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Abstract

In this paper education simultaneously affects growth and income inequality. More education does not necessarily decrease inequality when the latter is assessed by the Lorenz dominance criterion. Increases in education first increase and then decrease growth as well as income inequality, when measured by the Gini coefficient. There is no clear functional relationship between growth and measured income inequality. The model identifies regimes of this relationship which depend crucially on the production and schooling technology. Conventional growth regressions with human capital and inequality as regressors may miss the richness of the underlying nonlinearities, but viewed as approximations may still provide important information on the nonlinear relationship between growth and education.

KEYWORDS: Education, Growth, Inequality, Policy

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1 Introduction

This paper focuses on the dual role of education played in explanations of how income inequality and economic growth are associated.

It is often shown that human capital and education are potentially important driving forces in the determination of long-run growth. See, for instance, Lucas (1988), Barro (1991), Mankiw, Romer, and Weil (1992), Benhabib and Spiegel (1994), Fernandez and Rogerson (1995), or Bénabou (1996a).

Secondly, the link between distribution and growth has been analyzed in many contributions.[1] See, for example, Galor and Zeira (1993), Bertola (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), García-Peñalosa (1995) or Perotti (1996). The consensus emerging from these theoretical and empirical studies is that across countries inequality is negatively associated with growth.[2]

That consensus has recently been challenged by Deininger and Squire (1998), Li and Zou (1998), Forbes (2000), Barro (2000) and others who find non-robust or even positive associations, suggesting that income inequality might be good for growth, especially in rich countries.

However, these latter studies suffer from various methodological and data problems. For example, many of these studies do not use consistent *income* concepts and resort to so called "unadjusted inequality meansures" which are mixes of Gini coefficients for gross and net income.[3] The consequences for any

[1]That literature is surveyed by e.g. Bénabou (1996b), Bertola (2000), or Aghion, Caroli, and García-Peñalosa (1999).

[2]Recent results indicate that there does not seem to be a robust relationship between inequality and growth *within* countries over time. For instance, Li, Squire, and Zou (1998) show that there is little variation in within country income dispersions over time. Atkinson (1998), in turn, finds that for the G7 countries the income dispersions have changed significantly over time. However, based on compilations of inequality data from household surveys, it has been found that inequality varies substantially *across* countries.

[3]Sometimes they also add Gini indices for consumption expenditure to their "unadjusted" measure of income inequality. On the importance of income and recipient concepts in the

empirical investigation of the inequality-growth nexus, especially for linear, OLS-like models, has been pointed out by e.g. Rehme (1999), Rehme (2002), or Rehme (2003b). There it is found that the linear relationship is negative - at least for rich countries -, when income is measured consistently.[4]

Clearly, the usual focus on linear relationships in empirical work is the standard approximation argument, because tackling issues of non-linearity usually raises a whole set of complicated questions.[5] Against this background Banerjee and Duflo (2003) have recently presented theory and empirical evidence that the growth-inequality relationship appears to be nonlinear. This seems to be a step forward in untangling a possibly complex relationship. However, they also use "unadjusted" inequality measures (mixes of Gini coefficients) so that the results have to be interpreted with caution.

In this context the present paper makes the following points: First, it is argued that education simultaneously affects growth and (income) inequality.[6] Second, it is shown that the often used Gini coefficient generates certain predictions by construction which may have adverse effects for testing linear relationships between inequality and growth. Third, in the model the relationship between (measured) inequality and growth is highly nonlinear and depends on important structural parameters. This may have important consequences for empirical research.

In the model human capital is taken to be *lumpy* and can be identified with degrees. People are hired as high-skilled workers in the labour market only if

measurement of income inequality see, for example, Atkinson (1983), Lambert (1993), and Cowell (1995).

[4]Notice that in all these contributions the maintained theoretical relationship is also modelled as being nonlinear. They then focus on deriving some consequences for linear empirical models and present empirical evidence for the latter.

[5]In defence of linear models it may be argued that it may be quite interesting to know whereabouts we are on some possibly complicated (looking) curve.

[6]Thus, the paper builds on contributions such as Galor and Zeira (1993), Sylwester (2000) or Eicher and García-Peñalosa (2001).

they have obtained a degree. The source of income inequality lies in the production process, because high and low-skilled people are imperfect substitutes in production

That raises the question what forces determine the labour force mix in production. Tinbergen (1975), chpt. 6, has argued that there is a race between technological development and education so that differences in the human capital composition may be caused by the demand side of an economy (e.g. skill-biased technological change).[7] However, contributions such as Katz and Murphy (1992) or Murphy, Riddel, and Romer (1998) provide evidence that the dominating forces at work are more likely to be supply driven. Therefore, in this paper the supply of education is taken to win Tinbergen's race in the long-run.

In the model the government provides education and finances it by raising a tax on the resources (wealth) of all individuals.[8]

In equilibrium growth is positively related to human capital only up to a certain point, since the government takes resources away from the private sector in order to finance education, which discourages investment and reduces growth. On the other hand it generates more high-skilled people which exert a positive effect on production. For high growth taxes and so the number of high-skilled people must not be too high. Thus, there is an inverted U-shaped relationship between growth and education.[9]

Next, the effects of education on personal income inequality are analyzed. It is

[7]Thus, the paper should be viewed as complementary to recent models along the lines of, for instance, Galor and Tsiddon (1997), Acemoglu (1998), or Caselli (1999). For empirical evidence on skill-biased technological change see e.g. Krusell, Ohanian, Ríos-Rull, and Violante (2000), or Beaudry and Green (2000)

[8]Thus, even those who have not received education contribute to financing it. That is realistic in most public education systems and may be in the low-skilled people's interest as is e.g. shown by Johnson (1984), Creedy and Francois (1990), or Rehme (2003a).

[9]For instance, Castelló and Doménech (2002) analyze how human capital inequality bears on growth. They find that the relationship is negative.

shown that the well-known Lorenz curves cross when there are more high-skilled people. Thus, no clear welfare ranking is possible for the (wage or total factor) income distribution. See Atkinson (1970). The result highlights that it matters what assumptions one makes about welfare weights attached to bottom or top incomes in distributional analyses and what inequality measure one uses when assessing overall income inequality.

Due to the availability of inequality data from Deininger and Squire (1996) many recent studies have employed the Gini coefficient as the indicator of income inequality. Investigating the model's implications for the Gini coefficient reveals the following: Increases in the number of high-skilled people first increase and then decrease measured inequality in wages and personal factor incomes. Thus, there is also an inverted U-shaped relationship between inequality and education. This feature of the model is in line with a result in Fields (1987) who shows that the Gini coefficient often exhibits this shape by construction when incomes are rising.

The non-linear relationships between growth and education as well as inequality and education imply that it matters for empirical analyses, which investigate the link between inequality and growth, where each function attains its maximum. One can identify regimes under which growth would appear to be positively or negatively associated with inequality. These relationships would hold conditional on the number of high-skilled people present. If one considers increases in education as a development process, then the model predicts that in the early stages of development, when relatively few people are educated, growth, but also inequality would rise when education rises. Then there is a stage where inequality rises or falls and growth falls or rises. Thus, no clear prediction is possible in that case. Knowledge of the production and education technologies, and of the level of education where inequality and growth attain their maximum is necessary to determine which regime an economy or a cross-section of economies is in. Finally, when there is abundant education, growth and measured inequality would both decrease as the number of high-skilled people increases.

For empirical studies this implies that linear regression models may miss some of the non-linear relationships between measured inequality and growth. Many studies based on linear empirical models have found that inequality and growth are negatively associated in cross-sections of countries. This finding is in line with a particular regime in the model, but it only holds conditional on a particular level of education and a given production and education technology.

