Luxembourg Income Study Working Paper Series

Working Paper No. 32

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June 1989

(scanned copy)

Luxembourg Income Study (LIS), asbl

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RELATIVE DEPRIVATION AND ECONOMIC WELFARE: A STATISTICAL INVESTIGATION WITH GINI-BASED WELFARE INDICES

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June, 1989

Preliminary version: Not for quotation or publication. Comments appreciated.

RELATIVE DEPRIVATION AND ECONOMIC WELFARE: A STATISTICAL INVESTIGATION WITH GINI-BASED WELFARE INDICES

I. Introduction

The Gini coefficient, while perhaps the most widely used index of income inequality, has been subject to a great deal of criticism. As Atkinson (1970) and others have pointed out, the Gini puts greater weight on equalizing transfers near the middle of the income distribution. Newbury (1970), and Dasgupta, Sen and Starrett (1973) show the Gini coefficient is not consistent with any symmetric and strictly quasiconcave welfare function. Lambert (1985), extends these results and shows that the Gini coefficient is not consistent with any individualistic social welfare function which is increasing, symmetric, and differentiable.

This does not imply that the Gini is not consistent with any social welfare function. Sen (1974) examines the axiomatic basis of the Gini index. The key axiom is that welfare weights should be a decreasing function of the rank in the income distribution. Sheshinski (1972) and Sen (1976) both derive a welfare index based on the Gini coefficient.[1] The welfare function is non-additive, and (nonstrictly) quasiconcave. These analyses examine the ethical basis of the Gini from the point of view of individualistic social welfare functions, or, in Sen's (1979) terminology, "welfarism."

An alternative ethical basis for the Gini is the concept of relative deprivation (Runciman, 1966). Relative deprivation, and its converse, relative satisfaction, are the result of the individual's position relative to some reference group. Yitzhaki (1979, 1982) shows that the average level of satisfaction can be measured by the same welfare index derived by Sheshinski and Sen. Kakwani (1985, 1986) and Lambert (1985), also appealing to the concept of relative deprivation, propose a generalization of the Sheshinski-Sen-Yitzhaki welfare index. The Kakwani-Lambert index allows the weight given to income inequality, measured by the Gini, to vary.

1. The choice of an inequality index implicitly specifies a social welfare function, conversely, the choice of a welfare function implies a specific income inequality index. See Blackorby and Donaldson (1978, 1984), and Ebert (1987).

The purpose of this paper is twofold. First, we apply these Gini-based welfare indices empirically to compare relative deprivation and economic welfare across countries. These indices have been applied empirically to Indian (Sen, 1976), Australian (Kakwani, 1985, 1986) and US (Berrebi and Silber, 1987) data.[2] There has been no systematic cross-country comparison of these Gini-based indices. Our empirical analysis is carried out using data on nine countries from the Luxembourg Income Study (LIS). The countries are Australia, Canada, the Netherlands, Norway, Sweden, Switzerland, the UK, the US, and West Germany. Bishop, Formby and Smith (1989) compare income inequality across these countries, but do not consider welfare comparisons.

Second, we provide statistical inference procedures for the welfare indices based on the Gini. Many empirical studies of income inequality and welfare, including all of those cited above, are based on samples of the population. This implies that inequality and welfare indices, as well as mean incomes and income shares, calculated from the sample data are sample statistics. The calculated sample values of such statistics (i.e., point estimates) are frequently compared, and conclusions regarding comparative degrees of inequality and welfare drawn. However, these sample statistics are subject to sampling variability. The comparisons of point estimates are hypothesis tests for which the probability of Type I error is one.

We argue that the sampling variability of these inequality and welfare indices should be taken into account in making comparison of income inequality and welfare. In order to apply sound statistical inference procedures the sampling distribution of the statistic must be known. In this paper we derive large sample distributions for the Sheshinski-Sen-Yitzhaki welfare index and the Kakwani-Lambert index. The analysis draws on the asymptotic distribution theory of Lorenz curves and its recent generalizations.[3] The data employed in our empirical analysis are from

2. Sen (1976) compares Indian states using data for 1960-1962, Kakwani (1985, 1986) uses Australian data for 1975-1976, and Berrebi and Silber (1987) carry out an intertemporal comparison using US data for 1959, 1969, and 1979.

3. See Gail and Gastwirth (1977a), Beach and Davidson (1983), Beach and Richmond (1985), Gastwirth and Gail (1985), Beach and Kaliski (1986), Bishop, Chakraborti and Thistle (1988a, 1988b, 1989), and Bishop, Formby and Thistle (1989). On the closely related problem of the sampling distributions of inequality indices, see Gastwirth (1974), Gail and Gastwirth (1977b), and Thistle (1989).

weighted samples and we extend earlier distributional results to the case of weighted sampling. This leads to simple and easily applied results on the asymptotic distribution of the Sheshinski-Sen-Yitzhaki and Kakwani-Lambert indices.

Section II briefly discusses the Sheshinski-Sen-Yitzhaki and Kakwani-Lambert indices. Section III derives the sampling distributions and discusses inference procedures. In Section IV the empirical results are reported, and brief concluding remarks are offered in Section V.

