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**The Impact of Taxes and Transfer Payments on the  
Distribution of Income: A Parametric Comparison**

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**The Impact of Taxes and Transfer Payments on the Distribution of Income:  
a Parametric Comparison**

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**Abstract**

The Luxembourg Income Study data is used to explore the impact of taxes and transfer payments on the distribution of income across thirteen countries for different years. The five-parameter generalized beta distribution and ten of its special cases are considered as models for the size distribution of income. Maximum likelihood methods are used to estimate the model with corresponding measures of goodness of fit and inequality reported. These results identify the best-fitting two, three, and four- parameter models as well as describe the inter-temporal patterns of inequality corresponding to earnings, total income, and disposable income. A general pattern of increasing inequality is observed for almost all countries considered along with significantly different distributional impacts of taxes and transfer payments across countries.

**KEY WORDS:** distribution of income, Gini coefficient, generalized beta, Dagum, Singh-Madalla

\*The authors appreciate the comments of David Sims

## 1. INTRODUCTION

Issues related to the distribution of income are important considerations in discussions about appropriate fiscal policy. Questions about who benefits from changes in taxes and transfer payments are often at the core of the dialogue. While these issues have long been heatedly debated, there has recently been renewed discussion, prompted in part by the significant increases in income inequality observed since the 1980s in the United States and many other countries<sup>1</sup>. Possible explanatory factors considered in the United States include increased demand for a higher and more skilled work force (Murphy and Welch (1992), Katz and Murphy (1992)), the impact of technological change or reduced affirmative action (Bound and Johnson (1992)), increasing migration (Topel (1994)), and possibly tax reform in the 1980's (Auten and Carroll(1999) and Altig and Carlstrom(1999)). Other possible factors include the declining real value of the minimum wage and de-unionization (Dinardo, Fortin, and Lemieux, 1996), cohort supply side factors (Card and Lemieux, 2001), and long term economic trends (Gottschalk and Smeeding (1998)). While we do not expect identical structural changes in each country to impact the distribution of income, we are interested in accurately modeling the distribution of income and related distributional characteristics as inaccurate estimates can lead to misleading policy evaluations.

Parametric and nonparametric methods have been considered in describing the size distribution of income. This paper focuses on a consideration of parametric models, and provides an example of the importance of exercising care when selecting a distributional form for a given definition of income. Thus, poor fitting functional forms can lead to inaccurate measures of inequality and inappropriate economic policy.

Pareto first proposed a model of income distribution in 1895 which was found to accurately model the upper tail of the distribution, but did a poor job describing the lower tail. Pareto's analysis generated a debate on the effect of economic growth on income inequality. Gini disagreed with Pareto's opinion that economic growth leads to less inequality. Gini proposed a unit-free measure of income inequality known as the Gini coefficient that is still commonly used today (Gini, 1912).

Gibrat's (1931) law of proportionate effect provided a theoretical basis for the two-parameter lognormal distribution to be considered as a model for the size distribution of income. The lognormal was further examined by Aitchinson and Brown (1969). Another two-parameter distribution, the gamma, was proposed by Ammon (1895) and was more recently reintroduced and fit to US income data by Salem and Mount (1974). Bartels and van Metelel (1975) suggested the two-parameter Weibull distribution. While these two-parameter models provide increased flexibility in fitting empirical data, they do not allow for intersecting Lorenz curves sometimes observed with income data.

The introduction of a third parameter allows for intersecting Lorenz curves. Some three-parameter models which have been used to model the size distribution of income include the generalized gamma (Amoroso, 1924-25 and Taille, 1981) and beta (Thurow, 1970) as well as two closely related models which are members of the Burr family of distributions: the Singh-Maddala (1976), known in statistics literature as the Burr 12, and the Dagum (1977), known as the Burr 3.

The generalized beta of the first and second kinds(GB1 and GB2) are four-parameter distributions which have not only been very successful in fitting the data, but also include all of the previously

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<sup>1</sup> Föster (2000) gives an in-depth look at trends over time in OECD countries.

mentioned distributions as special or limiting cases, McDonald (1984). The empirical success of the GB2 was complemented by Parker's (1999) theoretical model of income generation, showing earnings to follow a GB2 distribution. Bordley, McDonald and Mantrala (1996) found that the GB2 distribution generally provided a significantly better fit than its nested distributions when fit to income data from the United States. Bandourian, McDonald, and Turley (2003) applied the generalized beta distributions to income data from 23 countries and various years from the mid 70's to the mid 90's. They found that the Weibull, Dagum, and generalized beta of the second kind were generally the best fitting models with two, three and four parameters when using earnings income data. Furthermore, estimated measures of inequality increased over time (1979-2000) for most countries.

This paper extends the analysis of Bandourian, McDonald, and Turley (2003) to include the impact of transfer payments and taxes on the distribution of income. To the extent that transfer payments act as a safety net for those with lower incomes and taxes redistribute from the rich to the poor, both taxes and transfer payments would be expected to reduce income inequality. Section 2 of this paper discusses the statistical models and methodology used in this study, section 3 describes the data sets being considered, section 4 reports the results, and section 5 summarizes the authors' conclusions.

## 2. STATISTICAL MODELS

### A. The Generalized Beta Distribution Family

The generalized beta (GB) distribution is defined by its probability density function (pdf),

$$GB(y; a, b, c, p, q) = \frac{|a| y^{ap-1} (1 - (1-c)(y/b)^a)^{q-1}}{b^{ap} B(p, q) (1 + c(y/b)^a)^{p+q}} \quad \text{for } 0 < y^a < \frac{b^a}{1-c}$$

and zero otherwise, where  $0 \leq c \leq 1$ ,  $b, p, q > 0$ , and  $B(p, q)$  denotes the beta function.

The GB includes all of the distributions mentioned in Section 1 as special or limiting cases, McDonald and Xu (1995). The four-parameter GB1 and GB2 correspond to the GB with the  $c$  parameter set equal to zero and one, respectively:

$$GB1(y; a, b, p, q) = \frac{|a| y^{ap-1} \left(1 - (y/b)^a\right)^{q-1}}{b^{ap} B(p, q)} = GB(y; a, b, c=0, p, q)$$

$$GB2(y; a, b, p, q) = \frac{|a| y^{ap-1}}{b^{ap} B(p, q) \left(1 + (y/b)^a\right)^{p+q}} = GB(y; a, b, c=1, p, q).$$

Similarly, the three-parameter Dagum<sup>2</sup> and Singh-Maddala distributions correspond to the cases

$$\begin{aligned} \text{DAGUM}(y; a, b, p) &= \text{GB2}(y; a, b, p, q=1), \\ \text{SM}(y; a, b, q) &= \text{GB2}(y; a, b, p=1, q). \end{aligned}$$

The generalized gamma (GG) distribution is a limiting case of the GB2 defined as

$$\begin{aligned} \text{GG}(y; a, \beta, p) &= \lim_{q \rightarrow \infty} \text{GB}(y; a, b = q^{1/a} \beta, c = 1, p, q)^3 \\ &= \frac{y^{ap-1} e^{-(y/\beta)^a}}{\beta^a \Gamma(p)} \end{aligned}$$

where  $\Gamma(\cdot)$  denotes the gamma function. The generalized gamma includes the Weibull and gamma distribution as special cases, corresponding to  $p=1$  and  $a=1$ , respectively. The lognormal pdf can be expressed as a limiting case of the generalized gamma as

$$\text{LN}(y; \mu, \sigma) = \lim_{a \rightarrow 0} \text{GG}\left(y; a, \beta = (\sigma^2 a^2)^{1/a}, p = (a\mu + 1)/\sigma^2 a^2\right).$$

A convenient way to visualize these relationships and some other special cases mentioned in the introduction is the distribution tree in figure 1. Expressions for all of the probability density functions considered in this paper are presented in Appendix 1. Additional detail can be found in McDonald and Xu (1995).