The main insights to be gained from the paper are the following. Increases in education first increase and then decrease growth as well as income inequality, when measured by the Gini coefficient. There is no clear functional relationship between growth and measured income inequality. If one conditions on inequality and human capital in growth regressions, the estimated effect of inequality on growth may be spurious, but may still provide important information on the nonlinear relationship between growth and education.

The paper is organized as follows: Section 2 presents the model. Section 3 analyzes income inequality. Section 4 derives the relationship between growth and inequality and discusses its empirical implications. Section 5 provides concluding remarks.

2 The Model

Consider an economy that is populated by N (large) members of two representative dynasties. The population is stationary and the two dynasties consist of *high-skilled* workers, L_h, and *low-skilled* workers, L_l, where L_h, L_l denote the total numbers of the respective agents in each dynasty. The difference between high and low-skilled labour is "lumpy", that is, *either* an individual has received higher education certified in the form of a degree and is then considered high-skilled *or* an individual has received basic education with no degree and remains in the low-skilled labour pool.[10]

Agents are assumed to have infinite lifetimes and skills depreciate in such a way that the government needs to maintain education spending in order to keep a constant proportion of skilled workers in the labour force.

Each (adult) worker supplies one unit of either high or low-skilled labour inelastically over time. All agents initially own an equal share of the total capital stock, which is held in the form of shares of many identical firms operating in a world of perfect competition. Thus, all agents receive wage and capital income and make investment decisions.[11] Furthermore, aggregate output is produced according to

$$Y_t = A_t \; K_t^{1-\alpha} \; H^{\alpha}, \; H^{\alpha} = \left[(L_h + L_l)^{\alpha} + L_h^{\alpha}\right], \qquad 0 < \alpha < 1, \tag{1}$$

[10]This assumes that agents are endowed by some *basic* ability and receive basic education which is produced and provided costlessly. In the paper education is always meant to be higher education. *Ex ante* everybody is a candidate for receiving (higher) education, that is, ex ante there is excess demand for education and once *in* the education process agents will complete their degrees. The education process is taken to be sufficiently productive in converting no skills into high skills. Even if people have the same innate abilities and the same initial endowments and although the capital market functions perfectly, there is inequality in the present value of lifetime earnings in the model. Thus it concentrates on a technology based explanation of inequality. For a recent model studying the effects of differences in wealth or ability on education, income inequality or growth see, for instance, Chiu (1998).

[11]Postulating an equal initial wealth (capital) distribution is a simplification, which serves to bring out clearly the effects of different policies on the income distribution. Alternatively, suppose a third type owns *all* the initial capital stock and there is no social mobility. Then one may verify that all the results for governments representing high or low-skilled workers hold. Also, as will be shown below, the workers' utility depends on the balanced growth rate so that analyzing high and low-skilled workers embodies a problem that is very similar to the one a pure capitalist class would have.

where K_t denotes the aggregate capital stock including disembodied technological knowledge,[12] H measures effective labour in production, and A_t is a productivity index at time t.

Modelling production in this way relates to work that distinguishes between tasks performed for a *given* educational attainment of the labour force and education mixes for *given* tasks. See e.g. Tinbergen (1975), chpt. 5, and Lindbeck and Snower (1996). More precisely, the production function is a reduced form of the following relationship: By assumption *effective labour* depends on tasks requiring *basic skills* and tasks requiring *high skills.* These tasks are *imperfect substitutes* in production. On the other hand it is assumed that low and high-skilled *people* are *perfect substitutes* in performing *basic* tasks. Thus, high-skilled people are multi-tasking and always perform the tasks of low-skilled people in the model, but low-skilled people can never execute tasks that require a degree. See Appendix A. Notice that each type of labour alone is *not* an essential input in production.

The government runs a balanced budget, uses its tax revenues to finance public education and commits itself to preannounced policies. It maintains a constant ratio of expenditure G_t to its tax base[13] and taxes the agents' wealth holdings at a constant rate τ. The capital stock per capita is $k_t = \frac{K_t}{N}$ so that $G_t = \tau k_t N = \tau K_t$ and $\frac{G_t}{K_t} = \tau$ for all t. Thus, real resources are taken from the private sector and used to finance public education, which generates high-skilled

[12] Thus, technological knowledge is taken to be a sort of capital good which is used to produce final output in combination with other factors of production. For an up-to-date discussion of these kinds of endogenous growth models see, for instance, Aghion and Howitt (1998), chpt. 1. Notice that the paper abstracts from the important phenomenon of skill-biased technological change and should, therefore, be viewed as complementary to recent models along the lines of, for instance, Galor and Tsiddon (1997), Acemoglu (1998), or Caselli (1999).

[13] Capital taxes keep the analysis simple and are supposed to capture a broad class of tax arrangements. Time consistency is imposed in order to focus on the long-run effects of education policies. For a similar approach in a different context see Alesina and Rodrik (1994).

workers.

The reason for concentrating on public education[14] is that secondary as well as tertiary education are primarily financed publicly. For example, in the OECD countries only 23 percent of the funds going into tertiary education are provided privately. For all levels of education it is only 13 percent. See e.g. OECD (2001), "Education at a Glance 2001", Table B3.

In the model all agents are identical ex ante so that innate ability or initial wealth differences are not important. Furthermore, problems arising from the time spent receiving education are ignored by assuming that education is provided as a public good and that all people spend the same time in school, but attend different courses leading to different degrees.[15]

In general, public education is 'produced' using government resources and other factors such as high-skilled labour itself. That is captured by the following *reduced form* of the education technology, which relates the percentage of high-skilled people to the education expenditures in terms of the tax base,

$$x = \tau^{\epsilon} \quad \text{where} \quad 0 < \epsilon \leq 1 \ , \tag{2}$$

x denotes the percentage of high-skilled people in the population, $x_{\tau} = \epsilon\tau^{\epsilon-1} > 0$ and $x_{\tau\tau} = \epsilon(\epsilon-1)\tau^{\epsilon-2} \leq 0$. Thus, if the government channels more resources into the education process, it will generate more high-skilled people, $x_{\tau} > 0$. However, doing this generally becomes more difficult at the margin. This is supposed to reflect that, if $x_{\tau\tau} < 0$, more public resources provided to the education sector

[14]The effects of public vs. private education on growth and inequality have, for instance, been analyzed by Glomm and Ravikumar (1992), Fernandez and Rogerson (1998), or Bräuninger and Vidal (2000).

[15]Opportunity costs of education might easily be introduced into the model by subtracting a fixed amount of happiness from a high-skilled person for having spent time in school. The paper's results would not change in that case.

lead to a decreasing marginal product of those resources due to congestion or other effects.

The parameter ϵ measures the productivity of the education sector for a given policy.[16] If $\epsilon < 1$, the education sector is relatively more productive than when $\epsilon = 1$. Thus, if $\epsilon < 1$, then a marginal increase in taxes increases education output relatively more. Underlying that is the description of an education sector with spillovers from, for instance, high-skilled to new high-skilled people or where the capital equipment such as computers makes the education technology very productive. For a justification see appendix B.

2.1 The Private Sector

There are as many identical firms as individuals and the firms face perfect competition and maximize profits. By assumption they are subject to knowledge spillovers, which take the form $A_t = \left(\frac{K_t}{N}\right)^\eta = k_t^\eta$ with $\eta \geq \alpha$. Thus, the *average* stock of capital, which includes disembodied technological knowledge, is the source of a positive externality.[17] Then simplify by setting $\eta = \alpha$ which allows one to concentrate on steady state behaviour. For a justification see Romer (1986). As the firms cannot influence the externality, it does not enter their decision

[16] The reduced form directly relates the percentage of high-skilled people (x) to the percentage of resources (wealth) going into the education sector (τ). Let $pr = \frac{x}{\tau}$ denote the productivity of the education sector. Then $pr = \tau^{\epsilon-1}$, which is decreasing in ϵ for given policy. For a justification of the reduced form set-up see Appendix B.