II. Gini-Based Welfare Indices and Relative Deprivation

The Sheshinski-Sen-Yitzhaki (SSY) welfare index is

$$S = \mu(1 - \gamma) \tag{1}$$

where μ is the mean, and γ is the Gini coefficient. There are two complementary interpretations of this index based on individualistic welfare functions. First, the index in (1) can be interpreted as the equally-distributed-equivalent income measure for the Gini coefficient. Second, Sen (1976) considers the welfare basis of indices of real national income. In this case the social preference relation is defined on the distribution of commodities, and only indirectly on the distribution of income. Sen shows that the index (1), calculated from the income distribution, is a subrelation of the social preference relation.

Yitzhaki (1979, 1982) shows that the index in (1) can be based on relative deprivation.[4] Define $I_A(y)$ as the indicator function for the set A, and let

$$d(y, x) = (x - y)I_{[0, x]}(y) \tag{2}$$

be the deprivation felt by an individual with income y, compared to an individual with income x.[5] Taking the population as the reference group, the relative deprivation at income y is

$$D(y) = \int_0^\infty d(y, x)\, dF(x) = \int_y^\infty (1 - F(x))\, dx.$$

4. See also Hey and Lambert (1980).
5. More generally, relative deprivation is an increasing function of the proportion of the population with incomes above y. Then, as Berrebi and Silber (1985) show, many inequality indices can be interpreted as indices of relative deprivation.

The converse of deprivation is satisfaction, $S(y) = \int_0^y (1 - F(x))\,dx$. Then the average level of satisfaction in society is $S = \int_0^\infty S(y)\,dF(y) = \mu(1 - \gamma)$.[6]

Kakwani (1985, 1986) and Lambert (1985), also appealing to relative deprivation, propose generalizations of the SSY index. Kakwani assumes the individual's welfare depends on own income, but is reduced by some fraction of the deprivation felt relative to the income level x. That is, the individual's welfare is $w(y, x) = y - \theta d(y, x)$, where $\theta \in [0, 1]$ and $d(y, x)$ is defined in (2). Then expected welfare, conditional on having income y, is $w(y) = y - \theta D(y)$. Lambert assumes directly that the individual's welfare depends positively on own income and negatively on relative deprivation.

Under either interpretation, social welfare is the average level of individuals' welfare. This yields the Kakwani-Lambert (KL) welfare index

$$S(\theta) = \int_0^\infty (y - \theta D(y))\,dF(y) = \mu(1 - \theta\gamma).$$

The KL index allows the weight given to the Gini to vary. Since $\theta \leq 1$, the KL index is less inequality averse than the SSY index. Of course, if one distribution has a larger mean and a smaller Gini than another, then it has a larger value of the KL index for all θ. The KL index can also be written as a convex combination of the SSY index and mean income,

$$S(\theta) = (1 - \theta)\mu + \theta S. \tag{3}$$

If $\theta = 1$, then welfare is measured by the SSY index, while if $\theta = 0$, no weight is given to inequality and welfare is simply measured by mean income. The SSY and KL indices can be interpreted as measures of the standard of living, either in terms of welfare percapita or the average level of relative satisfaction.

Both the SSY and KL indices have geometric interpretations which follow from the geometric interpretation of the Gini coefficient. Let Y(p) be the inverse distribution function. Then the Lorenz curve is $L(p) = (1/\mu)\int_0^p Y(\pi)d\pi$. As is well known, the Gini coefficient is twice the area between the Lorenz curve and the 45-degree "equality line," $\gamma = 2\int_0^1 (p - L(p))dp$. Then 1

6. If one income distribution has lower relative deprivation (equivalently, higher relative satisfation) at all income levels than a second income distribution, then the first distribution second degree stochastically dominates the second.

- γ is twice the area below the Lorenz curve. Rescaling the Lorenz curve by mean income yields the generalized Lorenz curve (Shorrocks, 1983; Kakwani, 1984). The generalized Lorenz curve is $G(p) = \int_0^p Y(\pi)d\pi$. The equality line is μp, and twice the area between the equality line and the generalized Lorenz curve equals $\mu\gamma$ or one-half of Gini's mean difference. The SSY index is twice the area below the generalized Lorenz curve,

$$S = 2\int_0^1 G(p)dp = 2\int_0^1\int_0^p Y(\pi)d\pi dp.$$

This is illustrated in Figures 1 and 2, which show, respectively, the Lorenz and generalized Lorenz curves for an income distribution. In Figure 1, γ equals 2A, and 1 - γ equals 2B. The generalized Lorenz curve is shown in Figure 2. The equality line in Fig. 2 is the line μp. The area between the equality line and the generalized Lorenz curve is $A' = \mu A$, and the area below the generalized Lorenz curve is $B' = \mu B$. Since the SSY index is twice the area below the generalized Lorenz curve, then, in terms of Fig. 2, we have $\gamma = A'/(A' + B')$ and $S = 2B'$.