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<sup>2</sup> This is actually a Dagum Type 1 distribution. Dagum's more general form has the cumulative probability function (cdf):  $F(y) = \alpha + (1 - \alpha) \left(1 + (y/b)^{-a}\right)^{-p}$ . Dagum's Types 1, 2, and 3 correspond to  $\alpha = 0$ ,  $0 < \alpha < 1$ , and  $\alpha < 0$ , respectively. Dagum's Type 2 model allows for non-positive values of Y with  $F(0) = \alpha$ . Type 3 is associated with a positive lower bound for Y,  $y_0$ . A generalization of this formulation is given by  $F(y) = \alpha + (1 - \alpha) F^*(y)$  where  $F^*(y)$  could denote any cdf for positive Y, such as a GB1, GB2, or GB. An alternative formulation could be viewed as arising from a "translation of the origin" to  $y_0$  where  $y_0$  can be negative, zero, or positive.  $y_0$  can be estimated from other information such as the fraction of negative and zero observations for Dagum's Type 2 model or can be estimated as a parameter. Bandourian, McDonald, and Turley (2003) include an example of including a translated origin in the estimation of the models considered in this paper.

<sup>3</sup> The GB2 can be expressed as a mixture of a generalized Gamma and an inverse generalized gamma

$$\text{distribution } \text{GB2}(y; a, b, p, q) = \int_0^\infty \text{GG}(y/\lambda; a, \beta, p) \text{IGG}(\lambda; a, b, q) d\lambda \text{ where the IGG}$$

distribution is just a GG with a negative value of the parameter a. This mixture interpretation can be used as a model for a multiplicative measurement error model where  $(\lambda)$  denotes the multiplicative measurement error and true income is distributed as a GG, (Israelsen and McDonald, 2003).

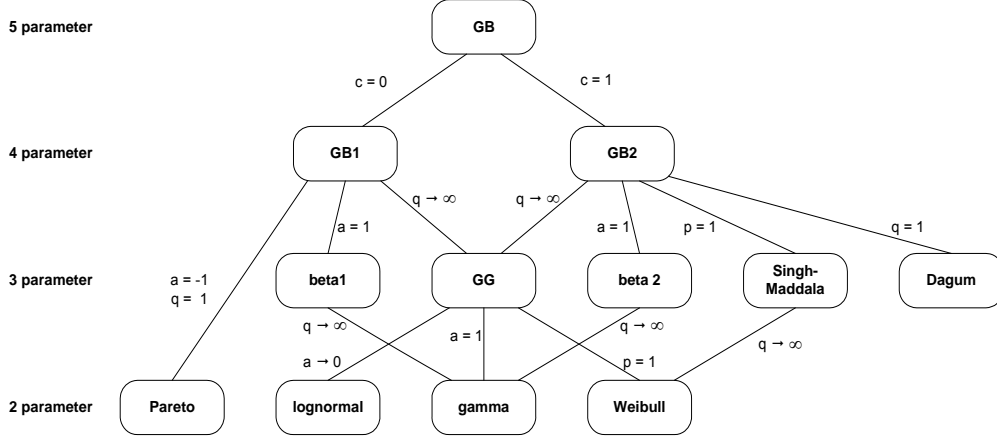


Figure 1: Distribution Tree

#### B. Parameter estimation and measures of goodness of fit.

Maximum likelihood estimation is used for all cases. For individual observations and for data reported in a grouped format, respectively, the distributional parameters are selected to maximize

$$\ell(\theta) = \sum_{i=1}^N \ell n(f_d(y_i; \theta))$$

$$\ell(\theta) = \ell n(N!) + \sum_{i=1}^g \{n_i \ell n[p_i(\theta)] - \ell n(n_i!)\}$$

where  $p_i(\theta) = F_d(Y_i; \theta) - F_d(Y_{i-1}; \theta)$ ,  $f_d(\cdot)$  and  $F_d(\cdot)$ , denote the pdf and cdf for distribution type  $d$ ,  $\theta$  is a vector containing the distributional parameters,  $Y_i$  and  $Y_{i-1}$  are the upper and lower bounds of the  $i^{\text{th}}$  of  $g$  data groups,  $n_i$  is the number of observations in the  $i^{\text{th}}$  group, and  $N$  is the total number of observations.

Numerical optimization methods are used to estimate the unknown parameters. Repeated applications of simplex and amoeba search algorithms, using the Matlab optimization tool kit, were used to obtain optimum values.

The likelihood ratio test statistic, defined by

$$LR = 2[\hat{\ell} - \hat{\ell}^*] \sim_a \chi^2(r),$$

can be used to compare nested distributions where  $\hat{\ell}$  and  $\hat{\ell}^*$  respectively represent the log-likelihood values corresponding to the unconstrained and nested models and  $r$  (the degrees of freedom for the asymptotic chi-square) is the difference in the number of estimated parameters in the two model specifications. Thus, the statistical improvement of the GB2 relative to the Dagum distribution can be tested using a chi-square distribution with one degree of freedom. Nested models on the boundary of the parameter space may compromise the appropriateness of  $\chi^2(r)$ .

For comparing non-nested models, such as the generalized gamma and the beta of the second kind, the LR statistic does not provide the basis for a test. For these cases we will compare the

values of the sum of squared errors (SSE), sum of absolute errors (SAE), and chi-square( $\chi^2$ ) goodness-of-fit measures which are defined by

$$\begin{aligned} \text{SSE} &= \sum_{i=1}^g \left( \frac{n_i}{N} - p_i(\hat{\theta}) \right)^2, \\ \text{SAE} &= \sum_{i=1}^g \left| \frac{n_i}{N} - p_i(\hat{\theta}) \right|, \text{ and} \\ \chi^2 &= N \sum_{i=1}^g \left[ \left( \frac{n_i}{N} - p_i(\hat{\theta}) \right)^2 / p_i(\hat{\theta}) \right], \end{aligned}$$

where  $\hat{\theta}$  denotes the estimated parameter vector. The  $\chi^2$  is asymptotically distributed as a chi-square with degrees of freedom equal to the one less than the difference between the number of groups and the number of estimated parameters, (Cox and Hinkley, 1974, p.316).

### C. Measures of inequality

Numerous measures of inequality have been considered in the literature, including the coefficient of variation (CV), the Pietra index (P), the standard deviation of logarithms (H), Theil's entropy measure (T), and the Gini coefficient (G). These inequality measures are defined by

$$\begin{aligned} \text{CV} &= \sigma / \mu \\ P &= E(|Y - \mu|) / 2\mu \\ H &= \sqrt{E(\ln(Y / \mu))^2} \\ T &= \int_0^\infty \left( \frac{y}{\mu} \right) \ln \left( \frac{y}{\mu} \right) f(y) dy \text{ and} \\ G &= \frac{1}{2\mu} \int_0^\infty \int_0^\infty |x - y| f(x) f(y) dx dy \end{aligned}$$

where  $\mu$  and  $\sigma^2$  denote the mean and variance of Y. Each of these measures can be expressed in terms of the underlying distributional parameters ( $\theta$ ).<sup>4</sup> The Gini coefficient is probably the most widely used measure of income inequality and will be used in this paper. The equations for expressing the Gini coefficient in terms of the distributional parameters are taken from Dagum (1977) and McDonald (1984) and are reported in the Appendix for all but the GB which has not been derived. These equations were used to estimate the Gini coefficient for the data sets and model specifications considered in this paper. Sarabia, Castillo, and Slottje (2002) consider Lorenz orderings for distributions in the GB2 family.

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<sup>4</sup> McDonald (1981) includes expressions for the P and T indices in terms of the distributional parameters for the Pareto, lognormal, gamma, beta, and Singh-Maddala distributions.