[17] Here the assumption is that regardless of the source of new ideas or blueprints production is undertaken so that all agents are affected relatively equally from knowledge spillovers. The results would not change if the externality depended on the *entire* capital stock instead.

directly so that

$$
\begin{aligned}
r &= (1-\alpha) k_t^{\alpha} K_t^{-\alpha} H^{\alpha}, \\
w_h &= \alpha k_t^{\alpha} K_t^{1-\alpha} \left[(L_h + L_l)^{\alpha-1} + L_h^{\alpha-1} \right], \\
w_l &= \alpha k_t^{\alpha} K_t^{1-\alpha} \left(L_h + L_l \right)^{\alpha-1}.
\end{aligned}
\tag{3}
$$

Notice that for a constant x the structure of the production function implies that r is constant.

The individuals have logarithmic utility and own all the assets which are collateralized one-to-one by physical capital. As each type of worker lends capital to firms in a competitive capital market, all individuals face the same interest rate. Each high- or low-skilled individual owns k_i units of the capital stock and takes the paths of r, w_h, w_l, τ as given and solves

$$\max_{c_i} \int_0^{\infty} \ln c_i \; e^{-\rho t} \; dt \tag{4}$$

$$s.t. \qquad \dot{k}_i = w_i + (r-\tau) k_i - c_i \qquad i = l, h \tag{5}$$

$$k_{i0} = k_0 = \text{given}, \;\; k_i(\infty) = \text{free}.$$

Equation (5) is the worker's dynamic budget constraint.[18] The condition $k_{i0} = k_0$ captures that the agents own the initial capital stock equally.

The solution for this problem is standard and involves[19]

$$\gamma = \frac{\dot{c}_i}{c_i} = r - \tau - \rho \tag{6}$$

$$\lim_{t\to\infty} k_i \, e^{-(r-\tau)t} = 0. \tag{7}$$

[18]This allows the workers to hold different shares of capital over time. I thank an anonymous referee for pointing this out to me.

[19]See e.g. Barro and Sala–i–Martin (1995), ch. 4.

The last equation requires that the agents's wealth holdings approach a value of zero asymptotically. It reflects the well-known transversality condition which rules out that individuals incur debts forever.

Equation (6) implies that consumption of all workers grows at the same rate in the optimum. We assume that the agents are sufficiently patient, namely, that $r-\tau > \rho$, which implies that consumption grows at a positive rate and will ensure that the utility functional converges. Thus, the consumption growth rate depends on the after-tax return on capital. The property of equal consumption growth reflects the fact that the workers are price takers in the labour market. This means that their wage income does not feature in their decision how consumption should grow. Thus, the model builds on Bertola (1993) who shows that incomes from the non-accumulated factor of production do in general not bear on growth in endogenous growth models.

2.2 Market Equilibrium

For equilibrium the labour markets for high and low-skilled workers clear and all people are employed. For the rest of the paper let $L_h \equiv xN$ and $L_l \equiv (1-x)N$ where x denotes the percentage of high-skilled people in the population N, and normalize by setting $N = 1$. Then the factor rewards in equilibrium (see eq. (3)) are given by

$$r = (1-\alpha)(1+x^{\alpha}) \ , \quad w_h = \alpha k_t(1+x^{\alpha-1}) \quad \text{and} \quad w_l = \alpha k_t. \tag{8}$$

For constant x the return on capital is constant over time and wages grow with the capital stock. As $w_h = w_l\,(1+x^{\alpha-1})$, high-skilled labour receives a premium over what their low-skilled counterpart gets. That reflects the fact that the high-skilled may always perfectly substitute for low-skilled labour so that both types of agents receive the same wage w_l for routine tasks and that performing high-skilled tasks is remunerated by the additional amount $w_l\, x^{\alpha-1}$. The premium depends on the percentage of high-skilled labour in the population, grows over time at the rate γ and is decreasing in x for a *given* capital stock.

Thus, in a perfectly competitive labour market the high-skilled, more productive workers get a wage *premium* over and above what their low-skilled colleagues receive. The relative wage premium, w_h/w_l, depends negatively on the number of high-skilled people, which captures an important and realistic aspect in the explanation of wage inequality. See, for instance, Freeman (1977), Bound and Johnson (1992), or Autor, Krueger, and Katz (1998).[20]

From the production function one immediately gets $\gamma_y = \gamma_k$ so that per capita output and the capital-labour ratio grow at the same rate. Thus, Y/K will be constant so that total output also grows at the same rate as the aggregate capital stock. In equilibrium $w_{it} = d_i k_t$ where $d_h \equiv \alpha(1+x^{\alpha-1})$ and $d_l \equiv \alpha$ are constant. Thus, wages grow at the same rate as k_t. In a steady state all variables grow at constant rates.

Let $\gamma_{k_i} \equiv \frac{\dot{k}_{it}}{k_{it}}$ and $\gamma_k \equiv \frac{\dot{k}_t}{k_t}$ where $k_t = \frac{K_t}{N} = K$ since $N = 1$ by our normalization. At each point in time $xk_{ht} + (1-x)k_{lt} = k_t = K_t$.

As all agents own the initial capital stock equally, $k_{h0} = k_{l0} = k_0$, this boils down to

$$\begin{aligned} xk_0 e^{\gamma_{k_h} t} + (1-x)k_0 e^{\gamma_{k_l} t} &= k_0 e^{\gamma_k t} \\ xe^{(\gamma_{k_h}-\gamma_k)t} + (1-x)e^{(\gamma_{k_l}-\gamma_k)t} &= 1. \end{aligned}$$

[20] Notice that w_l does not directly depend on x. It only does so indirectly through $k_t(x)$ in equilibrium. See Johnson (1984). Hence, more human capital is taken to have a stronger immediate impact on the wages of the high-skilled than on the wages of the low-skilled. On this see e.g. Büttner and Fitzenberger (1998).

It is not difficult to see that, as soon as at least one γ_{k_i} is not equal to γ_k, the last equation will not be satisfied for any t. It will, however, be satisfied when $\gamma_{k_h} = \gamma_{k_l} = \gamma_k$. Thus, when variables grow at constant rates, equilibrium requires that each high- or low-skilled agent accumulates wealth at the same rate γ_k.

In appendix C it is shown that the agents will accumulate wealth at the same rate as the consumption growth rate. Hence, in the steady state equilibrium we have balanced growth with $\gamma_{k_h} = \gamma_{k_l} = \gamma_k = \gamma_y = \gamma = \gamma_{c_h} = \gamma_{c_l} = r - \tau - \rho$.

That implies equal investment decisions of the individuals and so an unchanging wealth distribution. Thus, all agents continue to own equal shares of the total capital stock over time. This is because the model has no transitional dynamics, that is, the economy is always on a balanced growth path, and consumption, wealth and income all grow at the same common rate for both types of individuals.