III. Statistical Inference

The geometric interpretation of the SSY and KL indices leads to a straightforward asymptotic distribution theory. Let F be the distribution function for income, Y, a positive random variable. Let $\{p_1, ..., p_K\}$, be a set of K fractions, where $0 = p_0 < p_1 <, ..., < p_{K-1} < p_K = 1$. Let ξ_i be the income quantile at p_i, so $F(\xi_i) = p_i$. Let $\mu_i = E(Y \mid \xi_{i-1} \leq Y \leq \xi_i)$ and $\lambda_i^2 = E((Y - \mu_i)^2 \mid \xi_{i-1} \leq Y \leq \xi_i)$ be the i^{th} conditional mean and variance, that is, the mean and variance of incomes between ξ_{i-1} and ξ_i. The average height of the inverse distribution function on the interval $[p_{i-1}, p_i]$ is μ_i, and we approximate the inverse distribution on the intervals $[p_{i-1}, p_i]$ by the conditional means.

Using the geometric interpretation, the SSY index can be calculated directly from the conditional means. Letting $\Delta p_i = (p_i - p_{i-1})$ and $q_i = 1 - p_i$, the SSY index is

$$S = \sum_1^K a_i\mu_i, \qquad (4)$$

where $a_i = 2\Delta p_i(q_i + q_{i-1})$ for $i = 1, ..., K$.[7] The KL index can be calculated, using (3), as $S(\theta) = (1-\theta)\mu - \theta S$. Using the fact that $\mu = \sum_1^K \Delta p_i \mu_i$, the KL index can also be calculated directly from the conditional means as

$$S(\theta) = \sum_1^K b_i(\theta)\mu_i, \tag{5}$$

where $b_i(\theta) = (1-\theta)\Delta p_i + \theta a_i$, $i = 1, ..., K$.[8]

Now suppose that we have a random sample of n observations from F. Since most surveys use weighted sampling schemes, we assume the observations are independently, but not necessarily identically distributed. Let w_j be the weight for the observation Y_j; the weights are assumed to be fixed in repeated samples.

Let $Y_{(j)}$ be the j^{th} sample order statistic, with weight $w_{(j)}$. Let $r_i = [p_i \sum_1^n w_j]$, where [x] denotes the integer part of x; let $t_i = r_{i-1} + 1$. The sample conditional mean is $\hat{\mu}_i = (1/n_i^*)\sum w_{(j)}Y_{(j)}$, where $n_i^* = \sum w_{(j)}$, and the sums are from t_i to r_i. Letting $\mathbf{m} = (\mu_1, ..., \mu_K)'$ denote the vector of conditional means, the estimator of $\mathbf{m}$ is $\hat{\mathbf{m}} = (\hat{\mu}_1, ..., \hat{\mu}_K)'$. We then have the following important result on the sampling distribution of $\hat{\mathbf{m}}$.

Proposition 1. If F is continuously differentiable and strictly monotonic with finite variance, then $\sqrt{n}(\hat{\mathbf{m}} - \mathbf{m})$ has a limiting K-variate normal distribution $N(0, \Omega)$. Letting $\beta_i = p_i(\xi_i - \mu_i) + p_{i-1}(\mu_i - \xi_{i-1})$,

$$\omega_{ii} = [\Delta p_i(\lambda_i^2 + \mu_i^2) - (p_i\xi_i^2 - p_{i-1}\xi_{i-1}^2) + \beta_i(\xi_i - \xi_{i-1} - \beta_i)]/(\Delta p_i)^2,$$

and, for $i < j$,

$$\omega_{ij} = \beta_i(\xi_j - \xi_{j-1} + \beta_j)/\Delta p_i \Delta p_j,$$

and $\omega_{ji} = \omega_{ij}$.

Proof. Let N_i be the set of observation numbers for the observations in the i^{th} sample group; N_i contains n_i elements. Following Beach and Kaliski (1986), let $X_j = \mu + w_j(n_i/n_i^*)(Y_j - \mu)$, for all $j \in N_i$, and for $i = i, ..., K$. Then $\hat{\mu} = (1/n_i)\sum X_j = (1/n_i^*)\sum w_jY_j$, where the sums are over $j \in$

7. For equally spaced fractions, $a_i = 4(K - i + 1/2)/K2$.
8. The formulae in (4) and (5) are based on the trapezoidal integration formula.

N_i. Since the X_j are independently and identically distributed, the result follows from Theorem 3.8 of Bishop, Formby, and Thistle (1989). *Q.E.D.*

The expressions for the variances and covariances ω_{ij} depend only on the quantiles and conditional moments. The i^{th} weighted sample quantile, $\hat{\xi}_i$, is the r_i^{th} order statistic. The sample conditional variance is $\hat{\lambda}_i^2 = (1/n_i^*)\sum w_{(j)}(Y_{(j)} - \hat{\mu})^2$. The estimators $\hat{\xi}_i$, $\hat{\mu}_i$ and $\hat{\lambda}_i^2$ are all (strongly) consistent for the corresponding population quantities. Thus, **m** and **Ω** can be consistently estimated without assuming a specific parametric form for the distribution function.