### 3. THE LUXEMBOURG INCOME STUDY

Household income data was obtained from the Luxembourg Income Study (LIS) database for thirteen countries, including both developed and developing economies. European countries are somewhat over-represented within the LIS database because of data quality and availability. LIS data is classified into 5-year waves (Historical and Waves I, II, III, IV, and V), which, for a few countries, went back as far as the 1970's. The data used in this study correspond to Waves I to V and are summarized in Table 1. In all cases income was measured in nominal local currency units. Because of government regulations and privacy laws, income data with individual observations are usually not available. In this analysis, the data was obtained for positive income in a grouped format with twenty equal probability intervals, corresponding to the 5<sup>th</sup> through 95<sup>th</sup> percentiles.

An advantage of using the LIS data set is that the data from each country are formatted as uniformly as possible, particularly concerning the definition of income, so that comparisons across countries and time are more credible than studies using data obtained for each country separately (Gottschalk and Smeeding, 1998). This paper focuses on the household unit with different measures of income. The three measures of income are earnings (gross salary, farm and non-farm income before transfer payments and tax deductions), total income (earnings before taxes and after transfer payments and property income), and net disposable income (total income less taxes). A more detailed breakdown of the components of these three income measures is given in Table 2. The use of different definitions of income is associated with different observed distributions of income. Indeed, government redistribution has the potential to significantly alter the distribution of income. Not all component variables exist in all countries for all years and only observations reporting positive values of earnings, total income, and net disposable income are used. While the empirical results reported in this paper could be sensitive to possible alternative variable definitions, this possibility was not explored in this paper. Ervik (1998) uses LIS data for eight countries with several income concepts in his analysis.

TABLE 1: Income Data Obtained from the LIS used in this study

	Wave I Around 1980	Wave II Around 1985	Wave III Around 1990	Wave IV Around 1995	Wave V Around 2000
Australia	1981	1985	1989	1994	
Canada	1981	1987	1991	1994	2000
England	1979	1986	1991	1995	1999
Finland		1987	1991	1995	2000
Germany†	1981	1984	1989	1994	2000
Israel	1979	1986	1992	1997	2001
Italy		1986*	1991*	1995*	2000*
Luxembourg		1985*	1991*	1994*	2000*
Mexico		1984*	1989*	1994*	2000*
Poland		1986*	1992*	1995	1999
Sweden	1981	1987	1992	1995	2000
Taiwan	1981	1986	1991	1995	2000
United States	1979	1986	1991	1994	2000

† Germany waves I II and III for the former “West-Germany” only. Waves IV and V refer to reunified East and West.

\*Earnings data are available for all countries for all survey years. In the LIS data set data for total income and disposable income are available for all survey years and countries except for:

-Italy: 1986 total income not reported; 1991, 1995, 2000 tax information is not reported; the LIS reports identical values for total income and disposable income.

-Luxembourg: 1985 total income not reported; 1991, 1994, 2000 tax information is not reported; the LIS reports identical values for total income and disposable income.

-Mexico: 1984, 1989, 1994 tax information is not reported; the LIS reports identical values for total income and disposable income; 2000 total income not reported in the LIS.

-Poland: Available tax information in the LIS varies by year; 1986, 1992 only total income is available.

TABLE 2: Definitions of Income Measures<sup>5</sup>

Earnings (income before taxes and transfer payments)

- Gross salary income
- Farm self-employment income
- Non-farm self employment income

Total Income (income before taxes and after transfer payments)

- Earnings
- Cash property income: cash interest, dividends, rents, annuities, royalties, etc.
- Cash sickness insurance benefits (sick pay)
- Accident pay, includes short-term accident or injury pay
- Disability pay, long-term disability
- Social retirement pay
- Child or family allowances, alimony and child support
- Unemployment compensation
- Maternity allowances, cash payments for maternity or paternity
- Public and private pensions
- Military/war benefits
- Means-tested cash and near cash benefits
- Other social insurance, other regular private income and other cash income

Net Disposable Income (earnings after taxes and transfer payments)

- Total income

Minus

- Mandatory contributions for self-employed (includes social security, unemployment etc.)
- Income taxes
- Payroll taxes

## 4. RESULTS

Except for the Pareto, each of the distributions depicted in Figure 1 was fit to the 163 data sets (different countries, years, and definitions of income), and goodness-of-fit criteria were calculated

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<sup>5</sup> See <http://www.lisproject.org/techdoc/summary.pdf>

for each distribution, including the estimated Gini coefficient. Table 3 summarizes sample output corresponding to each of the data sets.<sup>6</sup>

For the 1987 Canadian Earnings data, of the two-parameter models (the gamma, lognormal, and Weibull), the Weibull is the best fitting model using any of the reported criteria. Similarly, the Dagum and GB2 are the best-fitting three and four parameter models. The SSE, SAE, and  $\chi^2$  criteria are generally, but not always, in very close agreement as to the best-fitting models.

Table 3: Sample Estimation Results: Canada Earnings Data for 1987

	Model	Log-L	SSE	SAE	$\chi^2$	Gini
Two-Parameter	Gamma	-377.2	.0032	.214	670.2	.386
	Lognormal	-1090.6	.0111	.380	2610.	.459
	Weibull	-245.2	.0018	.142	376.15	.366
Three-Parameter	Dagum	-141.0	.0007	.090	139.6	.368
	B1	-253.6	.0021	.165	371.4	.364
	B2	-377.2	.0032	.214	670.3	.386
	GG	-199.8	.0014	.136	259.0	.362
	SM	-245.2	.0018	.143	386.2	.366
Four-Parameter	GB1	-199.8	.0014	.136	259.0	.362
	GB2	-132.1	.0007	.088	120.2	.375
Five-Parameter	GB	-131.5	.0006	.088	118.7	

The nested relationships of the distributions depicted in Figure 1 guarantee that a distribution will fit the data at least as well, based on the estimation criterion, as any of its cases. However, this does not suggest their superiority as a descriptive model will be statistically significant. Nested models are commonly compared using the likelihood ratio (LR) test:

$$LR = 2[\ln L(\theta_{ML}) - \ln L(\theta_R)],$$

where  $\theta_{ML}$  and  $\theta_R$  represent parameter estimates of the general and of the restricted models, respectively. In cases in which the parameters are not on the boundary of the parameter spaces, the LR is asymptotically distributed as a  $\chi^2$  with degrees of freedom equal to the number of parameter restrictions imposed. Thus the GG can be seen to provide a statistically significant improvement relative to the Weibull and gamma distributions. The differences between the GB2 and the Dagum and SM distributions are statistically significant, while the differences between the GB and GB2 are not using conventional critical values.<sup>7</sup> The LR test cannot be used to compare non-nested models, e.g. the differences between the GB1 and GB2 can't generally be compared using the LR test statistic; however, in this particular case it might be argued that the GB2 is "better" than the GB1 because the GB provides a statistically significant improvement relative to the GB1, but not the GB2.

<sup>6</sup> The results reported in this paper for earned income may differ from those given in Bandourian et al. (2003) because prior to Version 7, SAS had a bug when computing weighted percentiles with the PROC UNIVARIATE command. LIS used SAS 8.2 in creating the data used for this paper.

<sup>7</sup> Both the GB1 and GB2 are on the boundary of the parameter space of the GB, thus raising questions about the exact distribution of the associated LR tests.

The choice of distribution function can impact the estimated level of inequality as measured by the Gini coefficient using the equations found in the Appendix. Generally, the Gini coefficients estimated by the lognormal are the highest for all data sets considered. The disagreement results from an inferior fit, particularly in the tails.

Rather than reporting the results in Table 3 for each data set, Tables 4, 5, and 6 report only the Log-L value for the “best fitting”<sup>8</sup> two, three, four, and five-parameter models earnings, total income, and disposable personal income respectively.

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<sup>8</sup> The “best fitting” is determined by comparing log-L values which tend to be highly correlated with the rankings based on SAE, SSE, and  $\chi^2$ .