The equilibrium also entails that agent i's consumption at each point in time is given by

$$c_{it} = c_{i0}\, e^{\gamma t} \quad = \quad d_i\, k_0\, e^{\gamma t} + \rho\, k_{i0}\, e^{\gamma t} = w_{it} + \rho k_t. \tag{9}$$

Hence, in equilibrium the relatively richer high-skilled agents consume more than the low-skilled individuals at each point in time. That also means that the welfare of the low-skilled is less than that of the high-skilled. For empirical evidence on this see, for example, Easterlin (2001). Second, notice that the marginal propensity to consume out of wage income is unity for both groups. But consumption in terms of wage income, c_{it}/w_{it}, is higher for the relatively poorer, low-skilled agents.[21]

[21]This raises the question whether, for example, the introduction of a consumption loans market could serve to eliminate different consumption levels. It can be shown, however, that in

From (6), (8) and $\tau = x^{\frac{1}{\epsilon}}$ one obtains $\gamma = (1-\alpha)(1+x^{\alpha}) - x^{\frac{1}{\epsilon}} - \rho$ and[22]

$$\frac{d\gamma}{dx} = \alpha(1-\alpha)x^{\alpha-1} - \frac{1}{\epsilon}x^{\frac{1}{\epsilon}-1}$$

and verifies $\lim_{x\to 0}\frac{d\gamma}{dx} = +\infty$, $\lim_{x\to 1}\frac{d\gamma}{dx} < 0$ since $\frac{1}{\epsilon} > \alpha(1-\alpha)$ so that the slope must change sign at least once. Furthermore, $\frac{d\gamma}{dx} = 0$ implies that

$$\hat{x} = [\epsilon\alpha(1-\alpha)]^{\frac{\epsilon}{1-\epsilon\alpha}}, \quad \text{and} \quad \hat{\tau} = [\epsilon\alpha(1-\alpha)]^{\frac{1}{1-\epsilon\alpha}}$$

maximize growth, which is strictly concave in x since for $\epsilon \leq 1$ and any $x \in [0,1]$

$$\frac{d^2\gamma}{(dx)^2} = -\alpha(1-\alpha)^2 x^{\alpha-2} - \frac{1}{\epsilon}\left(\frac{1}{\epsilon}-1\right)x^{\frac{1-2\epsilon}{\epsilon}} < 0.$$

Thus, $\frac{d\gamma}{dx}$ changes sign only once so that growth is first increasing then decreasing in x. This captures that it is possible that an economy has high-skilled workers, but does not necessarily do better than another economy with less high-skilled people. The reason for growth to decline with skill abundance are congestive circumstances and increased education costs. For a description of this phenomenon

this model no perfect loans market would come into being. This is because in a perfect market equilibrium the rates of return on assets would have to be equal. Otherwise, one group would end up with all the assets in the economy and the others would mortgage their entire assets and wage income. This is unrealistic, especially because there is always the return on capital for loans to firms which must be equal for both groups in any equilibrium. Also, in any optimum the agents would suffer reductions in consumption when borrowing and when intertemporal solvency is required. Hence, no such market would survive into the future or would come into being in the first place - at least not when agents are not concerned about the other group. A more formal argument for this reasoning is available from the author on request.

[22]The model, of course, implies $\gamma(x(\tau,\epsilon),\alpha,\rho)$. However, policies differ widely across countries and α, ϵ or ρ are difficult to measure so that x may be a good observable proxy for the underlying differences. As regards endogeneity Caselli, Esquivel, and Lefort (1996) argue that at a more abstract level, "*... we wonder whether the very notion of exogenous variables is at all useful in a growth framework (the only exception is perhaps the morphological structure of a country's geography).*" However, there may be other exceptions one may think of such as differences in willful actions, social fabrics, languages, or historical incidents. In the logic of this model such differences lead to different policies (τ) and so human capital and growth.

see, for instance, Temple (1999), p. 140.

The effect of a change in the productivity of the education sector for a given $x \in (0,1)$ is given by $\frac{d\gamma}{d\epsilon} = \frac{\ln(x)\ x^{\frac{1}{\epsilon}}}{\epsilon^2} < 0$. Hence, a reduction in ϵ, that is, making the education technology more productive, raises growth.

Lemma 1 *The growth rate is first increasing and then decreasing in the number of high-skilled people. A more productive education technology raises growth.*

3 Income Inequality

In the model all income differences are due to differences in wages. This is justified by the fact that empirically the main source of inequality stems from differences in wage incomes. See, for example, Atkinson (1998), p. 19.

As growth is often related to measures of gross income inequality, the paper also concentrates on the distribution of (personal) gross (of tax) income at a particular time. This is motivated by current research that has employed data for inequality measures of current, personal incomes.[23] See, for example, Deininger and Squire (1998), Forbes (2000), or Barro (2000).

In this section we will first analyze the effect of changes in human capital on wage inequality. In a second step we look at the distribution of personal factor income, that is, personal wage plus capital income.[24]

The section will establish that for wage as well as total personal income no clear inequality ranking will be possible according to the Lorenz dominance cri-

[23]When one relates growth to income inequality one should ideally look at an average of personal incomes over time. But data for the calculation of inequality of lifetime incomes do not exist for a large number of countries.

[24]Personal income so defined, of course, ignores other sources of income like rents. As these other sources play a small role compared to the wage and capital income component of personal incomes we take these two sources as the prime indicators of personal income. See, for instance, O'Higgins, Schmaus, and Stephenson (1989).

terion. Furthermore, it will be shown that when wage or personal factor income inequality is measured by the Gini coefficient, there is an inverted U-shaped relationship between inequality and the fraction of high-skilled people in the population.

3.1 Inequality in Wage Incomes

A common concept for the evaluation of income inequality is the Lorenz curve. A Lorenz curve (LC) relates population shares to income shares.

From equation (8) we obtain the relative wage premium as

$$\pi \equiv \frac{w_h}{w_l} = (1 + x^{\alpha-1}) \quad \text{with} \quad \frac{d\pi}{dx} = (\alpha - 1)x^{\alpha-2} < 0.$$

Clearly, π is decreasing in x. The mean wage income at time t is given by

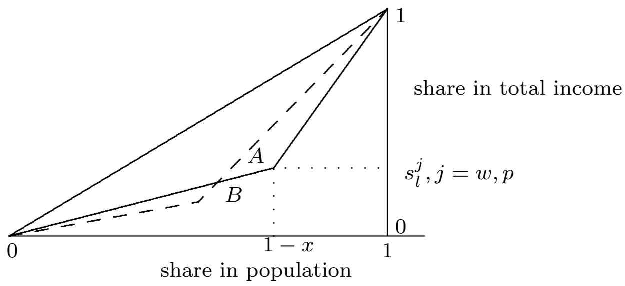
$$\mu^w \equiv (1 - x)w_l + xw_h = (1 - x + x\pi)w_l = (1 + x^\alpha)w_l,$$

where the time subscript has been and will be dropped in what follows for convenience. Total wage income is equal to mean wage income, μ^w, since $N = 1$.

The share of the total wage income going to the low-skilled is $s_l^w \equiv \frac{w_l(1-x)}{\mu^w}$ so that the Lorenz curve looks like figure 1 below.

The LC has a kink at the point A at which $(1 - x)$ percent of the population receive s_l^w percent of total wage income. An increase in x implies that the new kink at B will be to the left and below A. The movement down follows because s_l^w is decreasing in x. The new LC will cut the old one as shown in the figure by the following reasoning: For any Lorenz curve the slope of each line segment is given by the ratio of each group's average income to the overall average income.

Figure 1: Lorenz Curve



See Bourguignon (1990), p. 219.

Thus, the slope of the lower line segment of the Lorenz curve for wage income, denoted λ^w, is given by

$$\lambda^w \equiv \frac{w_l}{\mu^w} = \frac{1}{1+x^\alpha}$$

which is less than 1. For this segment one finds

$$\frac{d\lambda^w}{dx} = -\frac{\alpha x^{\alpha-1}}{1+x^\alpha} < 0 \text{ and } \frac{d^2\lambda^w}{dx^2} = -\frac{\alpha(\alpha-1)x^{\alpha-2}(1+x^\alpha) - \alpha^2 x^{2\alpha-2}}{1+x^\alpha} > 0. \quad (10)$$

Thus, λ^w is decreasing in x.