Proposition 1 leads to the distributions of the income shares, Lorenz ordinates and the Gini coefficient. The income share of the i^{th} group is $\psi_i = \Delta p_i \mu_i/\mu$; let $\boldsymbol{\psi} = (\psi_1, \ldots, \psi_K)'$ be the vector of income shares. The i^{th} Lorenz ordinate is the cumulative income share of the poorest $100p_i$ percent of the population, $L_i = \sum_{j=1}^{i} \psi_j$. Let $\mathbf{L} = (L_1, \ldots, L_K)'$ be the vector of Lorenz ordinates. Let **J** be the KxK lower triangular matrix with ones on and below the diagonal and zeros above, so that $\mathbf{L} = \mathbf{J}\boldsymbol{\psi}$. The Gini can be calculated from the Lorenz ordinates as

$$\gamma = \sum_1^K (p_i - L_i + p_{i-1} - L_{i-1})/\Delta p_i$$

where $L_K \equiv 1$. Letting $c = \sum_1^{K-1}(p_i + p_{i-1})/\Delta p_i + p_{K-1}/\Delta p_K$, and $\mathbf{h} = ((\Delta p_1^{-1} + \Delta p_2^{-1}), \ldots, (\Delta p_{K-1}^{-1} + \Delta p_K^{-1}), p_{K-1}/\Delta p_K)'$ yields $\gamma = c + \mathbf{h'L}$.

Proposition 2. (a) Let $\mathbf{D} = [d_{ij}]$, where $d_{ij} = (\delta_{ij}\psi_j - \psi_i\psi_j)/\mu_j$, and δ_{ij} is the Kronecker delta. Then $\sqrt{n}(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi})$ has a limiting normal distribution $N(0, \boldsymbol{\Sigma}_1)$, where $\boldsymbol{\Sigma}_1 = \mathbf{D\Omega D'}$.

(b) Let $\mathbf{E} = \mathbf{JD}$, where $e_{ij} = (\psi_j/\mu_j)(\sum_h^i \delta_{hj} - \psi_h)$. Then $\sqrt{n}(\hat{\mathbf{L}} - \mathbf{L})$ has a limiting normal distribution $N(0, \boldsymbol{\Sigma}_2)$, where $\boldsymbol{\Sigma}_2 = \mathbf{E\Omega E'}$.

(c) $\sqrt{n}(\hat{\gamma} - \gamma)$ has a limiting normal distribution $N(0, \Gamma)$, where $\Gamma = \mathbf{h'\Sigma_2 h}$.

This follows from Proposition 1 and Rao's Theorem.[9] Since the income shares sum to one (hence the K^{th} Lorenz ordinate is identically one), the matrices $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ both have rank K-1.

The asymptotic distribution of $\hat{S}$ follows directly from the asymptotic distribution of the sample conditional means.

Proposition 3. Let $\mathbf{a} = (a_1, ..., a_K)'$. Then $\sqrt{n}(\hat{S} - S)$ has a limiting normal distribution $N(0, \nu)$, where $\nu = \mathbf{a}'\boldsymbol{\Omega}\mathbf{a}$

This follows from the fact that $\hat{S}$ is a linear function of the sample conditional means. Thus, an asymptotically distribution-free confidence interval around S is given by

$$\{\hat{S} \pm Z_\alpha \sqrt{(\hat{\nu}/n)}\}, \qquad (6)$$

where Z_α is the upper α percentage point of the standard normal distribution.

These results can be readily extended to the KL index.

Proposition 4. Let $\mathbf{b}(\theta) = (b_1(\theta), ..., b_K(\theta))'$. Then $\sqrt{n}(\hat{S}(\theta) - S(\theta))$ has a limiting normal distribution, $N(0, \nu(\theta))$, where $\nu(\theta) = \mathbf{b}(\theta)'\boldsymbol{\Omega}\mathbf{b}(\theta)$.

Since $\hat{S}(\theta)$ is a linear function of the sample conditional means, this also follows directly from Proposition 1. Proposition 3 is, of course, the special case where $\theta = 1$. For any given θ, asymptotically distribution free confidence intervals can be constructed as in (6)

$$\{\hat{S}(\theta) \pm Z_\alpha \sqrt{(\hat{\nu}(\theta)/n)}\}.$$

If two independent random samples of n_1 and n_2 observations are available, an asymptotically distribution-free test of the null hypothesis $H_0: S_1(\theta) = S_2(\theta)$, for given θ, can be based on the statistic

$$Z(\theta) = (\hat{S}_1(\theta) - \hat{S}_2(\theta))/(\hat{\nu}_1(\theta)/n_1 + \hat{\nu}_2(\theta)/n_2)^{1/2}. \qquad (7)$$

9. See Beach and Davidson (1983) and Gastwirth and Gail (1985) for alternative proofs of Proposition 2 under simple random sampling, and Beach and Kaliski (1986) for a proof under weighted sampling.

Then, for example, if the alternative hypothesis is one sided, $H_A: S_1(\theta) > S_2(\theta)$, an approximately size α test is to reject H_0 in favor of H_A if $Z(\theta) > Z_\alpha$, where Z_α is the upper α quantile of the standard normal distribution.

Proposition 4 is based on the assumption that the value of θ is given. However, the choice of a value for θ embodies a specific tradeoff between mean income and the Gini coefficient. The choice of θ will rarely be uncontroversial. This suggests estimating $S(\theta)$ for a set of values, say, $\Theta = \{\theta_1, ..., \theta_r\}$, $r \geq 2$.