Table 4: Best-Fitting Models--Earnings (* Denotes 5% Statistical Significance)										
Country	Year	Two-Parameter		Three-Parameter		Four-Parameter		Five-Parameter		Gini
		model	Log-L	model	Log-L	model	Log-L	model	Log-L	
Australia	1981	Weibull	-350.4	Dagum	-248.2	GB2	-245.5*	GB	-234.1*	0.332
	1985	Weibull	-215.1	Dagum	-152.0	GB2	-146.4*	GB	-144.5	0.336
	1989	Weibull	-241.4	Dagum	-173.7	GB2	-172.9	GB	-172.5	0.345
	1994	Weibull	-190.5	Dagum	-145.8	GB2	-145.1	GB	-144.1	0.362
Canada	1981	Weibull	-306.6	Dagum	-145.8	GB2	-145.7	GB	-143.9*	0.359
	1987	Weibull	-245.1	Dagum	-141.0	GB2	-132.1*	GB	-131.5	0.375
	1991	Weibull	-244.1	Dagum	-130.8	GB2	-130.3	GB	-128.9	0.379
	1994	Weibull	-419.8	Dagum	-143.1	GB2	-140.7*	GB	-127.6*	0.389
	2000	Weibull	-405.8	Dagum	-156.9	GB2	-156.5	GB	-152.1*	0.407
Finland	1987	Weibull	-705.3	Dagum	-328.9	GB2	-315.1*	GB	-313.9	0.398
	1991	Weibull	-709.8	Dagum	-340.2	GB2	-335.3*	GB	-334.9	0.406
	1995	Weibull	-679.9	Dagum	-286.6	GB2	-283.2*	GB	-283.2	0.445
	2000	Weibull	-688.9	Dagum	-319.5	GB2	-315.9*	GB	-315.9	0.422
Germany	1981	gamma	-114.4	Dagum	-93.0	GB2	-92.8	GB	-92.1	0.294
	1984	Weibull	-216.7	Dagum	-171.8	GB2	-167.2*	GB	-162.8*	0.344
	1989	Weibull	-169.0	Dagum	-135.5	GB2	-133.0*	GB	-132.5	0.334
	1994	Weibull	-195.5	Dagum	-144.5	GB2	-141.4*	GB	-134.9*	0.367
	2000	Weibull	-333.9	Dagum	-203.6	GB2	-200.4*	GB	-198.2*	0.395
Israel	1979	Weibull	-76.6	GG	-76.1	GB2	-75.0	GB	-75.0	0.367
	1986	gamma	-78.3	B2	-74.7*	GB2	-74.7	GB	-74.5	0.389
	1992	gamma	-89.3	SM	-75.6	GB2	-75.0	GB	-73.9	0.415
	1997	gamma	-84.3	SM	-80.7	GB2	-80.0	GB	-77.0*	0.426
	2001	gamma	-109.4	SM	-100.6	GB2	-99.1	GB	-99.1	0.461
Italy	1986	gamma	-502.0	SM	-489.2	GB2	-488.5	GB	-482.9*	0.309
	1991	lognormal	-382.8	GG	-382.5	GB2	-382.5	GB	-382.4	0.284
	1995	gamma	-219.7	SM	-207.1	GB2	-204.7*	GB	-195.7*	0.341
	2000	gamma	-364.4	Dagum	-337.6	GB2	-337.1	GB	-336.2	0.335
Luxembourg	1985	lognormal	-85.7	GG	-85.8	GB2	-84.8	GB	-84.7	0.272
	1991	gamma	-84.0	Dagum	-75.1	GB2	-75.0	GB	-75.0	0.294
	1994	gamma	-72.9	B2	-71.4	GB2	-71.0	GB	-71.0	0.292
	2000	Weibull	-76.6	SM	-74.1	GB2	-73.8	GB	-73.7	0.321
Mexico	1984	gamma	-126.9	Dagum	-86.0	GB2	-85.0	GB	-82.6*	0.496
	1989	lognormal	-256.6	Dagum	-116.8	GB2	-116.8	GB	-116.8	0.499
	1994	lognormal	-330.8	Dagum	-136.0	GB2	-121.1*	GB	-116.3*	0.568
	2000	lognormal	-289.1	Dagum	-136.3	GB2	-126.2*	GB	-125.3	0.564
Poland	1986	Weibull	-875.6	Dagum	-436.4	GB2	-421.3*	GB	-419.2*	0.382
	1992	gamma	-162.7	SM	-149.4	GB2	-147.1*	GB	-143.7*	0.341
	1995	lognormal	-276.3	B2	-166.0	GB2	-164.8	GB	-164.8	0.370
	1999	gamma	-198.2	SM	-146.7	GB2	-146.6	GB	-146.6	0.372
Sweden	1981	Weibull	-584.7	Dagum	-271.9	GB2	-271.9	GB	-271.8	0.398
	1987	Weibull	-491.1	Dagum	-212.8	GB2	-212.2	GB	-212.1	0.413
	1992	Weibull	-886.8	Dagum	-361.3	GB2	-360.2	GB	-359.8	0.443
	1995	Weibull	-1059.8	Dagum	-428.2	GB2	-427.6	GB	-427.0	0.451
Taiwan	1981	Weibull	-976.5	Dagum	-526.7	GB2	-518.1*	GB	-518.1	0.439
	1981	lognormal	-248.7	Dagum	-182.9	GB2	-182.9	GB	-169.3*	0.293
	1986	gamma	-346.4	Dagum	-175.7	GB2	-165.7*	GB	-150.2*	0.314
	1991	gamma	-242.9	Dagum	-156.4	GB2	-156.3	GB	-136.3*	0.314
	1995	Weibull	-351.4	Dagum	-231.6	GB2	-227.3*	GB	-198.9*	0.338
United Kingdom	2000	Weibull	-426.3	Dagum	-303.4	GB2	-291.4*	GB	-278.2*	0.354
	1979	Weibull	-182.8	Dagum	-139.1	GB2	-136.0*	GB	-131.1*	0.329
	1986	Weibull	-95.2	Dagum	-77.9	GB2	-77.9	GB	-76.8	0.351
	1991	Weibull	-130.2	Dagum	-97.8	GB2	-96.0	GB	-94.2	0.372
	1995	Weibull	-111.1	Dagum	-85.5	GB2	-84.6	GB	-84.2	0.378
United States	1999	Weibull	-226.6	Dagum	-117.9	GB2	-108.9*	GB	-107.8	0.395
	1979	Weibull	-256.3	Dagum	-150.7	GB2	-141.2*	GB	-136.6*	0.391
	1986	Weibull	-232.0	Dagum	-157.5	GB2	-152.3*	GB	-147.7*	0.405
	1991	Weibull	-178.5	Dagum	-104.1	GB2	-102.9	GB	-101.1	0.419
	1994	Weibull	-389.1	Dagum	-203.9	GB2	-203.8	GB	-200.2*	0.425
	2000	gamma	-436.1	Dagum	-284.7	GB2	-282.6*	GB	-282.6	0.426