The slope of the upper line segment is given by

$$\Lambda^w \equiv \frac{w_h}{\mu^w} = \frac{\pi w_l}{\mu^w} = \pi \cdot \lambda^w.$$

This is also decreasing in x, since both π and λ^w are decreasing functions in x. Thus, the new LC must cut the old one from below.

This implies that no clear welfare ranking is possible according to Atkinson (1970), because the new LC does not Lorenz-dominate the old LC. Lorenz dominance would be given if the new LC lay entirely above the old one. Under that condition the new LC would then be unambiguously preferred on welfare grounds.

Thus, the effect of more human capital on (wage) income inequality may be positive or negative depending on whether one looks at the upper or lower part of the LCs. In order to assess this one would have to use an inequality measure that puts particular welfare weights on the bottom and top parts of the income distribution. Thus, extra assumptions would be needed to justify the use of an inequality measure that deals with this problem.

Instead of discussing the possible inequality measures that one may use we will now focus on the Gini coefficient which has been used widely in the recent growth-inequality literature. The Gini coefficient measures the area between the Lorenz Curve and the 45^o degree line as a fraction of the total area under the 45^o degree line. A Gini coefficient of 0 (1) reports perfect equality (inequality).

From the Lorenz curve one may calculate the Gini coefficient for wage incomes as

$$G^w = 1 - 2\left[\frac{(1-x)\,s_l^w}{2} + xs_l^w + \frac{(1-s_l^w)\,x}{2}\right] = 1 - (s_l^w + x) \tag{11}$$

where the expression in square brackets represents the area under the LC. Recall that $s_l^w = \frac{w_l(1-x)}{\mu^w} = \lambda^w(1-x) = \frac{1-x}{1+x^\alpha}$. Then the effect of an increase in x on G^w is given by

$$\frac{dG^w}{dx} = -\frac{ds_l^w}{dx} - 1 = -\left[\frac{d\lambda^w}{dx}(1-x) - \lambda^w\right] - 1.$$

For low x this expression is positive, because $\lim\limits_{x\to 0} = -\dfrac{d\lambda^w}{dx} = -\dfrac{-\alpha x^{\alpha-1}}{1+x^\alpha} = +\infty$, whereas for higher x we have $\lim\limits_{x\to 1}\dfrac{dG^w}{dx} = 0 + \dfrac{1}{1+1} - 1 = -\dfrac{1}{2} < 0$. Furthermore, for all $x \in [0,1]$ we have

$$\frac{d^2G^w}{dx^2} = \frac{d\lambda^w}{dx} - (1-x)\frac{d^2\lambda^w}{dx^2} + \frac{d\lambda^w}{dx} < 0$$

since $\frac{d\lambda^w}{dx} < 0$ and $\frac{d^2\lambda^w}{dx^2} > 0$ by (10). Thus, for all $x \in [0,1]$ we have that $\frac{dG^w}{dx}$ is strictly decreasing in x and so changes sign once. This means that for low x an increase in x raises G^w, whereas for higher values of x a higher x reduces it. Hence, G^w is inverted U-shaped in x.

3.2 Inequality in Personal Factor Incomes

Recently the relationship between the distribution of personal incomes and growth has been analyzed in a number of contributions. Here we concentrate on the two prime components of personal incomes, namely, the sum of wage and capital income per person. Thus, each individual has income $w_{it}+rk_{it}$ at time t for $i = h, l$. In the model all agents accumulate capital at the same rate so that k_{it} and, thus, rk_{it} is the same for all agents. Recall that $r = (1-\alpha)(1+x^\alpha)$. In what follows we will drop the time subscript again for convenience. Then mean personal factor income is

$$\mu^p = \mu_w + rk = (1 + x^\alpha)k$$

where the superscript p denotes variables relating to personal (factor) incomes.

Again we consider the effects of changes in x on the Lorenz curve.[25] To this end we look at the slopes of the lower and upper line segment of the LC. For the

[25]Of course, now the y-axis in figure 1 denotes the share of personal incomes in total personal incomes.

lower segment we define in analogy to the previous subsection

$$\lambda^p \equiv \frac{w_l + rk}{\mu^p} = \frac{w_l}{\mu^p} + (1-\alpha) = \frac{\alpha}{1+x^\alpha} + (1-\alpha)$$

because $\frac{rk}{\mu^p} = \frac{(1-\alpha)(1+x^\alpha)k}{(1+x^\alpha)k} = 1-\alpha$. Then

$$\frac{d\lambda^p}{dx} = -\frac{\alpha^2 x^{\alpha-1}}{1+x^\alpha} < 0 \text{ and } \frac{d^2\lambda^p}{dx^2} = -\frac{\alpha^2(\alpha-1)x^{\alpha-2}(1+x^\alpha) - \alpha^3 x^{2\alpha-2}}{1+x^\alpha} > 0. \quad (12)$$

Thus, λ^p is decreasing in x.

Similarly, for the upper segment of the LC, Λ^p, we get

$$\Lambda^p \equiv \frac{w_h + rk}{\mu^p} = \frac{w_h}{\mu^p} + (1-\alpha) = \frac{\pi w_l}{\mu^p} + (1-\alpha) = \pi\lambda^p + (1-\alpha),$$

because $w_h = \pi w_l$ and $\lambda^p = \frac{w_l}{\mu^p}$. The segment reacts negatively to a change in x, since

$$\frac{d\Lambda^p}{dx} = \frac{d\pi}{dx}\lambda^p + \pi\frac{d\lambda^p}{dx} < 0,$$

as both derivatives on the RHS are negative.

Thus, the new Lorenz curve with a higher x cuts the old Lorenz curve from below as in figure 1. This implies that no Lorenz dominance for the distribution of personal incomes holds.

For the analysis of the Gini coefficient for personal incomes, G^p, we define the share of personal factor incomes of the low-skilled in total personal factor incomes as

$$s_l^p \equiv \frac{(w_l + rk)(1-x)}{\mu^p} = (1-x)\lambda^p + (1-x)(1-\alpha).$$

With this the Gini coefficient for factor incomes is given by

$$G^p = 1 - 2\left[\frac{(1-x)\,s_l^p}{2} + xs_l^p + \frac{(1-s_l^p)\,x}{2}\right] = 1 - (s_l^p + x). \tag{13}$$

Changing x implies

$$\frac{dG^p}{dx} = -\frac{ds_l^p}{dx} - 1 = -\left[-\lambda^p + (1-x)\frac{d\lambda^p}{dx}\right] - 1.$$

Again, we find $\lim_{x\to 0}\frac{dG^p}{dx} = +\infty$, $\lim_{x\to 1}\frac{dG^p}{dx} < 0$ and

$$\frac{d^2G^p}{dx^2} = \frac{d\lambda^p}{dx} - (1-x)\frac{d^2\lambda^p}{dx^2} + \frac{d\lambda^p}{dx} < 0$$

for all $x \in [0,1]$, since $\frac{d\lambda^p}{dx} < 0$ and $\frac{d^2\lambda^p}{dx^2} > 0$ by (12). Hence, by the same arguments as in the previous subsection G^p is also inverted U-shaped in x.

3.3 Implications

The properties of the model have implications for empirical work. As regards the main theoretical predictions it is convenient to summarize them in the following propositions.