The estimates $\hat{S}(\theta_i)$ for different values of θ_i will be correlated. Let $\mathbf{S} = (S(\theta_1), ..., S(\theta_r))'$ be the vector of values of the KL indices for the $\theta_i \in \Theta$. Define $\mathbf{B}$ as the rxK matrix having $\mathbf{b}(\theta_i)$ as the i^{th} row, so that $\mathbf{S} = \mathbf{Bm}$. Then we have the following result.

Proposition 5. $\sqrt{n}(\hat{\mathbf{S}} - \mathbf{S})$ has a limiting normal distribution $N(0, \mathbf{V})$, where $\mathbf{V} = \mathbf{B\Omega B}'$.

Again, this follows from the linearity of S in the conditional means. Then the covariance between $\hat{S}(\theta_i)$ and $\hat{S}(\theta_j)$ is $\nu_{ij} = \mathbf{b}(\theta_i)'\mathbf{\Omega b}(\theta_j)$. For any θ_i, $\theta_j \in \Theta$ and $\theta \in [0, 1]$ there is a ζ such that

$$\mathbf{b}(\theta) = \zeta\mathbf{b}(\theta_i) + (1 - \zeta)\mathbf{b}(\theta_j). \tag{8}$$

This implies that V is singular, with rank 2. This is a essentially a consequence of the fact (recall (3)) that, for any θ, $S(\theta)$ can be written as a convex combination of the mean and the SSY index.

Since the estimators $\hat{S}(\theta_i)$ are linear functions of asymptotically jointly normal variates, the technique of Richmond (1982) can be applied to construct simultaneous confidence intervals. Applying Richmond's results and Proposition 5, it follows that

$$\Pr\{\sqrt{n}\,|\hat{S}(\theta_i) - S(\theta_i)|\,/\sqrt{\hat{\nu}(\theta)} \leq M_\alpha(r, \infty), i = 1, ..., r\} \geq 1 - \alpha, \tag{9}$$

where $M_\alpha(r, \infty)$ is the upper α point of the Studentized Maximum Modulus (SMM) distribution with r and ∞ "degrees of freedom." In fact, (8) implies that (9) can be strengthened to

$$\Pr\{\sqrt{n}\,|\hat{S}(\theta) - S(\theta)|\,/\sqrt{\hat{\nu}(\theta)} \leq M_\alpha(r, \infty), \forall\ \theta \in [0, 1]\} \geq 1 - \alpha.$$

The $100(1 - \alpha)$ percent simultaneous confidence intervals are given by

$$\{\hat{S}(\theta) \pm M_{\alpha}(r, \infty)\surd(\hat{\nu}(\theta)/n)\}. \quad (10)$$

Thus, Richmond's procedure allows the construction of simultaneous confidence intervals with confidence level of at least $1 - \alpha$ for $S(\theta)$, $\forall\ \theta \in [0, 1]$, while minimizing the length of the confidence intervals for the preselected values $S(\theta_i)$, $\theta_i \in \Theta$, of primary interest. Stoline and Ury (1979) show that $M_{\alpha}(r, \infty) = Z_{\delta/2}$, where $\delta = 1 - (1 - \alpha)^{1/r}$, so the SMM critical points can be calculated from the standard normal distribution.

Now consider the set of r hypotheses $H_0{:}S_1(\theta_i) = S_2(\theta_i)$, $\theta_i \in \Theta$. These hypotheses can be tested simultaneously using the statistics $Z(\theta_i)$ in (7). However, since there are r hypotheses, these statistics should be tested as SMM(r, ∞) variates. If, for example, the alternative hypotheses are all one sided, an approximately size α test of the i^{th} null hypothesis is to reject H_0 in favor of H_A if $Z(\theta_i) > M_{\alpha}(r, \infty)$. This test is asymptotically distribution-free.

IV. International Comparisons

In this section we compare the KL indices across nine countries. The KL indices are estimated at several parameter values, allowing the full range of variation in the tradeoff between mean income and the Gini.

The empirical analysis employs data from the Luxembourg Income Study (LIS). The LIS data consists of national survey data from ten countries during the period 1979 to 1983. Table 1 describes the original surveys on which the LIS is based and gives the sample sizes for the countries included in our empirical analysis.[10] The national survey data are corrected for differences in the definitions of income and income recipient units, and, except for the UK, are weighted to more accurately represent the populations. More detailed descriptions of the data are provided by O'Higgins, et. al. (1987), and Buhmann, et. al. (1988).

The income measure used is net cash income, defined as market income plus public and private transfer payments, less direct (income and payroll) taxes. Indirect taxes, in-kind transfers, and government provided goods and services are not included. Net cash income is the most

10. We exclude Israel from our analysis because of non-comparability, as the Israeli data does not cover the rural and the non-voting populations.

comprehensive income measure available in the LIS data. The data are analyzed on both family and percapita bases. Since the results are virtually identical for both income recipient units, only the results for percapita income are reported.

Incomes are rescaled so that the ratio of percapita net cash income to national GDP percapita measured in 1979 US dollars equals the US ratio of percapita net cash income to GDP percapita.[11] This accomplishes two things. First, the SSY and KL indices are denominanted in the same units as income, so this procedure converts all incomes to 1979 US dollars as the common unit of measurement. Second, the countries examined rely to different degrees on indirect taxes, and on in-kind transfers and government provision of goods and services. These forms of income are not included in net cash income, which tends to bias the results against countries that rely more heavily on direct taxes, and in favor of countries that rely more heavily on cash transfers. Since these items are included in GDP, rescaling incomes partially corrects for these omissions.[12] Table 2 reports GDP percapita in 1979 US dollars, mean net cash income (in the national currency), the conversion factors, and percapita income in 1979 US dollars.