Table 5: Best-Fitting Models--Total Income (* Denotes 5% Statistical Significance)										
country	year	two-parameter		three-parameter		four-parameter		five-parameter		Gini
		model	Log-L	model	Log-L	model	Log-L	model	Log-L	
Australia	1981	Weibull	-497.8	B1	-485.6	GB1	-474.9*	GB	-474.9	0.367
	1985	gamma	-389.6	B1	-382.8*	GB1	-369.7*	GB	-369.7	0.377
	1989	gamma	-407.2	B1	-405.8	GB1	-383.5*	GB	-383.4	0.322
	1994	gamma	-278.5	B1	-277.1	GB1	-259.0*	GB	-259.0	0.393
Canada	1981	Weibull	-233.6	B1	-224.6*	GB1	-218.2*	GB	-218.2	0.358
	1987	gamma	-165.9	B1	-160.8*	GB1	-157.1*	GB	-157.1	0.358
	1991	gamma	-208.4	B1	-207.6	GB1	-191.4*	GB	-180.7*	<b>0.360</b>
	1994	gamma	-454.6	GG	-447.7*	GB1	-403.9*	GB	-347.5*	<b>0.368</b>
	2000	gamma	-165.3	GG	-164.9	GB1	-147.0*	GB	-147.0	0.369
Finland	1987	gamma	-215.1	B1	-200.3*	GB1	-188.4*	GB	-188.4	0.344
	1991	gamma	-232.2	B1	-222.1*	GB1	-208.2*	GB	-208.2	0.343
	1995	gamma	-193.4	GG	-184.5*	GB1	-182.5*	GB	-138.5*	0.352
	2000	gamma	-255.0	GG	-229.0*	GB1	-227.2	GB	-173.1*	0.364
Germany	1981	gamma	-86.4	B1	-83.9*	GB1	-82.6	GB	-82.1	0.338
	1984	gamma	-99.1	GG	-95.0*	GB1	-94.9	GB	-94.1	0.366
	1989	gamma	-89.4	GG	-88.8	GB1	-86.3*	GB	-86.2	0.350
	1994	gamma	-135.6	GG	-121.8*	GB1	-121.8	GB	-118.0*	0.323
	2000	gamma	-145.7	GG	-115.3*	GB1	-113.8	GB	-90.5*	0.380
Israel	1979	gamma	-79.3	B1	-76.9*	GB1	-74.7*	GB	-74.7	0.380
	1986	gamma	-88.4	GG	-73.1*	GB1	-73.0	GB	-72.9	0.396
	1992	gamma	-114.3	B2	-84.9*	GB2	-84.8	GB	-84.8	0.403
	1997	gamma	-99.4	GG	-80.3*	GB1	-80.3	GB	-79.7	0.427
	2001	lognormal	-96.6	GG	-84.6*	GB2	-84.5	GB	-84.2	0.444
Italy	1986									
	1991	gamma	-121.2	GG	-115.5*	GB2	-115.5	GB	-115.5	0.328
	1995	gamma	-153.5	GG	-133.0*	GB1	-131.8	GB	-131.8	<b>0.357</b>
	2000	lognormal	-153.6	B2	-118.9*	GB2	-118.4	GB	-118.4	0.360
Luxembourg	1985									
	1991	gamma	-75.7	SM	-74.3	GB2	-73.9	GB	-73.9	0.295
	1994	gamma	-77.6	B2	-74.9*	GB2	-74.8	GB	-74.8	0.291
	2000	lognormal	-74.8	GG	-74.8	GB2	-74.8	GB	-74.7	0.305
Mexico	1984	lognormal	-114.8	Dagum	-85.2	GB2	-85.1	GB	-84.9	0.494
	1989	lognormal	-159.9	SM	-111.1	GB2	-110.7	GB	-110.7	0.488
	1994	lognormal	-184.4	Dagum	-131.3	GB2	-130.6	GB	-125.9*	0.562
	2000									
Poland	1986									
	1992	gamma	-92.0	B2	-87.3*	GB2	-86.7	GB	-86.7	0.310
	1995	gamma	-240.8	B2	-140.7*	GB2	-139.0	GB	-139.0	0.319
	1999	lognormal	-211.5	B2	-113.4	GB2	-106.8	GB	-106.8	0.314
Sweden	1981	gamma	-217.6	GG	-213.7*	GB1	-212.5	GB	-149.3*	0.318
	1987	gamma	-217.6	B1	-211.9*	GB1	-200.0*	GB	-200.0	0.337
	1992	gamma	-242.2	GG	-241.7	GB1	-233.0*	GB	-233.0	0.344
	1995	Weibull	-380.6	SM	-357.7*	GB2	-357.6	GB	-327.4*	0.344
	2000	gamma	-190.8	GG	-180.9*	GB1	-176.0*	GB	-175.9	0.362
Taiwan	1981	lognormal	-136.1	Dagum	-101.3	GB2	-100.0	GB	-92.3*	0.294
	1986	lognormal	-270.3	Dagum	-131.4	GB2	-124.7*	GB	-113.0*	0.315
	1991	gamma	-233.2	Dagum	-134.0	GB2	-134.0	GB	-115.9*	0.315
	1995	gamma	-175.8	Dagum	-111.4	GB2	-111.4	GB	-97.8*	0.333
	2000	gamma	-159.8	Dagum	-102.3	GB2	-102.3	GB	-93.6*	0.353
United Kingdom	1979	Weibull	-303.1	B1	-290.7	GB1	-279.9*	GB	-279.4	0.356
	1986	lognormal	-249.8	GG	-239.9*	GB1	-237.2*	GB	-237.2	<b>0.393</b>
	1991	gamma	-275.1	GG	-243.0*	GB1	-235.2*	GB	-235.2	<b>0.407</b>
	1995	lognormal	-205.7	GG	-202.3*	GB2	-202.4	GB	-202.1	0.423
	1999	lognormal	-356.4	GG	-333.8*	GB2	-334.1	GB	-332.4	0.422
United States	1979	gamma	-156.8	SM	-152.2	GB2	-152.2	GB	-152.2	0.392
	1986	gamma	-131.4	GG	-129.5*	GB1	-129.4	GB	-128.2	0.404
	1991	gamma	-123.7	GG	-108.4*	GB1	-107.9	GB	-105.2*	0.410
	1994	gamma	-403.5	GG	-227.6*	GB1	-227.6	GB	-226.6	0.424
	2000	gamma	-302.3	GG	-163.2*	GB2	-162.9	GB	-162.9	0.427

