	0.08	1.54	0.04	1944	38871	Yes
United States	1992	3.14	0.07	1.49	0.03	1907	38137	Yes
United States	1993	2.51	0.06	1.46	0.03	1863	37252	Yes

Table 3: Hypothesis testing of  $\hat{\alpha}_{\text{lab}} = \hat{\alpha}_{\text{cap}}$ .

Country	Year	$\hat{\alpha}_{\text{lab}}$	s.e.	$\hat{\alpha}_{\text{cap}}$	s.e.	$k$	$N$	Reject $H_0$ ?
United States	1994	2.43	0.06	1.45	0.03	1868	37355	Yes
United States	1995	2.45	0.06	1.48	0.04	1604	32076	Yes
United States	1996	2.34	0.06	1.51	0.04	1558	31165	Yes
United States	1997	2.29	0.06	1.47	0.04	1546	30913	Yes
United States	1998	2.32	0.06	1.45	0.04	1557	31144	Yes
United States	1999	2.41	0.06	1.44	0.04	1566	31320	Yes
United States	2000	2.19	0.05	1.4	0.03	2348	46961	Yes
United States	2001	2.23	0.05	1.31	0.03	2308	46168	Yes
United States	2002	2.29	0.05	1.35	0.03	2198	43963	Yes
United States	2003	2.35	0.05	1.37	0.03	2170	43410	Yes
United States	2004	2.39	0.05	1.33	0.03	2118	42354	Yes
United States	2005	2.27	0.05	1.42	0.03	2081	41624	Yes
United States	2006	2.27	0.05	1.46	0.03	2013	40264	Yes
United States	2007	2.34	0.05	1.49	0.03	1966	39312	Yes
United States	2008	2.35	0.05	1.32	0.03	2026	40523	Yes
United States	2009	2.26	0.05	1.33	0.03	1922	38446	Yes
United States	2010	2.56	0.06	1.41	0.03	1869	37383	Yes
United States	2011	2.29	0.05	1.4	0.03	1836	36728	Yes
United States	2012	2.33	0.05	1.56	0.04	1843	36865	Yes
United States	2013	2.4	0.07	1.4	0.04	1225	24494	Yes
United States	2014	2.37	0.05	1.41	0.03	2316	46312	Yes
United States	2015	2.32	0.05	1.36	0.03	2213	44260	Yes
United States	2016	2.29	0.05	1.42	0.03	2270	45401	Yes
United States	2017	2.42	0.05	1.45	0.03	2255	45102	Yes
United States	2018	2.26	0.05	1.34	0.03	2265	45307	Yes
Uruguay	2004	2.53	0.32	1.81	0.23	62	1241	No
Uruguay	2007	3.26	0.22	1.53	0.11	212	4236	Yes
Uruguay	2010	3.1	0.22	1.63	0.11	204	4085	Yes
Uruguay	2013	3.69	0.28	1.8	0.14	175	3497	Yes
Uruguay	2016	3.53	0.26	1.69	0.13	182	3632	Yes
Vietnam	2011	3.26	0.4	1.23	0.15	68	1362	Yes
Vietnam	2013	2.61	0.31	1.64	0.2	69	1384	No

Note:  $\hat{\alpha}_{\text{lab}}$  (resp.  $\hat{\alpha}_{\text{cap}}$ ): point estimate of Pareto exponent for labor (resp. capital) income; s.e.: standard error ( $\hat{\alpha}(k)/\sqrt{k}$ );  $k$ : the number of order statistics used for the Hill-estimator;  $N$ : the number of positive capital income observations; the last column denotes if  $H_0$  is rejected.