Table 3 reports estimates and comparisons of the Gini coefficients of percapita income for all nine countries. Panel A of Table 3 gives the estimates of the Gini's (based on deciles) and the upper and lower ninety-five percent confidence bounds.[13] West Germany and the Netherlands have relatively small samples, and consequently the confidence intervals are quite wide. Even excluding West Germany and the Netherlands, the width of the confidence intervals ranges from two to eight percent of the Gini, which is not negligible.

Panel B of Table 3 reports the comparisons of the Gini's across the nine countries. A "+" (-) indicates that the country in the row (column) of Table 3 has a larger values of the Gini than the country listed in the column (row), and this difference is statistically significantly at the five percent level. A "0" indicates the values of the Gini's are not statistically significantly different. Sweden has

11. The GDP percapita data are from Summers and Heston (1988).
12. This procedure implicitly assumes excluded income items are distributed proportionally to net cash income, so that the distributional effects of these items is not captured..
13. The sample correlation between mean income ($1979) and Gini coefficient is 0.3740, and the sample rank correlation is 0.5294, neither of which is significantly different from zero.

a Gini that is significantly smaller than that for any other country. Norway and the UK have smaller Ginis than any other country except Sweden; the Ginis for Norway and the UK are not significantly different. The US has a Gini that is at least as large as that of any other country. Excluding West Germany, there is no significant difference between the Ginis in nearly one-third (nine of twenty-eight) of the comparisons

Estimates of the KL indices for five selected values of the parameter θ are reported in Table 4 . The values of θ employed are 0.00 (mean income), 0.25, 0.50, 0.75, and 1.00 (the SSY index). Table 4 also reports the upper and lower 95 percent simultaneous confidence bounds, calculated from (10).[14] The width of the confidence intervals is not negligible, but ranges from two percent to over six percent of the value of the index.

Table 5 contains the results of the international comparison of the KL indices for all five parameter values. It is apparent from the results in Table 5 that taking account of sampling variability can affect the conclusions drawn from the comparisons.[15] A clear example of the effect of sampling variability are the comparisons of Canada with the US. Comparing the point estimates in Table 4, the US has larger values of the KL index for $\theta = 0.00$, 0.25 and 0.50, while Canada has larger index values for $\theta = 0.75$ and 1.00. Once the sampling variability of these estimates is taken into account, then, for all values of θ, the KL indices are not significantly different for Canada and the US. Similarly, the KL indices for West Germany are not significantly different from those the Nehterlands, Sweden or Switzerland for all values of θ. Overall, there are 41 cases (22.8 percent) where the KL indices for two countries are not significantly different.

It is also apparent from Table 5 that, for some countries, the conclusions that can be drawn from the comparisons depend on the value of θ, that is, on the weight given to inequality or to relative deprivation. An example of this is the comparison of Norway and the US. The US has a higher percapita income and a higher Gini than Norway. The US has a significantly larger value of the KL index for $\theta = 0.00$, 0.25 and 0.50. The KL indices are not significantly different for $\theta = 0.75$,

14. The five percent critical value of the SMM(5, ∞) distribution is approximately 2.57. Since the sampling unit in the surveys is the family, we take the number of families as the sample size.
15. For additional evidence on this point, see Bishop, Formby and Thistle (1989), and Bishop, Formby and Smith (1989).

and Norway has a larger SSY index (θ = 1.00). There are thirty-six possible pairwise comparisons of the nine countries. In ten of the thirty-six comparisons, or 27.8 percent, the result of the comparison depends on the weight given to inequality or relative deprivation.[16]

Canada, Norway and the US fare relatively well in these comparisons. No country ever has a significantly larger KL index than Canada. That is, Canada has a standard of living that is at least as high or higher than that of the other eight countries. Except for Norway's SSY index, no country ever has a significantly larger KL index than the US. Similarly, with the exception of Canada and the US, which have higher KL indices at low values of θ (0.00 and 0.25), Norway has values of the KL index as large as or larger than those of any other country.

The Netherlands, Sweden, Switzerland and West Germany never have values of the KL index that are greater than those for Canada, Norway or the US, or smaller than those for Australia and the UK. The Netherlands never has a larger value of the KL index than Switzerland or West Germany, although for some values of θ the indices are not significantly different. Sweden has smaller values of the index than Switzerland for low values of θ (0.00, 0.25, and 0.50), but for θ = 0.75 and 1.00 the KL indices for Sweden and Switzerland are not significantly different.

Australia and the UK fare relatively poorly in the comparisons. Australia always has as small a value or a smaller value of the KL index as any other country. The UK has larger values of the KL index than Australia for θ = 0.75 and 1.00. Otherwise, every other country always has as large a value or a larger value of the KL index than the UK. The implication is that, according to these measures, Australia and the UK have the lowest standards of living.