Table 6: Best-Fitting Models--DPI (* Denotes 5% Statistical Significance)										
country	year	two-parameter		three-parameter		four-parameter		five-parameter		
		model	Log-L	model	Log-L	model	Log-L	model	Log-L	Gini
Australia	1981	Weibull	-423.6	B1	-410.3	GB1	-406.3*	GB	-406.3	0.327
	1985	Weibull	-312.5	B1	-297.9	GB1	-290.4*	GB	-287.8*	0.330
	1989	gamma	-318.5	B1	-306.9*	GB1	-295.7*	GB	-295.7	0.384
	1994	gamma	-225.6	B1	-220.2*	GB1	-210.5*	GB	-210.5	0.350
Canada	1981	Weibull	-192.9	B1	-186.0	GB1	-185.1	GB	-185.1	0.333
	1987	gamma	-153.7	GG	-151.1*	GB2	-151.1	GB	-151.1	0.329
	1991	gamma	-160.1	B1	-158.0*	GB1	-151.6*	GB	-149.8	0.327
	1994	gamma	-314.4	B1	-313.7	GB1	-288.7*	GB	-288.6	<b>0.331</b>
	2000	gamma	-125.0	GG	-122.8*	GB1	-121.6	GB	-121.6	0.340
Finland	1987	gamma	-311.9	B1	-281.9*	GB1	-264.6*	GB	-264.6	0.302
	1991	gamma	-285.8	B1	-261.0*	GB1	-244.6*	GB	-244.6	0.306
	1995	gamma	-270.4	GG	-264.3*	GB1	-263.5	GB	-175.2*	0.308
	2000	gamma	-304.1	GG	-281.1*	GB1	-278.3*	GB	-192.8*	0.323
Germany	1981	gamma	-80.3	SM	-79.6	GB2	-79.4	GB	-79.4	0.304
	1984	gamma	-86.5	GG	-85.0	GB1	-85.0	GB	-84.8	0.319
	1989	gamma	-79.5	GG	-79.2	GB1	-79.1	GB	-79.1	0.307
	1994	gamma	-96.5	GG	-87.7*	GB1	-87.6	GB	-87.3	0.328
	2000	gamma	-108.2	GG	-93.4*	GB1	-93.1	GB	-91.4	0.327
Israel	1979	Weibull	-76.3	B1	-73.0	GB1	-72.5	GB	-72.4	0.329
	1986	gamma	-79.7	GG	-79.0	GB2	-78.9	GB	-78.6	0.335
	1992	gamma	-75.4	B2	-75.4	GB2	-75.4	GB	-75.4	0.341
	1997	gamma	-78.0	GG	-76.0*	GB1	-76.0	GB	-76.0	0.369
	2001	gamma	-92.6	B2	-86.5*	GB2	-86.5	GB	-86.8	0.376
Italy	1986	gamma	-171.8	B2	-150.2*	GB2	-149.6	GB	-145.4*	0.336
	1991	gamma	-121.2	GG	-115.5*	GB2	-115.5	GB	-115.5	0.328
	1995	gamma	-153.5	GG	-133.0*	GB1	-131.8	GB	-131.8	<b>0.357</b>
	2000	lognormal	-153.6	B2	-118.9	GB2	-118.4	GB	-118.4	0.360
Luxembourg	1985	lognormal	-83.9	B2	-78.7	GB2	-78.3	GB	-78.3	0.299
	1991	gamma	-75.7	SM	-74.3	GB2	-73.9	GB	-73.9	0.295
	1994	gamma	-77.6	B2	-74.9	GB2	-74.8	GB	-74.8	0.291
	2000	lognormal	-74.8	GG	-74.8	GB2	-74.8	GB	-74.7	0.305
Mexico	1984	lognormal	-114.8	Dagum	-85.2	GB2	-85.1	GB	-84.9	0.494
	1989	lognormal	-159.9	SM	-111.1	GB2	-110.7	GB	-110.7	0.488
	1994	lognormal	-184.4	Dagum	-131.3	GB2	-130.6	GB	-125.9*	0.562
	2000	lognormal	-159.1	SM	-116.1	GB2	-116.0	GB	-113.4*	0.559
Poland	1986	gamma	-136.6	B1	-130.2	GB1	-123.8*	GB	-123.8	0.323
	1992	gamma	-92.0	B2	-87.3*	GB2	-86.7	GB	-86.7	0.310
	1995	lognormal	-278.3	B2	-155.8	GB2	-150.8*	GB	-150.8	0.324
	1999	lognormal	-234.1	SM	-128.2	GB2	-123.5*	GB	-123.5	0.318
Sweden	1981	lognormal	-311.6	GG	-304.5*	GB1	-304.5	GB	-304.5	<b>0.288</b>
	1987	gamma	-302.4	B1	-301.4	GB1	-289.5*	GB	-242.3*	<b>0.313</b>
	1992	gamma	-319.1	GG	-315.2*	GB1	-303.7*	GB	-303.7	<b>0.321</b>
	1995	Weibull	-591.0	GG	-576.5*	GB2	-575.8	GB	-506.6*	0.314
	2000	gamma	-327.0	GG	-293.5*	GB1	-292.0	GB	-244.5*	0.333
Taiwan	1981	lognormal	-136.0	SM	-99.8	GB2	-98.7	GB	-90.5*	0.291
	1986	lognormal	-271.8	Dagum	-132.4	GB2	-127.8*	GB	-109.3*	0.309
	1991	gamma	-223.7	Dagum	-128.2	GB2	-128.2	GB	-114.1*	0.310
	1995	gamma	-173.8	Dagum	-108.8	GB2	-108.8	GB	-95.5*	0.330
	2000	gamma	-148.4	Dagum	-99.6	GB2	-99.5	GB	-90.0*	0.352
United Kingdom	1979	gamma	-206.9	B1	-203.7*	GB1	-194.5*	GB	-174.2*	0.332
	1986	lognormal	-158.8	GG	-154.8*	GB1	-154.8	GB	-154.8	<b>0.328</b>
	1991	gamma	-196.8	GG	-168.1*	GB1	-167.2	GB	-163.8*	0.374
	1995	lognormal	-135.0	GG	-132.9*	GB2	-132.9	GB	-132.9	0.376
	1999	lognormal	-180.6	GG	-155.2*	GB1	-153.9	GB	-153.8	<b>0.385</b>
United States	1979	Weibull	-130.7	GG	-130.0	GB1	-130.0	GB	-130.0	0.344
	1986	gamma	-115.9	B1	-113.6*	GB1	-113.6	GB	-113.6	0.365
	1991	gamma	-100.8	B1	-99.1	GB1	-93.4*	GB	-93.4	0.365
	1994	gamma	-227.0	GG	-183.1*	GB1	-172.9*	GB	-172.9	<b>0.380</b>
	2000	gamma	-183.2	B2	-170.4*	GB2	-169.6	GB	-169.6	0.381

The *asterisked* Log-likelihood entries in Tables 4, 5, and 6 correspond to cases in which the more general distribution provides a statistically significant improvement (at the five percent level) relative to its nested distributions. Thus we see that in 23, 15, and 16 of the cases (earnings, total income, and disposable personal income, respectively) the GB provides a statistically significant improvement relative to either the GB1 or GB2 based on conventional critical values which may be compromised because the GB1 and GB2 correspond to the parameter  $c$  being on the boundary of the parameter space. Similarly, in the remaining cases either the GB1, GB2 or both are “observationally” very close to the GB. In 6 (earnings), 15 (total income), and 23 (disposable income) cases the GB1 and GB2 are observationally equivalent to the GB.

Table 7 Summarizes the Goodness of fit results from Tables 4, 5, and 6. The best fitting distributions appear to depend on the definition of income being used. Among two-parameter distributions, the Weibull is the best fitting for earnings data but does not fit as well as either the gamma or lognormal for other income definitions, whereas the gamma is best fitting for total income and disposable personal income. The Dagum is clearly the best fitting three-parameter distribution for earnings data, but likewise is among the distributions not fitting as well for other income definitions, for which the GG is most often the best fitting with the B1 the next most frequent best fit. Finally, while the GB2 is clearly the better fitting of the four-parameter distributions for earnings data, it is not clear which between the GB1 and GB2 is a better fit for the other definitions, although the GB1 slightly outperforms the GB2 for total income and disposable personal income<sup>9</sup>. Results for two- three- and four-parameters all highlight the observed result that transfer payments and taxes do alter the shape of the distribution of income. Thus when modeling the distribution of income it may be important to explore alternative distributional forms rather than relying on a single distribution.

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<sup>9</sup> It should be noted that these general results may be sensitive to the time frame of the data; for wave I and II data, the Weibull and B1 distributions are best-fitting for total income and disposable personal income.

<b>Table 7: Summary of Best-Fitting Models*</b>			
	Earnings	Total Income	Disposable Personal Income
Two-Parameter			
gamma	16	39	38
Weibull	36	4	6
lognormal	7	12	15
Three-Parameter			
Dagum	45	7	6
SM	8	4	6
B2	3	6	9
B1	0	13	15
GG	3	25	23
Four-Parameter			
GB2	59	23	27
GB1	0	32	32

\*Total income has 55 cases while earnings and disposable income have 59 due to data limitations described in footnote to Table 1

Table 8 summarizes the estimated Gini coefficients for each country from the best-fitting four-parameter distribution. Annual estimated Gini coefficients and over time average Gini coefficients are reported. Looking at average levels, Mexico is seen to have the least egalitarian distribution of earnings followed by Sweden, and Finland. Transfer payments (comparing total income to earnings) are generally seen to be associated with increased inequality for Australia, Italy, West Germany (“Germany” waves I-III), and the United Kingdom. Transfer payments tend to decrease inequality in Canada, Finland, Unified Germany (waves IV, V), Mexico (slightly), Poland, and Sweden with the other countries (Israel, Luxembourg, Taiwan, and the United States) seeing little impact from transfer payments. Comparing the net impact of transfer payments and taxes on the distribution of income, Canada, Finland, Germany, Israel, Sweden, and the United States are seen to have a more egalitarian distribution of disposable income than for earnings.<sup>10</sup> Not surprisingly, Finland and Sweden are associated with the most significant reductions in income inequality from a combination of both transfer payments and taxes. While the addition of transfer payments to earnings for Germany, Israel, and the United States do not appear to significantly impact inequality, the inclusion of taxes lead to relatively large reductions in income inequality. The net impact of taxes and transfer payments appear to have little effect on income inequality as measured by the Gini coefficient in Australia, Taiwan and the United Kingdom. These results may be sensitive to only working with positive observations. Increasing the number of countries considered or considering different time periods would be of interest, but the cases considered in this paper are sufficient to show the possibility of diverse impact of fiscal policy on efforts to redistribute income.