Proposition 1 *Wage and personal factor income inequality changes due to an increase in x cannot unambiguously be ranked according to the Lorenz-dominance criterion. Wage and personal income inequality does not necessarily decrease when the number of high-skilled people increases and inequality is assessed by the Lorenz dominance criterion.*

Proposition 2 *When wage or personal factor income inequality is measured by the Gini coefficient, there is an inverted U-shaped relationship between inequality* *and the fraction of high-skilled people in the population.*

A further result follows, when one compares the Gini coefficients for wage and for factor income. In this context, one finds $G^p < G^w$.[26] Thus,

Proposition 3 *For positive x, inequality in wage incomes is higher than inequality in factor incomes when inequality is measured by the Gini coefficient, i.e. $G^w > G^p$.*

These results raise three issues for empirical analyses.

First, The Gini coefficient for personal factor incomes in equation (13) has been derived under the assumption of equal capital ownership and income. In reality, the capital income component of the distribution of personal gross (factor) incomes affects (often reduces) measured inequality. The model's Gini coefficients, thus, capture that the main source of inequality stems from wage inequality.

In this context some snap-shot evidence may be useful. The following table presents Gini indices for the distribution of households' total factor income (proxied in the model by G^p), the distribution of households' factor incomes minus incomes from property (proxied in the model by G^w), which corresponds to the sum of gross wages, salaries and self employment income, and the distribution of households' factor income minus property and self employment income (denoted G_2^w), which (mostly) corresponds to gross wages and salaries, for six of the G7 countries around 2000, calculated from micro data provided by the

[26]We have $G^p < G^w \Leftrightarrow s_l^w < s_l^p$. But

$$s_l^w < s_l^p \quad \Leftrightarrow \frac{(w_l + rk)(1-x)}{(1+x^\alpha)k} < \frac{(1-x)w_l}{(1+x^\alpha)\alpha k} \quad \Leftrightarrow (w_l + rk) > k$$

and so $\alpha k + (1-\alpha)(1+x^\alpha)k > k \Leftrightarrow 1 + (1-\alpha)x^\alpha > 1$ which is true.

Luxembourg Income Study (LIS).[27]

Table 1: Gini coefficients based on LIS

Gini	United States	Canada	France	Germany	Italy	United Kingdom
G^p	0.462	0.419	0.466	0.458	0.451	0.546
G^w	0.469	0.432	0.485	0.468	0.453	0.559
G_2^w	0.483	0.450	0.514	0.504	0.549	0.583

Cross-country comparisons of the coefficients are problematic, because some countries' data refer to net rather than gross wages. However, the Gini coefficients are based on consistent concepts for each country. As one can see moving from measured inequality in wages to inequality in factor income (roughly wages and capital income) reduces inequality when measured by the Gini coefficient for each country considered. This appears to corroborate the feature of the model captured in proposition 3.

Second, households, which are often the unit of analysis in cross-country work, may consist of people with different educational backgrounds. However, when household surveys are based on observations of individual units, the Gini coefficient would not change its informational content if there was a rearrangement of persons into high or low-skilled groups.

Third, two values of x may be associated with the same value of the Gini coefficient and the income distribution with the higher x has a higher mean income. This reflects that the Lorenz curves intersect so that clear rankings

[27] I have calculated the Gini coefficients using standard routines provided by LIS at *www.lisproject.org*. The income recipient is the household. The income unit is household equivalized income, where the equivalence scale is the square root of household members. The income data have been bottom and top-coded for each country, following suggestions by LIS for obtaining a range of inequality measures including the Gini index. The data for the United States, Canada, Germany and Italy are for the year 2000, those for the United Kingdom for the year 1999 and the French data are for the year 1994. The wages and salaries data for France and Italy are for net rather than gross wages. Property income is given by LIS as cash property income which includes cash interest, rent, dividends, annuities, royalties, etc., but excludes e.g. capital gains. A detailed description of how the coefficients were obtained and what LIS variables have been used is provided in appendix D.

of income distributions would in general not be possible. See Atkinson (1970) and, in particular, Fields (1987) or Amiel and Cowell (1999), chpt. 6, who show that the Gini coefficient usually generates a Kuznets curve *by construction*, when incomes are rising. Thus, measurement issues are important and may not have received enough attention in the macroeconomics and growth literature.

For the model that raises an important point. Suppose the economies were identical except for their composition of human capital. Then countries should have a higher mean income and possibly lower or higher inequality if the number of high-skilled people increases and if (wage or total factor) income inequality is measured by the Gini coefficient.

4 Growth and measured income inequality

In this section we focus on the model's predictions on the relationship between growth and income inequality when x changes within a particular country. This relationship is not clear when income inequality is measured by the Gini coefficient. Both growth and measured inequality, are inverted U-shaped functions of x. For what follows we need not distinguish between the Gini coefficient for wages, G^w, or for personal factor incomes, G^p, because all arguments will apply to both of them. Thus, in this section we will denote the Gini coefficient simply by G.

Now, we know that the growth rate attains its maximum when $x = \hat{x} \equiv [\epsilon\alpha(1-\alpha)]^{\frac{\epsilon}{1-\alpha\epsilon}}$. Depending on a country's α and ϵ the growth maximizing $\hat{x}$ may be to the left or to the right of the x where the Gini coefficient attains its maximum. Let that point be denoted by $\tilde{x}$. The following figures visualize the possible cases, assuming that $\hat{x} = \tilde{x}$ is in general very unlikely.

Figure 2: Growth and the Gini I

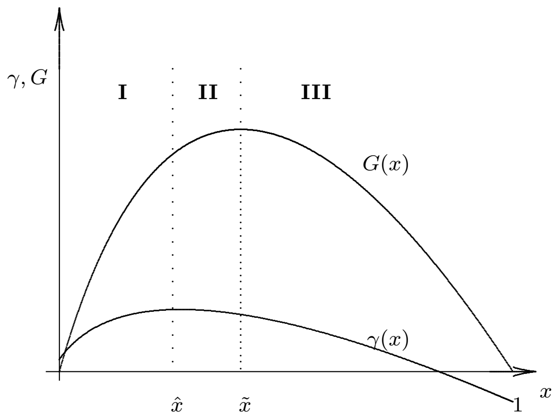
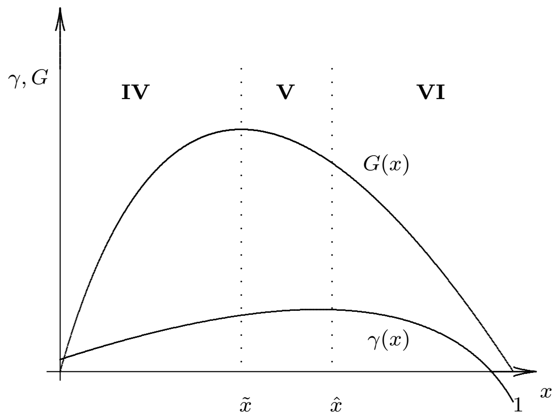


Figure 3: Growth and the Gini II



Figures 2 and 3 show that for sufficiently low x (regimes I and IV) the growth rate and the Gini coefficient are increasing in x. Thus, for a country that is sufficiently poor in human capital we should observe that income inequality, measured by the Gini coefficient, and growth are positively associated. Similarly, for sufficiently high x the growth rate and income inequality are both decreasing in x (regimes III and VI). Thus, in a country that overinvests in human capital more of it reduces growth and measured income inequality.

In regime II, where $\hat{x} < \tilde{x}$, a country has more human capital than is good for growth, but inequality is still rising with an increase in x. Thus, the association between growth and income inequality would be observed to be negative in this case. In regime V, where $\hat{x} < \tilde{x}$, a country has not attained the number of high skilled people necessary to maximize growth. But any increase in x would lower inequality and increase growth. Thus, the observed relation between inequality and growth would also be negative.