V. Conclusion

We compare the Sheshinski-Sen-Yitzhaki and Kakwani-Lambert indices for nine countries. The SSY and KL indices are based on the Gini coefficient of income inequality, and allow the tradeoff between mean income and the Gini to vary. These indices can be interpreted as measures

16. The comparisons are: AUS v. UK and WG, CAN v. NOR and WG, NLD v. SWE and WG, NOR v. US and WG, SWE v. SWI, UK v. WG, and US v. WG.

of the standard of living in the sense of the average level of relative satisfaction; the SSY index can also be interpreted as a measure of percapita welfare.

The SSY and KL indices are estimated using data from the Luxembourg Income Study. The LIS data set contains national survey data, adjusted for differences in the definitions of income and income recipients. The data are weighted samples. The use of sample data implies the sample SSY and KL indices are subject to sampling variability. We derive the asymptotic distributions of estimators of the SSY and KL indices under weighted sampling.

We employ asymptotically distribution-free statistical inference procedures for the SSY and KL indices. While the SSY and KL indices have been used in other empirical studies, this is the first study using these indices to apply statistical inference procedures. The use of statistical inference has an important effect on the conlcusions. For example, we find no significant differences in the SSY and KL indices for the US and Canada.

Empirically, the nine countries can be divided into three groups. Canada, Norway and the US have the highest standards of living as measured by the SSY and KL indices. The second group contains the Netherlands, Sweden, Switzerland, and West Germany. The third group consists of Australia and the UK, which have the lowest standard of living according to the SSY and KL indices. Within each group, comparisons depend on the tradeoff between mean income and the Gini coefficient.

Table 1
Summary of LIS Data

Country	Data Source	Sample Size*
Australia	Income and Housing Survey, 1981-82	15,985
Canada	Survey of Consumer Finances, 1981	15,136
Netherlands	Survey of Income of Program, 1983	4,833
Norway	Norwegian Tax Files, 1979	10,414
Sweden	Swedish Income Distrib. Survey, 1981	9,625
Switzerland	Income and Wealth Survey, 1982	7,036
UK	Family Expenditure Survey, 1979	6,888
US	Current Population Survey, 1979	15,225
WG	Transfer Survey, 1981	2,727

* Number of families

Table 2

Country	GDP $1979	LIS Mean	Conversion Factor	Mean $1979
AUS	8152	5542	0.70	3879
CAN	11358	8099	0.67	5428
NLD	8964	11342	0.38	4309
NOR	10708	27143	0.19	5156
SWE	8750	34268	0.12	4112
SWI	9628	19405	0.24	4656
UK	8094	1990	1.94	3860
US	11602	5543	1.00	5543
WG	9619	14549	0.32	4654

Table 3
Gini Coefficients for Nine Countries:
Estimates, Confidence Intervals and Comparisons

A. Estimates

	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
Gini	.3156	.3190	.3311	.2689	.2289	.3346	.2814	.3461	.3335
Upper bound	.3229	.3226	.3444	.2965	.2360	.3483	.2934	.3534	.4417
Lower bound	.3083	.3114	.3178	.2413	.2218	.3209	.2694	.3388	.2273

B. Comparisons

	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
AUS	*	0	0	+	+	0	+	-	0
CAN		*	0	+	+	0	+	-	0
NLD			*	+	+	0	+	-	0
NOR				*	+	-	0	-	0
SWE					*	-	-	-	0
SWI						*	+	0	0
UK							*	-	0
US								*	0

All tests at five percent significance level:
+ indicates row country has larger Gini
- indicates column country has larger Gini
0 indicates no significant difference in Ginis

Table 4
Kakwani-Lambert Indices for Nine Countries: Estimates and Confidence Intervals for Selected Parameter Values

$\theta = 0.0$	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
KL Index	3879	5428	4309	5156	4112	4656	3860	5543	4654
Upper bound	3928	5501	4415	5249	4166	4803	3922	5621	5815
Lower bound	3830	5354	4202	5063	4057	4510	3762	5462	3492

$\theta = 0.25$	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
KL Index	3573	4994	3952	4809	3876	4267	3588	5062	4266
Upper bound	3612	5053	4036	4884	3922	4383	3642	5128	5166
Lower bound	3534	4934	3867	4735	3830	4151	3535	4998	3365

$\theta = 0.50$	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
KL Index	3267	4560	3595	4463	3641	3877	3317	4583	3878
Upper bound	3300	4608	3665	4525	3688	3973	3361	4637	4517
Lower bound	3234	4511	3525	4400	3594	3781	3273	4528	3238

$\theta = 0.75$	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
KL Index	2961	4126	3238	4116	3406	3488	3045	4103	3490
Upper bound	2993	4172	3306	4177	3464	3579	3087	4159	3872
Lower bound	2929	4079	3171	4055	3347	3397	3003	4047	3107

$\theta = 1.00$	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
KL Index	2655	3692	2882	3769	3170	3098	2774	3624	3102
Upper bound	2692	3745	2959	3840	3244	3203	2821	3690	3246
Lower bound	2618	3638	2804	3699	3097	2993	2726	3557	2957

Table 5
Comparisons of Kakwani-Lambert Indices
at Selected Parameter Values for Nine Countries