A number of interesting observations arise from an inspection of the inter-temporal behavior of the entries in Table 8. First, there appears to be generally increasing inequality for earnings over

<sup>10</sup> Italy, Luxembourg, Mexico, and Poland are not included in this comparison because of data problems.

time for most countries over the time periods. Comparing  $Gini_v / Gini_t$ <sup>11</sup> for the various countries ranges from .97 for Poland to 1.09 for the United States and Australia to 1.20 and 1.21 for the United Kingdom and Taiwan to 1.26 and 1.34 for Israel and Germany and compensate for large differences in the *base* Gini (e.g. Mexico and Taiwan), Gottschalk and Smeeding (1998). Thus, the widely publicized increases in inequality in observed in the United States are relatively modest compared to the increases in a number of other countries. The inclusion of transfer payments to yield total income actually increases inequality for four countries (Germany (waves I-III), Israel, Italy, and the United Kingdom), reduces inequality for Canada, Finland, Germany waves IV and V, Poland, and Sweden and yields a mixed or insignificant impact for the other countries. Not surprisingly, the impact of taxes leads to a moderate to significant reduction in inequality of disposable income. Of particular significance is the large reduction in inequality in Finland and Sweden, followed by Israel, Canada, and the United States. While the levels of income inequality for disposable income are generally lower (except for Taiwan and the United Kingdom) than for earnings, the rates of increase are approximately the same except for Germany (1.34 vs. 1.08) and Israel (1.26 vs. 1.14) where they are lower and Sweden (1.10 vs. 1.21) where they are higher. Estimates of the redistributive effect observed among the three income definitions are given as percentage changes of Gini coefficients. Because of the lack of consistent tax data it is difficult to comment on the results for Italy, Luxembourg, Mexico, and Poland<sup>12</sup>.

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<sup>11</sup>  $Gini_t$  and  $G_v$ , respectively, denote the Gini coefficients for Wave I and Wave V.

<sup>12</sup> See footnotes to Table 1 for data limitations.

Table 8: Estimated Gini Coefficients; Mean and by Year; Redistributive Effects

Country	Year	Earnings	Total Income	Disposable Personal Income	%Δ Total Income from Earnings	%Δ DPI from Total Income	%Δ DPI from Earnings
<b>Australia</b>	Mean	0.344	0.365	0.348	6.0%	-4.7%	1.1%
	1981	0.332	0.367	0.327	10.5%	-10.9%	-1.6%
	1985	0.336	0.377	0.330	12.1%	-12.4%	-1.8%
	1989	0.345	0.322	0.384	-6.8%	19.3%	11.1%
	1994	0.362	0.393	0.350	8.6%	-11.0%	-3.4%
<b>Canada</b>	Mean	0.382	<b>0.363</b>	<b>0.332</b>	-5.0%	-8.5%	-13.1%
	1981	0.359	0.358	0.333	-0.2%	-7.1%	-7.3%
	1987	0.375	0.358	0.329	-4.7%	-8.1%	-12.4%
	1991	0.379	<b>0.360</b>	0.327	-5.1%	-9.2%	-13.8%
	1994	0.389	<b>0.368</b>	<b>0.331</b>	-5.5%	-10.1%	-15.0%
	2000	0.407	0.369	0.340	-9.3%	-7.9%	-16.5%
<b>Finland</b>	Mean	0.418	0.351	0.310	-16.0%	-11.7%	-25.9%
	1987	0.398	0.344	0.302	-13.6%	-12.3%	-24.3%
	1991	0.406	0.343	0.306	-15.4%	-10.8%	-24.5%
	1995	0.445	0.352	0.308	-21.0%	-12.5%	-30.9%
	2000	0.422	0.364	0.323	-13.8%	-11.3%	-23.5%
<b>Germany</b>	Mean	0.347	0.351	0.317	1.3%	-9.7%	-8.5%
	1981	0.294	0.338	0.304	14.7%	-10.0%	3.3%
	1984	0.344	0.366	0.319	6.5%	-12.8%	-7.1%
	1989	0.334	0.350	0.307	4.9%	-12.1%	-7.9%
	1994	0.367	0.323	0.328	-12.0%	1.5%	-10.7%
	2000	0.395	0.380	0.327	-3.8%	-13.8%	-17.1%
<b>Israel</b>	Mean	0.412	0.410	0.350	-0.4%	-14.6%	-14.9%
	1979	0.367	0.380	0.329	3.5%	-13.4%	-10.4%
	1986	0.389	0.396	0.335	1.7%	-15.3%	-13.9%
	1992	0.415	0.403	0.341	-2.9%	-15.4%	-17.9%
	1997	0.426	0.427	0.369	0.3%	-13.6%	-13.3%
	2001	0.461	0.444	0.376	-3.7%	-15.3%	-18.4%
<b>Italy</b>	Mean	0.317	<b>0.349</b>	<b>0.345</b>	10.1%	-1.1%	8.8%
	1986	0.309		0.336	-100.0%		8.8%
	1991	0.284	0.328	0.328	15.8%	0.0%	15.8%
	1995	0.341	<b>0.357</b>	<b>0.357</b>	4.8%	0.0%	4.8%
	2000	0.335	0.360	0.360	7.7%	0.0%	7.7%
<b>Luxembourg</b>	Mean	0.295	0.297	0.297	0.8%	0.1%	1.0%
	1985	0.272		0.299	-100.0%		9.7%
	1991	0.294	0.295	0.295	0.6%	0.0%	0.6%
	1994	0.292	0.291	0.291	-0.4%	0.0%	-0.4%
	2000	0.321	0.305	0.305	-4.9%	0.0%	-4.9%
<b>Mexico</b>	Mean	0.532	0.515	0.526	-3.2%	2.1%	-1.1%
	1984	0.496	0.494	0.494	-0.5%	0.0%	-0.5%
	1989	0.499	0.488	0.488	-2.2%	0.0%	-2.2%
	1994	0.568	0.562	0.562	-0.9%	0.0%	-0.9%
	2000	0.564		0.559			-1.0%
<b>Poland</b>	Mean	0.366	0.314	0.319	-14.2%	1.4%	-12.9%
	1986	0.382		0.323			-15.3%
	1992	0.341	0.310	0.310	-9.2%	0.0%	-9.2%
	1995	0.370	0.319	0.324	-13.7%	1.7%	-12.2%
	1999	0.372	0.314	0.318	-15.5%	1.2%	-14.5%
<b>Sweden</b>	Mean	0.429	<b>0.340</b>	<b>0.323</b>	-20.6%	-5.0%	-24.6%
	1981	0.398	0.318	<b>0.288</b>	-20.1%	-9.5%	-27.6%
	1987	0.413	0.337	<b>0.313</b>	-18.4%	-7.1%	-24.2%
	1992	0.443	<b>0.341</b>	<b>0.321</b>	-23.0%	-5.9%	-27.5%
	1995	0.451	0.344	0.314	-23.6%	-8.9%	-30.4%
<b>Taiwan</b>	Mean	0.439	0.362	0.333	-17.6%	-8.0%	-24.2%
	1981	0.323	0.322	0.318	-0.2%	-1.1%	-1.3%
	1986	0.293	0.294	0.291	0.3%	-1.2%	-0.9%
	1991	0.314	0.315	0.309	0.3%	-1.9%	-1.6%
	1995	0.314	0.315	0.310	0.2%	-1.5%	-1.3%
<b>United Kingdom</b>	Mean	0.338	0.333	0.330	-1.5%	-0.9%	-2.4%
	1991	0.338	0.333	0.330	-1.5%	-0.9%	-2.4%
	1995	0.378	0.423	0.376	12.0%	-11.2%	-0.5%
	1999	0.395	0.422	<b>0.385</b>	6.7%	-8.7%	-2.5%
	2000	0.354	0.353	0.352	-0.2%	-0.3%	-0.5%
<b>United States</b>	Mean	0.365	<b>0.395</b>	<b>0.362</b>	8.2%	-8.4%	-0.8%
	1979	0.329	0.356	<b>0.328</b>	8.2%	-7.9%	-0.3%
	1986	0.351	<b>0.393</b>	0.348	11.8%	-11.5%	-1.1%
	1991	0.372	<b>0.407</b>	0.374	9.5%	-8.0%	0.7%
	1995	0.378	0.423	0.376	12.0%	-11.2%	-0.5%
<b>United States</b>	Mean	0.395	0.422	<b>0.385</b>	6.7%	-8.7%	-2.5%
	1979	0.391	0.392	0.344	-0.4%	-10.8%	-11.2%
	1986	0.405	0.404	0.365	0.1%	-12.3%	-12.1%
	1991	0.419	0.410	0.365	-0.3%	-9.6%	-9.8%
	1994	0.425	0.424	<b>0.380</b>	-1.9%	-11.1%	-12.9%
<b>United States</b>	Mean	0.425	0.424	<b>0.380</b>	-0.2%	-10.4%	-10.5%
	2000	0.426	0.427	0.381	0.1%	-10.7%	-10.7%