To summarize the possible regimes, we have for an increase in x

I	II	III
$\gamma \uparrow, G \uparrow$	$\gamma \downarrow, G \uparrow$	$\gamma \downarrow, G \downarrow$

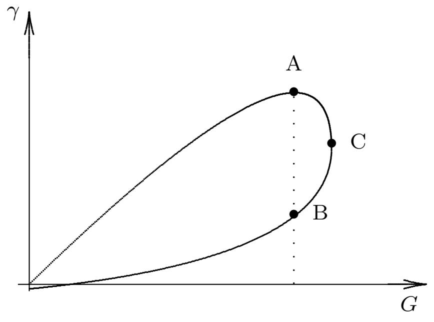
IV	V	VI
$\gamma \uparrow, G \uparrow$	$\gamma \uparrow, G \downarrow$	$\gamma \downarrow, G \downarrow$

If one views increases in x as reflecting the development process it becomes clear that for a given production (α) and education (ϵ) technology the relationship between growth and income inequality would in general be very difficult to predict. For instance, the same value of the Gini coefficient may be associated with two different growth rates. Furthermore, the parameters for the production and schooling technologies have to be taken as constant. Thus, no clear functional relationship would in general hold between growth and income inequality, when measured by the Gini coefficient.

This becomes clearer, if we plot the growth rate against the Gini coefficient, implied by figures 2 and 3. As can be seen, the same Gini coefficient can be associated with relatively higher or lower growth. For example, at point C in figure 4 inequality is maximal. If we then observed lower inequality (a decrease in inequality), this may be associated with higher or lower growth.[28] But whether one moves in the direction of A (higher growth) or B (lower growth) would depend on how x changes and whether we are moving between regimes II and III, or between regimes IV and V.

[28]A similar result is obtained by Banerjee and Duflo (2003) who look at the effect of (net) changes in inequality on the growth rate. They find that changes in inequality (in any direction) are associated with lower (future) growth rates. In this paper, however, no such clear relationship between changes in inequality and the growth rate holds.

Figure 4: The growth-inequality relationship



The lack of a clear functional relationship between measured inequality and economic growth may have important consequences for empirical work. Many studies find that growth and indicators of the inequality of personal incomes, including the Gini coefficient, are negatively associated in cross-sections of countries. This could be explained by the model when assuming that most countries are in regime V, figure 3, and when assuming that α and ϵ are the same for all countries. In that regime more human capital reduces income inequality and increases growth. Thus, some empirical studies may be interpreted as providing evidence that the majority of countries are indeed in that regime - under the maintained assumption of equal technological parameters and a distribution of x supporting that regime.

As the analysis shows one would in general need to know for empirical work where the turning points of the Gini coefficient and the growth rate would be. In particular, this would imply knowledge of parameters governing the production and schooling technology. To obtain that knowledge for a particular country would require a careful analysis of these technologies. For a cross-section of countries it would additionally require an analysis of how x, α and ϵ are distributed empirically.

A key implication of the analysis is that linear, empirical models (e.g. simple growth regressions with human capital and Gini coefficients as regressors) may miss the richness that may underlie the nonlinear relationship between economic growth and income inequality, when both are jointly determined by the level of human capital (or, ultimately, by the political economy considerations that lead to a particular level of human capital).

However, even in light of the present analysis it may be argued that linear empirical models are still of considerable value. If interpreted as approximations to an underlying nonlinear relationship, conventional growth regressions, which include indicators of human capital and the income distribution as regressors, may still provide important information.

5 Concluding Remarks

It is commonly argued that there is a link from education to income inequality and growth. The paper focuses on a supply-driven explanation of how that link may operate.

In the model education directly affects income inequality and growth. It is found that the effects of more education on income inequality cannot be unambiguously ascertained when inequality is assessed by the Lorenz dominance criterion. Increases in education first increase and then decrease growth as well as income inequality, when measured by the Gini coefficient. There is no clear functional relationship between growth and measured income inequality. If one conditions on inequality and human capital in growth regressions, the effect of inequality on growth may be spurious, but may still provide important information on the nonlinear relationship between growth and education.

In the model this relationship depends on the level of human capital as well as structural parameters for the education and production technology. The paper argues that determination of these may be crucial when analyzing the inequality-growth nexus.

Clearly, differences in educational level in time or across countries may be due to many things such as policy, history, labour market conditions, physical and human capital equipment used in schooling, laws, school financing (fees) etc. Furthermore, the differences may also reflect different demand conditions.

Untangling the precise demand-supply relationships between human capital, production and education technologies, and institutions in the explanation of growth or inequality is interesting ongoing research. An all comprehensive analysis of these links has been beyond the scope of this paper. These and other problems are left for future research.

A Technology

By assumption $Y_t = A_t H_t^{\alpha} K_t^{1-\alpha}$, where the index of effective labour H depends on labour requiring *basic skills* (B) and labour requiring *high skills* (S). *Labour* requiring basic skills is performed by high and low-skilled *persons*, $B = B(L_l, L_h)$, whereas high-skilled labour is only performed by high-skilled persons, $S = S(L_h)$. High and low-skilled people are perfect substitutes to each other when performing basic skill (routine) tasks, i.e. $B(L_l, L_h) = L_l + L_h$. Thus, high-skilled people also perform those routine tasks a low-skilled person may do.[29] On the other hand, only high-skilled people can perform high-skilled tasks (labour) and for simplicity let $S(L_h) = L_h$. To capture the relationship between labour inputs assume $H = [B^{\rho} + S^{\rho}]^{\frac{1}{\rho}} = \left[(L_h + L_l)^{\rho} + L_h^{\rho}\right]^{\frac{1}{\rho}}$. For $\rho < 1$ labour requiring basic skills (B) and labour requiring high skills (S) are imperfect (less than perfect) substitutes. For ease of calculations let $\rho = \alpha < 1$ which yields equation (1).

B Discrete Time Justification for $x = \tau^{\epsilon}$

Equation (2) is compatible with many models that also use high-skilled labour as an input generating education. For instance, let h_t denote the *total* stock of human capital in the economy in a discrete time model. Assume that human capital evolves according to $h_{t+1} = f(G_t, K_t, h_t)\, h_t$ where new human capital h_{t+1} is produced by non-increasing returns. Here human capital formation would depend on the level of the stock of knowledge h_t, government resources provided for education G_t and the tax base K_t. The function $f(\cdot)$ governs the evolution of human capital. Assume that it is separable in the form $f(g(G_t, K_t), h_t)$. Let $g = c\left(\frac{G_t}{K_t}\right) = c(\tau)$ and for simplicity

$$h_{t+1} = c(\tau)\ h_t^{\beta}, \quad \text{where} \quad c \geq 0,\ c' > 0,\ c'' \leq 0,\ 0 < \beta < 1.$$

where β measures the productivity of the education sector and $c(\tau)$ captures the quality of education, depending on the government resources channelled into education. For a similar expression see, for example, Glomm and Ravikumar (1992) eqns. (1), (2) and many other contributions.

In the model human capital is carried discretely so $h_t = x_t N$. Normalize population by setting $N = 1$. Then total human capital at date t is given by x_t. In steady state

[29] For instance, Lindbeck and Snower (1996) show that firms may organize production so that people perform one particular task (Tayloristic organization) or various tasks (holistic organization). In the model only high-skilled people are capable of performing several tasks and firms use a mixture of Tayloristic and holistic organization.