$\theta = 0.00$	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
AUS	*	-	-	-	-	-	0	-	0
CAN		*	+	+	+	+	+	0	0
NLD			*	-	+	-	+	-	0
NOR				*	+	+	+	-	0
SWE					*	-	+	-	0
SWI						*	+	-	0
UK							*	-	0
US								*	0

$\theta = 0.25$	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
AUS	*	-	-	-	-	-	0	-	0
CAN		*	+	+	+	+	+	0	0
NLD			*	-	0	-	+	-	0
NOR				*	+	+	+	-	0
SWE					*	-	+	-	0
SWI						*	+	-	0
UK							*	-	0
US								*	0

$\theta = 0.50$	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
AUS	*	-	-	-	-	-	-	-	-
CAN		*	+	0	+	+	+	0	+
NLD			*	-	-	-	+	-	0
NOR				*	+	+	+	0	+
SWE					*	0	+	-	0
SWI						*	+	-	0
UK							*	-	-
US								*	+

$\theta = 0.75$	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
AUS	*	-	-	-	-	-	-	-	-
CAN		*	+	0	+	+	+	0	+
NLD			*	-	-	-	+	-	0
NOR				*	+	+	+	0	+
SWE					*	0	+	-	0
SWI						*	+	-	0
UK							*	-	-
US								*	+

Table 5
(Continued)
Comparisons of Kakwani-Lambert Indices
at Selected Parameter Values for Nine Countries

θ = 1.00	AUS	CAN	NLD	NOR	SWE	SWI	UK	US	WG
AUS	*	-	-	-	-	-	-	-	-
CAN		*	+	0	+	+	+	0	+
NLD			*	-	-	-	+	-	0
NOR				*	+	+	+	+	+
SWE					*	0	+	-	0
SWI						*	+	-	0
UK							*	-	-
US								*	+

All tests at five percent significance level:
+ indicates row country has larger index value
- indicates column country has larger index value
0 indicates no significant difference in index values.

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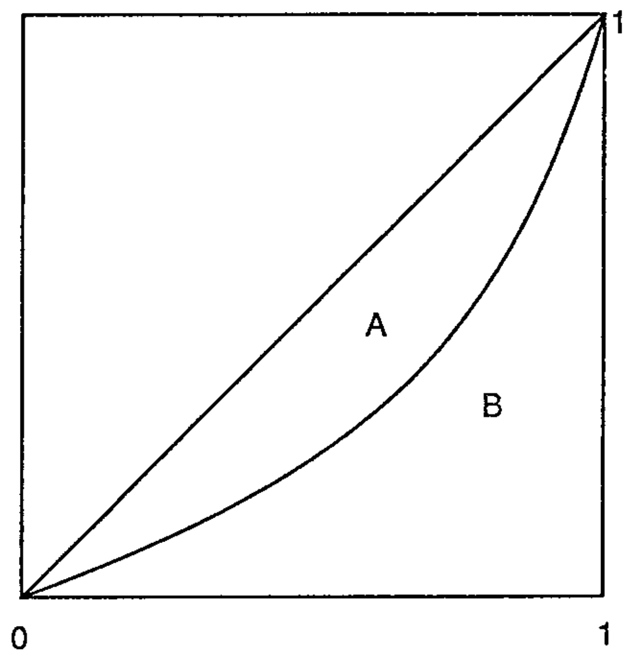

Fig. 1. Lorenz Curve

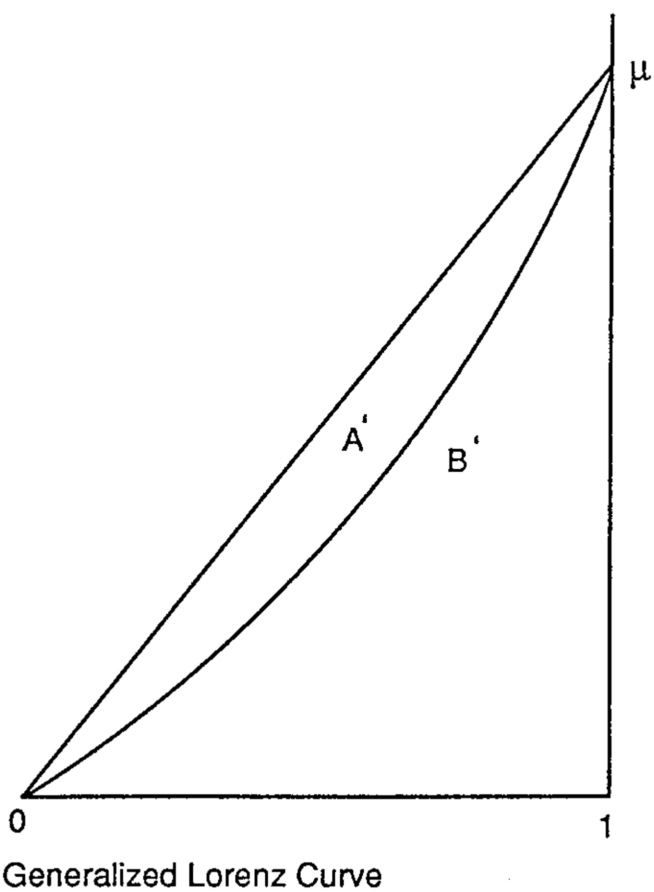

Fig. 2. Generalized Lorenz Curve