## 5. CONCLUSIONS

This paper compares the ability of eleven probability distribution functions to fit income data for thirteen countries over time, using three measures of income from the LIS: earnings, total income, and disposable personal income. In each case various goodness of fit measures and Gini coefficients were calculated. Concerning functional form, the best fitting two-parameter distribution is the Weibull for earnings data and the gamma for total income and disposable personal income. The Dagum was the best fitting three-parameter distribution for earnings, with the generalized gamma fitting total income and disposable personal income better. The GB2 fit the earnings data better than the GB1 in every case, but for total income and disposable personal income, the GB1 had a slight edge. An additional finding is that the inter-temporal behavior of the estimated Gini coefficients reveals a generally increasing trend towards inequality for almost all countries considered, regardless of the income measure used. While income inequality of earnings was generally larger than for disposable personal income, there was also a significantly different impact of government redistribution programs (transfer payments and taxes) across countries with Finland and Sweden being associated with the greatest distributional impact and Australia and Taiwan having the least. Finally, it should be mentioned that poor-fitting distributional forms can lead to poor estimates of inequality and questionable policy implications. The best fitting functional forms may change from one income definition to another, e.g. earnings, total income, or disposable income.

## *Appendix I*

### Probability Density Functions and Gini Coefficients for different models of Income

Generalized Beta 1	$GB1(y; a, b, p, q) = \frac{ a  y^{ap-1} \left(1 - (y/b)^a\right)^{q-1}}{b^{ap} B(p, q)}$	$G_{GB1}$
Generalized Beta 2	$GB2(y; a, b, p, q) = \frac{ a  y^{ap-1}}{b^{ap} B(p, q) \left(1 + (y/b)^a\right)^{p+q}}$	$G_{GB2}$
Beta 1	$B1(y; b, p, q) = \frac{y^{p-1} (1 - y/b)^{q-1}}{b^p B(p, q)}$	$\frac{B(p+q, 1/2) B(p+1/2, 1/2)}{B(q, 1/2) \pi}$
Generalized Gamma	$GG(y; a, b, p) = \frac{ a  y^{ap-1}}{b^{ap} \Gamma(p)} \exp\left(-(y/b)^a\right)$	$G_{GG}$
Beta 2	$B2(y; b, p, q) = \frac{y^{p-1}}{b^p B(p, q) \left(1 + (y/b)^a\right)^{p+q}}$	$\frac{2B(2p, 2q-1)}{pB^2(p, q)}$
Singh-Maddala	$SM(y; a, b, q) = \frac{ a  q y^{a-1}}{b^a (1 + y/b)^{q+1}}$	$1 - \frac{\Gamma(q) \Gamma(2q-1/a)}{\Gamma(q-1/a) \Gamma(2q)}$
Dagum	$DAGUM(y; a, b, p) = \frac{ a  p y^{ap-1}}{b^{ap} \left(1 + (y/b)^a\right)^{p+1}}$	$\frac{\Gamma(p) \Gamma(2p+1/a)}{\Gamma(p+1/a) \Gamma(2p)} - 1$
Pareto	$PARETO(y; b, p) = \frac{pb^p}{y^{p+1}},$	$\frac{1}{2p-1}$

Lognormal	$LN(y; \mu, \sigma) = \frac{1}{y\sigma\sqrt{2\pi}} \exp\left(-\left(\frac{\ln y - \mu}{\sigma\sqrt{2}}\right)^2\right)$	$2N[\sigma/\sqrt{2}; 0, 1] - 1$
Gamma	$GAM(y; b, p) = \frac{y^{ap-1}}{b^p \Gamma(p)} \exp(-y/b)$	$\frac{\Gamma(p+1/2)}{\Gamma(p+1)\sqrt{\pi}}$
Weibull	$W(y; a, b) = \frac{ a y^{a-1}}{b^a} \exp(-(y/b)^a)$	$1 - (1/2^{1/a})$

$$\text{where } G_{GB1} = \frac{B(2p+1/a, q)}{B(p, q)B(p+1/a, q)p(ap+1)} {}_4F_3 \left[ \begin{matrix} 2p+1/a, & p, & p+1/a, & 1-q; & 1 \\ 2p+q+1/a, & p+1, & p+1/a+1; & \end{matrix} \right]$$

$$G_{GB2} = \frac{B(2q-1/a, 2p+1/a)}{B(p, q)B(p+1/a, q-1/a)} \left\{ \left(\frac{1}{p}\right) {}_3F_2 \left[ \begin{matrix} 1, & p+q, & 2p+1/a; & 1 \\ p+1, & 2(p+q); & \end{matrix} \right] - \left(\frac{1}{p+1/a}\right) {}_3F_2 \left[ \begin{matrix} 1, & p+q, & 2p+1/a; & 1 \\ p+1/a+1, & 2(p+q); & \end{matrix} \right] \right\}$$

$$G_{GG} = \frac{1}{2^{2p+1/a} B(p, p+1/a)} \left\{ \left(\frac{1}{p}\right) {}_2F_1 \left[ \begin{matrix} 1, & 2p+1/a; & \frac{1}{2} \\ p+1; & \end{matrix} \right] - \left(\frac{1}{p+1/a}\right) {}_2F_1 \left[ \begin{matrix} 1, & 2p+1/a; & \frac{1}{2} \\ p+1/a+1; & \end{matrix} \right] \right\}$$

$$\text{and } {}_pF_q \left[ \begin{matrix} a_1 & \dots & a_p; & x \\ b_1 & \dots & b_q; & \end{matrix} \right] = \sum_{i=0}^{\infty} \frac{(a_1)_i \dots (a_p)_i x^i}{(b_1)_i \dots (b_q)_i i!} \text{ is the generalized hypergeometric series with } (a)_i = (a)(a+1)\dots(a+i-1), \text{ Rainville (1960).}$$

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