$\bar{x} = x_t = x_{t+1}$ and so $\bar{x} = c(\tau)^{\frac{1}{1-\beta}}$. Next suppose that the efficiency of the education sector is described by $c(\tau) = \tau^{\mu}$ where $0 < \mu < 1$. For non-increasing returns to scale it is necessary that $\mu + \beta \leq 1$. Let $\frac{\mu}{1-\beta} \equiv \epsilon$ then the more explicit set-up would be equivalent to (2) in steady state. As $\bar{x}_\epsilon < 0$, any increase in ϵ would mean that less human capital is generated in steady state. From non-increasing returns to scale it follows that $\mu \leq 1 - \beta$ so that $\epsilon \leq 1$. Hence, $\epsilon = 1$ would represent a relatively unproductive human capital formation process.

C Derivation of the Steady State Equilibrium

Let $R \equiv (r - \tau)$ and notice that R is constant in equilibrium. Consumption grows at $\gamma \equiv \frac{\dot{c}_{it}}{c_{it}} = R - \rho$. As R is the same for high- and low-skilled individuals $\gamma = \frac{\dot{c}_{ht}}{c_{ht}} = \frac{\dot{c}_{lt}}{c_{lt}}$. Furthermore, $c_{it} = c_{i0}\ e^{(R-\rho)t}$ where c_{i0} remains to be determined.

Integrating the budget constraint (5) from time 0 to some time T gives

$$\int_0^T c_{it}\ e^{Rt} dt + k_{iT} = \int_0^T w_{it}\ e^{Rt} dt + k_{i0} e^{Rt} dt. \tag{C1}$$

Multiplying both sides by e^{-Rt}, that is, discounting to time 0, letting T go to ∞, and using the transversality condition (7) with constant R yields

$$\int_0^\infty c_{it}\ e^{-Rt} dt = \int_0^\infty w_{it}\ e^{-Rt} dt + k_{i0}. \tag{C2}$$

Thus, the individuals set their present value of consumption equal to the sum of their initial wealth and the present value of their wages.

As $c_{it} = c_{i0}\ e^{(R-\rho)t}$ for a given value of initial consumption c_{i0}, we can substitute in (C2) to get

$$\begin{aligned} \int_0^\infty c_{i0}\, e^{(R_s-\rho)t}\ e^{-Rt} dt &= \int_0^\infty w_{it}\, e^{-Rt} dt + k_{i0} \\ c_{i0} &= \rho \int_0^\infty w_{it}\, e^{-Rt} dt + \rho\, k_{i0} \end{aligned} \tag{C3}$$

Thus, in the optimum the agents' initial consumption depends on initial wealth holdings and the present value value of their labour income.

Next, we divide the individual's budget constraint (5) by k_{it} to get

$$\gamma_{k_i} = \frac{\dot{k}_{it}}{k_{it}} = \frac{w_{it}}{k_{it}} + \frac{R k_{it}}{k_{it}} - \frac{c_{it}}{k_{it}}. \tag{C4}$$

In equilibrium $c_{it} = c_{i0}\, e^{\gamma t}$ where $\gamma = \frac{\dot{c_{it}}}{c_{it}}$. Substituting this, equation (C3), and the expressions for k_{it}, w_{it} and k_t under the condition $\gamma_{k_h} = \gamma_{k_l} = \gamma_k$ and $k_{h0} = k_{l0} = k_0$ in (C4) yields

$$\begin{aligned} \gamma_{k_i} &= \frac{d_i\, k_0 e^{\gamma_k t}}{k_0 e^{\gamma_k t}} + R - \frac{\rho k_0\, e^{\gamma t}}{k_0 e^{\gamma_k t}} - \frac{\rho \int_0^\infty w_{it}\; e^{-Rt} dt\, e^{\gamma t}}{k_0 e^{\gamma_k t}} \\ &= d_i + R - \rho e^{(\gamma-\gamma_k)t} - \rho d_i \int_0^\infty e^{(\gamma_k - R)t} dt\, e^{(\gamma-\gamma_k)t}, \end{aligned} \tag{C5}$$

where $w_{it} = d_i k_t$, and $d_h \equiv \alpha(1 + x^{\alpha-1})$ and $d_l \equiv \alpha$ are constant.

The transversality condition (7) requires that $\gamma_k < R$ for the integral expression in the equation above to converge.

Suppose $\gamma_k < \gamma < R$. Then γ_{k_i} would decrease over time, and would eventually become negative. Production would fall and wages as well as capital income would decline. This is incompatible with a plan to have growing consumption and cannot be optimal.

Suppose $\gamma < \gamma_k < R$. Then γ_{k_i} would increase over time. As $t \to \infty$, it would clearly be larger than R which violates the transversality condition. That cannot be an equilibrium either.

Suppose $\gamma = \gamma_k$. In that case (C4) becomes

$$\gamma_{k_i} = d_i + R - \rho - \rho \frac{d_i}{\rho} = R - \rho = r - \tau - \rho \;(= \gamma)$$

which is indeed true. Hence, $\gamma_{k_h} = \gamma_{k_l} = \gamma_k = \gamma_y = \gamma = \gamma_{c_h} = \gamma_{c_l} = r - \tau - \rho$ in equilibrium.

As $c_{it}, w_{it}, k_{it}, k_t$ and $Y_t = N k_t(1 + x^\alpha)$ begin at the values $c_{i0} = d_i k_0, w_{i0} = d_i k_0, k_{i0}, k_0$ and $Y_0 = N k_0 (1+x^\alpha)$, where $d_i, N = 1, (1+x^\alpha)$ are constant, these variables grow at the same rate from time 0 onwards. Thus, the model has no transitional dynamics. See also e.g. Barro and Sala–i–Martin (1995), p. 143.

D Data Appendix

D.1 The Calculation of the Gini coefficients

The Luxembourg Income Study (LIS) at *www.lisproject.org* provides micro data for the following income variables:

Income Variable	Variable Name
Factor Income	FI
Cash Property Income	V8
Farm self-employment income	V4
Non-farm self-employment income	V4
Gross wages and salaries	V1
SELFI	V4+V5

The Gini coefficients for G^p were calculated from FI. G^p was calculated from $FI - V8$, treating self-employment income as a component of wages, and G_2^p from $FI - V8 - SELFI$, treating self-employment income as capital owning individuals' remuneration (income). The cash property income variable $V8$ comprises cash interest, rent, dividends, annuities, royalties, etc., but it excludes capital gains, lottery winnings, inheritances, insurance settlements, and all other one-off lump sum payments. See *www.lisproject.org/techdoc/variables.htm* for more details.

Thus, G^w and G_2^w capture wage incomes, although they are not always for gross wages, since the data for France and Italy refer to net wages. Thus, comparisons across countries may be problematic, but comparisons within countries are possible.

The income data were transformed into equivalized income, by adjusting the income per household by the square root of household members. The resulting data were then bottom- and top-coded. The bottom-code was set at 0.01% of equivalent mean income. (This allows for very low incomes to be included in the calculations. LIS suggests 1%.) The STATA programme first creates the variable *botlin* equal to 0.01% of average equivalized income, then replaces the equivalised income by that value in case it is lower than it. The top-code was set at 1000 times median unequivalized wage or factor income. (This allows for very high incomes to be included in the calculations. LIS uses the factor 10.) First a variable *toplin* is created which is equal to 1000 times the median unequivalized income (eg. FI), then replaces equivalized income by the equivalized value of *toplin* in case FI etc. is higher than *toplin*. With these adjustments the programme then calculated a variety of inequality indices, including the Gini coefficient.

More detailed information on these procedures can be found at *www.lisproject.org*. The STATA programme code and the resulting detailed output for this paper's Gini coefficients are available from the author on request.

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